Matter-antimatter asymmetry?

Jean-Pierre Gazeau, Hamed Pejhan

Matter-antimatter asymmetry?

Jean-Pierre Gazeau[1*] and Hamed Pejhan[2†]
[1] *Université de Paris, CNRS, Astroparticule et Cosmologie, F-75013 Paris, France and*
[2] *Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Acad. G. Bonchev Str. Bl. 8, 1113, Sofia, Bulgaria*

(Dated: February 6, 2024)

The widely held expectation of equal matter and antimatter production during the Big Bang sharply contrasts with the observable Universe's current predominant composition of matter. This persistent asymmetry poses an intriguing puzzle and remains a paramount challenge in modern physics. In this study, we delve into the enigma of antimatter, focusing on the matter-antimatter asymmetry within the context of de Sitter spacetime as the classical background. De Sitter spacetime, representing a scenario where the cosmological constant governs, leading to accelerated expansion, emerges as a pivotal model essential for comprehending the dynamics of the present Universe. Our analysis, rooted in a fundamental requirement of the quantum field theory description of de Sitterian elementary systems — specifically, the analyticity requirement in the complexified de Sitter manifold, offers a novel perspective. It suggests that the matter-antimatter asymmetry is an observational anomaly discernible exclusively by local observers with causal access to a specific segment of de Sitter spacetime. In contrast to local observations, the global structure of this spacetime maintains a perfect matter-antimatter symmetry. This unconventional insight challenges established perspectives, ushering in a paradigm shift in our understanding of the observed asymmetry in the distribution of matter and antimatter throughout the Universe.

In this letter, we systematically present our rationale in four distinct steps, maintaining a technical approach by consistently employing natural units, where the fundamental constants $c, \hbar$ are set equal to 1.

I. The d-dimensional de Sitter (dS_d) spacetime

Recent observations, as reported, for instance, in Refs. [1–4], indicate an accelerating cosmic expansion, hinting at the presence of a small yet nonvanishing positive cosmological constant. This implies that our Universe could presently be entering a phase reminiscent of $\mathrm{dS}_{d=4}$, approaching a pure $\mathrm{dS}_{d=4}$ spacetime.

The dS_d spacetime represents the maximally symmetric solution to the vacuum Einstein's equations with a positive cosmological constant. Topologically, it is defined by $\mathbb{R}^1 \times \mathbb{S}^{d-1}$. Geometrically, the most intuitive depiction of dS_d spacetime is as a one-sheeted hyperboloid embedded within a $(1+d)$-dimensional Minkowski spacetime $\mathbb{R}^{1+d}$:

$$\mathrm{dS}_d = \left\{ x \in \mathbb{R}^{1+d} \; ; \; (x)^2 = x \cdot x = \eta_{\alpha\beta} x^\alpha x^\beta = -H^{-2} \right\}, \tag{1}$$

where $\alpha, \beta = 0, 1, \ldots, d$, $\eta_{\alpha\beta} = \mathrm{diag}(1, -1, \ldots, -1)$, and H stands for the Hubble constant.

The dS_d (relativity) group is $\mathrm{SO}_0(1, d)$, indicating the connected subgroup of $\mathrm{O}(1, d)$, or any of its covering groups. A recognizable manifestation of the corresponding Lie algebra is attained through the linear span of the Killing vectors:

$$K_{\alpha\beta} = x_\alpha \partial_\beta - x_\beta \partial_\alpha \, . \tag{2}$$

II. The necessity of analyticity within the complexified dS_d manifold ($\mathrm{dS}_d^{(\mathbb{C})}$)

The entire argument we present in this letter directly emerges from the imperative need for analyticity requirement within the complexified dS_d manifold ($\mathrm{dS}_d^{(\mathbb{C})}$), particularly in the context of describing elementary systems in dS_d through quantum field theory (QFT):

$$\mathrm{dS}_d^{(\mathbb{C})} = \Big\{ z = x + \mathrm{i}y \in \mathbb{C}^{1+d} \; ; \; (z)^2 = \eta_{\alpha\beta} z^\alpha z^\beta = -H^{-2}$$
$$\text{or equivalently} \; ; \; (x)^2 - (y)^2 = -H^{-2}, \; x \cdot y = 0 \Big\} . \tag{3}$$

