



**HAL**  
open science

# Matter-antimatter (a)symmetry and cosmological constant

Jean-Pierre Gazeau, Hamed Pejhan

► **To cite this version:**

Jean-Pierre Gazeau, Hamed Pejhan. Matter-antimatter (a)symmetry and cosmological constant. 2024. hal-04445771

**HAL Id: hal-04445771**

**<https://hal.science/hal-04445771v1>**

Preprint submitted on 8 Feb 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Matter-antimatter (a)symmetry and cosmological constant

Jean-Pierre Gazeau<sup>1\*</sup> and Hamed Pejhan<sup>2†</sup>

<sup>1</sup>*Université de Paris, CNRS, Astroparticule et Cosmologie, F-75013 Paris, France and*

<sup>2</sup>*Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,  
Acad. G. Bonchev Str. Bl. 8, 1113, Sofia, Bulgaria*

(Dated: August 10, 2024)

We investigate the matter-antimatter asymmetry within the framework of quantum field theory in de Sitter spacetime. While conventional perspectives often attribute this asymmetry to dynamical mechanisms such as CP violation in the early Universe, our analysis proposes a novel kinematical origin. Our findings suggest that this asymmetry may be an observational anomaly, observable only by local observers with causal access to a specific segment of the Universe. This unconventional insight, while grounded in de Sitter cosmology (an idealized scenario for real-world cosmology), has the potential to challenge established perspectives and drive a paradigm shift in our understanding of the observed asymmetry in the distribution of matter and antimatter throughout the Universe.

In this letter, we systematically present our rationale in four distinct steps, maintaining a technical approach by consistently employing natural units, where the fundamental constants  $c, \hbar$  are set equal to 1.

## I. The $d$ -dimensional de Sitter ( $dS_d$ ) spacetime

Recent observations, as reported, for instance, in Refs. [1–4], indicate an accelerating cosmic expansion, hinting at the presence of a small yet nonvanishing positive cosmological constant. This implies that our Universe could presently be entering a phase reminiscent of  $dS_{d=4}$ , approaching a pure  $dS_{d=4}$  spacetime.

Furthermore,  $dS_d$  spacetime and anti- $dS_d$  spacetime form a fundamental class of curved spacetimes on which the quantum field theory (QFT) description of elementary systems (or free fields) can be developed with a robust mathematical foundation. This aligns with Fronsdal’s 1965 notion [5]:

*“A physical theory that treats spacetime as Minkowskian flat must be obtainable as a well-defined limit of a more general physical theory, for which the assumption of flatness is not essential.”*

The  $dS_d$  and anti- $dS_d$  spacetimes are indeed maximally symmetric solutions to the vacuum Einstein equations, corresponding to positive and negative cosmological constants, respectively. In this context, Einstein-Poincaré relativity can be viewed as the idealized null-curvature limit of both  $dS_d$  and anti- $dS_d$  relativities. For further details, see Refs. [6, 7] and the references therein.

Topologically,  $dS_d$  spacetime examined in this study is homeomorphic to  $\mathbb{R}^1 \times \mathbb{S}^{d-1}$ . Geometrically, the most expository depiction of  $dS_d$  spacetime is as a one-sheeted hyperboloid embedded within a  $(1 + d)$ -dimensional Minkowski spacetime  $\mathbb{R}^{1+d}$ :

$$dS_d = \left\{ x \in \mathbb{R}^{1+d} ; (x)^2 = x \cdot x = \eta_{\alpha\beta} x^\alpha x^\beta = -H^{-2} \right\}, \quad (1)$$

where  $\alpha, \beta = 0, 1, \dots, d$ ,  $\eta_{\alpha\beta} = \text{diag}(1, -1, \dots, -1)$ , and  $H$  is a positive constant. In the real four-dimensional case ( $dS_{d=4}$ ),  $H$  corresponds to the Hubble constant.

The  $dS_d$  (relativity) group is  $SO_0(1, d)$  — indicating the connected subgroup of  $O(1, d)$  — or any of its covering groups. A recognizable manifestation of the corresponding Lie algebra is attained through the linear span of the Killing vectors:

$$K_{\alpha\beta} = x_\alpha \partial_\beta - x_\beta \partial_\alpha. \quad (2)$$

---

\* gazeau@apc.in2p3.fr

† pejhan@math.bas.bg

## II. The necessity of analyticity within the complexified $dS_d$ manifold ( $dS_d^{(C)}$ )

The entire argument we present in this letter stems from the imperative need for analyticity within the complexified  $dS_d$  manifold ( $dS_d^{(C)}$ ):

$$dS_d^{(C)} = \left\{ z = x + iy \in \mathbb{C}^{1+d} ; (z)^2 = \eta_{\alpha\beta} z^\alpha z^\beta = -H^{-2} \right. \\ \left. \text{or equivalently ; } (x)^2 - (y)^2 = -H^{-2}, x \cdot y = 0 \right\}. \quad (3)$$

This analyticity requirement is crucial for describing elementary systems in the global structure of