A foundational argument strongly supports this necessity. Expanding beyond the familiar flat Minkowski spacetime to dS_d introduces a fundamental challenge to the QFT description of elementary systems. The challenge lies in the absence of a well-defined spectral condition [5]. Regardless of the chosen QFT approach to field quantization on dS_d spacetime, formulating requirements for locality (microcausality) and covariance is a relatively straightforward task. However, establishing any condition on the spectrum of the "energy" operator proves to be a formidable challenge. Strictly speaking, in dS_d spacetime, granted that no globally timelike Killing vector exists, neither time nor energy can be globally defined. This inherent ambiguity results in the emergence of numerous inequivalent QFTs (vacuum states) for a single field model on dS_d spacetime. Each of these theories is typically linked to a specific choice of the time coordinate, leading to associated frequency splitting.

*gazeau@apc.in2p3.fr
†pejhan@math.bas.bg

Interestingly, within the Minkowskian framework, a notable observation highlights a direct link between the characteristics of analytic continuation in complexified spacetime and the spectral property of the model under consideration. Specifically, in the context of $(1+d)$-dimensional Minkowski spacetime $\mathbb{R}^{1+d}$, the spectral property can be expressed as follows: for each $n>1$, the Wightman n-point function corresponds to the boundary value, in the distribution sense, of a function that is holomorphic within the tube $T^{+(n)}$ defined as:

$$T^{+(n)} = \Big\{(z_1,\dots,z_n)\in\mathbb{C}^{n(1+d)}\ ;\ \mathrm{Im}(z_{j+1}-z_j)\in V^+\Big\}, \tag{4}$$

where $1\leqslant j\leqslant n-1$ and $V^\pm=\big\{y\in\mathbb{R}^{1+d}\ ;\ (y)^2>0,\ y^0\gtrless 0\big\}$.

In the context of dS_d spacetime (embedded in $\mathbb{R}^{1+d}$), a natural alternative to this property is to propose that the Wightman n-point function $\mathcal{W}_n(x_1,\dots,x_n)$ acts as the boundary value, in the distributional sense, for a function that is holomorphic within:

$$\mathcal{T}^{+(n)} = T^{+(n)}\cap\big[\mathrm{dS}_d^{(\mathbb{C})}\big]^n. \tag{5}$$

As a matter of fact, studies by Bros et al. [6, 7] have shown that $\mathcal{T}^{+(n)}$ forms a domain and a tuboid — bounded by reals — in a manner that maintains the relevance of the concept of the "distribution boundary value of a holomorphic function from this domain". Consequently, it becomes feasible to impose:

- *Weak spectral condition.* For each $n>1$, the Wightman n-point function $\mathcal{W}_n(x_1,\dots,x_n)$ is derived from the distributional boundary value of a function $W_n(z_1,\dots,z_n)$, which is holomorphic within the domain $\mathcal{T}^{+(n)}$.

In the simplest scenario, considering a general dS_d two-point function, the assertion of normal analyticity can be explicitly expressed as follows:

- The two-point function $\mathcal{W}_2(x_1,x_2)$ arises as the distributional boundary value of a function $W_2(z_1,z_2)$ that is analytically defined within the tuboid domain $\mathcal{T}^{+(2)}$ given by:

$$\mathcal{T}^{+(2)} = \Big\{(z_1,z_2)\ ;\ z_1\in\mathcal{T}^-,\ z_2\in\mathcal{T}^+\Big\}, \tag{6}$$

  where $\mathcal{T}^\pm=\big\{\mathbb{R}^{d+1}+\mathrm{i}V^\pm\big\}\cap\mathrm{dS}_d^{(\mathbb{C})}$. Note that $\mathcal{T}^\pm$ are referred to as the forward and backward tubes of $\mathrm{dS}_d^{(\mathbb{C})}$, respectively.

This elegant resolution effectively circumvents all ambiguities in dS_d QFTs, which arise from the absence of a genuine spectral condition in dS_d spacetime [6, 7]. It leads to the derivation of "vacua" that, despite exhibiting thermal properties, serve as precise analogs of Minkowski vacuum representations. The latter emerges as the limit of the former when the curvature tends to zero.

For additional arguments supporting the necessity of analyticity in the complexified dS_d manifold, readers are referred to Refs. [8, 9] and the references therein.

III. The dS_d static patch and the analyticity requirement

Following the approach outlined in Ref. [10], let us adopt the viewpoint of an observer (on dS_d) in motion along the geodesic $g(x_\bullet)$ that traverses through the point $x_\bullet=(0,\dots,0,x^d=H^{-1})$:[1]

$$g(x_\bullet) = \Big\{x=x(t)\ ;\ x^0=H^{-1}\sinh Ht,\quad \boldsymbol{x}=(x^1,\dots,x^{d-1})=0,\quad x^d=H^{-1}\cosh Ht,\quad t\in\mathbb{R}\Big\}. \tag{7}$$

Note that the chosen point $x_\bullet$ is located in the (x^0,x^d)-plane.

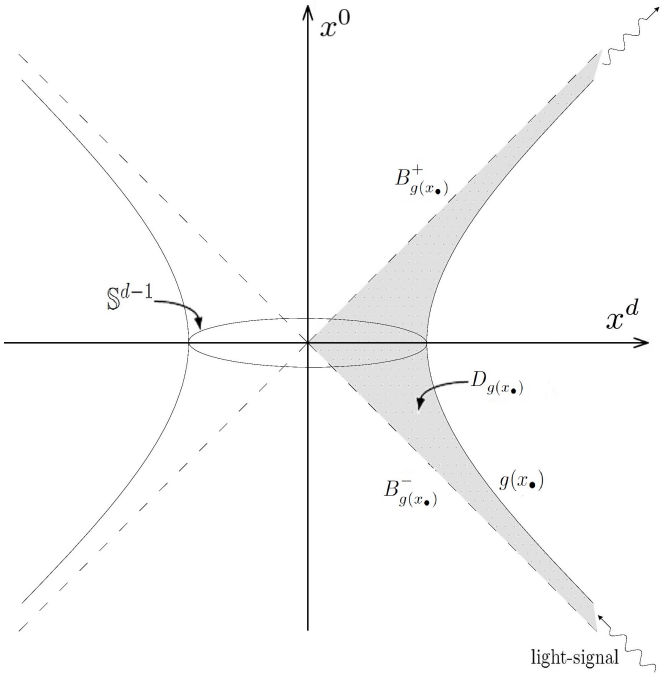

FIG. 1: The static patch of dS_d, linked with an observer traveling along the geodesic $g(x_\bullet)$.

The set of all "events" in dS_d that can be linked to the observer via the reception and emission of light signals is given by:

$$D_{g(x_\bullet)} = \Big\{x\in\mathrm{dS}_d\ ;\ x^d>|x^0|\Big\}. \tag{8}$$

This domain is delineated by two distinct boundaries:

$$B^\pm_{g(x_\bullet)} = \Big\{x\in\mathrm{dS}_d\ ;\ x^0=\pm x^d,\ x^d>0\Big\}. \tag{9}$$

[1] It is worth noting that the choice of this point is entirely arbitrary, given the $\mathrm{SO}_0(1,d)$ symmetry of dS_d.

These boundaries are respectively termed the "future horizon" and "past horizon" of the observer following the geodesic $g(x_\bullet)$.

The parameter t in expression (7) signifies the proper time experienced by the observer moving along the geodesic $g(x_\bullet)$. This enables us to designate the "time-translation group linked with $g(x_\bullet)$" as the one-parameter subgroup $\mathrm{T}_{g(x_\bullet)}$ (isomorphic to $\mathrm{SO}_0(1,1)$) of the dS_d group $\mathrm{SO}_0(1,d)$. The $\mathrm{T}_{g(x_\bullet)}$'s transformations involve hyperbolic rotations parallel to the (x^0,x^d)-plane. To characterize how these transformations operate on the domain $D_{g(x_\bullet)}$, let $x = x(t,\widetilde{\boldsymbol{x}})$ represent an arbitrary point within this domain:

$$x(t,\widetilde{\boldsymbol{x}}) = \begin{cases} x^0 &= \sqrt{H^{-2}-(\widetilde{\boldsymbol{x}})^2}\,\sinh Ht\,, \\ \widetilde{\boldsymbol{x}} &= (x^1,\dots,x^{d-1})\,, \\ x^d &= \sqrt{H^{-2}-(\widetilde{\boldsymbol{x}})^2}\,\cosh Ht\,, \quad t\in\mathbb{R}\,, \end{cases} \tag{10}$$

while $(\widetilde{\boldsymbol{x}})^2 = (x^1)^2+\ldots+(x^{d-1})^2 < H^{-2}$. The operation of $\mathrm{T}_{g(x_\bullet)}$ on $x(t,\widetilde{\boldsymbol{x}})$ is expressed as follows:

$$\mathrm{T}_{g(x_\bullet)}(\tau)\diamond x(t,\widetilde{\boldsymbol{x}}) = x(t+\tau,\widetilde{\boldsymbol{x}}) = x^\tau\,, \quad \tau\in\mathbb{R}\,. \tag{11}$$

It defines a group of isometric automorphisms within the domain $D_{g(x_\bullet)}$. The associated orbits $g_{\widetilde{\boldsymbol{x}}}(x_\bullet)$ depict separate hyperbolic branches in $D_{g(x_\bullet)}$. These hyperbolas lie in two-dimensional plane sections that are parallel to the (x^0,x^d)-plane. Within these orbits, the singular one that defines a geodesic of dS_d is $g(x_\bullet) = g_{\widetilde{\boldsymbol{x}}=\boldsymbol{0}}(x_\bullet)$. Hence, understanding the group $\mathrm{T}_{g(x_\bullet)}$ as time translation is particularly applicable to observers in motion on or near $g(x_\bullet)$, with the assumption that the proximity is significantly smaller than the dS_d radius of curvature.

Now, let us evaluate the depicted scenario by acknowledging the necessity of analyticity within the complexified dS_d manifold ($\mathrm{dS}_d^{(\mathbb{C})}$). The complex orbits of $\mathrm{T}_{g(x_\bullet)}$ are denoted as:

$$g^{(\mathbb{C})}_{\widetilde{\boldsymbol{x}}}(x_\bullet) = \Big\{ z^\tau = z(t+\tau,\widetilde{\boldsymbol{x}}),\ \tau\in\mathbb{C} \Big\}\,. \tag{12}$$

All nonreal points linked to the complex orbits $g^{(\mathbb{C})}_{\widetilde{\boldsymbol{x}}}(x_\bullet)$ reside in $\mathcal{T}^\pm$, the very domains of analyticity required for the QFT description of dS_d elementary systems. In this context, due to the analytic nature of dS_d QFTs, a notable connection emerges between the domain $D_{g(x_\bullet)}$ and its corresponding mirror region:

$$D_{g(-x_\bullet)} = \Big\{ x=(x^0,\widetilde{\boldsymbol{x}},x^d)\in \mathrm{dS}_d\,; (-x^0,\widetilde{\boldsymbol{x}},-x^d)\in D_{g(x_\bullet)} \Big\}\,, \tag{13}$$

To see the point, with the coordinate system (10) in consideration, setting $\mathrm{Im}(\tau)=\pi$ in (12) suffices. This connection inherently establishes a significant and highly nontrivial link between the QFT description within $D_{g(x_\bullet)}$ and its mirrored counterpart in the region $D_{g(-x_\bullet)}$. [For a comprehensive exploration of the two-point correlation functions in the domain $D_{g(x_\bullet)}$ and their mirrored counterparts in the region $D_{g(-x_\bullet)}$, refer to Ref. [7].] This pivotal link forms the cornerstone of our reasoning, as elaborated further in the subsequent step.

IV. Matter-antimatter asymmetry?

So far, in the initial two steps, we have articulated the foundational assumptions guiding our reasoning. Firstly, we assumed that our Universe presently reflects a phase akin to $\mathrm{dS}_{d=4}$, converging toward a pure $\mathrm{dS}_{d=4}$ spacetime — a proposition bolstered by observational data [1–4]. Secondly, we emphasized the imperative of analyticity within the complexified dS_d manifold ($\mathrm{dS}_d^{(\mathbb{C})}$), underscored by the inherent requirement for a natural substitution of the spectral condition. This necessity, in turn, facilitates a coherent QFT depiction of elementary systems within the dS_d framework [5–9]. Based on these foundational assumptions in the third step, we demonstrated that the physics in a region observable to a local observer on the dS_d manifold is intricately connected to the physics residing in the corresponding mirror region.

Referring to the mathematical content presented in the preceding step, it is essential to highlight the fundamental concept that the intrinsic time variable pertinent to an observer moving along the geodesic $g(-x_\bullet)$ (within the mirror region $D_{g(-x_\bullet)}$) is equivalent to $-t$. This arises from the fact that the associated time-translation group $\mathrm{T}_{g(-x_\bullet)}$ validates the relationship $\mathrm{T}_{g(-x_\bullet)}(\tau) = \mathrm{T}_{g(x_\bullet)}(-\tau)$; it is noteworthy that $\mathrm{T}_{g(-x_\bullet)}$ is derived from $\mathrm{T}_{g(x_\bullet)}$, for instance, through conjugation in the form $\mathrm{T}_{g(-x_\bullet)} = \mathtt{R}\,\mathrm{T}_{g(x_\bullet)}\,\mathtt{R}^{-1}$, where $\mathtt{R}$ represents a rotation of angle π in a plane orthogonal to the x^0-axis. [This characteristic serves as yet another indication of the fact that dS_d spacetime lacks a globally timelike Killing vector.] This feature remarkably implies that any matter/antimatter elementary system observed by a local observer on the dS_d manifold has an anti-counterpart in the corresponding mirror region; recall that the Feynman–Stueckelberg interpretation states that antimatter and antiparticles behave exactly like regular particles but travel backward in time.

Considering our basic assumptions, this intriguing property suggests that the matter-antimatter asymmetry is an observational anomaly discernible exclusively by local observers with causal access to a specific segment of the Universe. In contrast to local observations, the global structure of the Universe maintains a perfect matter-antimatter symmetry; our observable Universe has a mirror, nonobservable anti-Universe!

Final remarks: *First.* The above result serves as yet another illustration of how even a small, nonvanish-

ing cosmological constant can have highly nontrivial implications for our comprehension of elementary systems. It is crucial to highlight that this observation becomes entirely irrelevant when considering elementary systems within the context of flat Minkowski spacetime, where the cosmological constant vanishes.

Second. In the domain of inflationary models [11, 12], where the effective cosmological constant surpasses the currently observed value by several orders of magnitude, particles, including those responsible for mass, were effectively massless. Consequently, it is reasonable to expect that during this inflationary period, the observation would not align with the notion of "our observable Universe having a mirror, nonobservable anti-Universe".

Third. The (a)symmetry we demonstrate is based on analyticity extensions valid for massive systems (in the de Sitterian sense). It is crucial to emphasize, however, that the quantization approach, rooted in the criterion of analyticity, encounters limitations when applied to specific fields within the dS_d framework. Notably, this constraint becomes apparent in the case of the so-called "tachyonic" scalar fields, for instance. As a result, a departure from the established framework becomes essential, prompting the adoption of an alternative approach grounded in a Krein structure (endowed with an indefinite inner product), diverging from the conventional Hilbertian approach. [See Ref. [8] for more details.] The examination of such fields, particularly in the context of matter-antimatter (a)symmetry, is deferred to future work.

Akcnowledgment

Hamed Pejhan is supported by the Bulgarian Ministry of Education and Science, Scientific Programme 'Enhancing the Research Capacity in Mathematical Sciences (PIKOM)', No. DO1-67/05.05.2022.

[1] S. Perlmutter et al., Astrophys. J., 483, 565 (1997).
[2] B. Schmidt et al., Astrophys. J., 507, 46 (1998).
[3] A.J. Riess et al., Astron. J., 116, 1009 (1998).
[4] Richard Panek, *The Most Shocking Discovery in Astrophysics Is 25 Years Old*, Scientific American (2023).
[5] R.F. Streater and A.S. Wightman, *PCT, Spin and Statistics, and All That*, W.A. Benjamin, New York (1964).
[6] J. Bros, J.P. Gazeau, and U. Moschella, Phys. Rev. Lett., 73, 1746 (1994).
[7] J. Bros and U. Moschella, Rev. Math. Phys., 08, 327 (1996).
[8] M. Enayati, J.P. Gazeau, H. Pejhan, and A. Wang, *The de Sitter (dS) Group and its Representations; An Introduction to Elementary Systems and Modeling the Dark Energy Universe*, Springer Nature (2022).
[9] J.P. Gazeau, M.A. del Olmo, and H. Pejhan, Phys. Lett. B 848, 138402 (2024).
[10] G.W. Gibbons and S.W. Hawking, Phys. Rev. D, 15, 2738 (1977).
[11] A. Guth, Phys. Rev. D 23, 347 (1981).
[12] A. Linde, *Particle Physics And Inflationary Cosmology*, Chur: Harwood (1990).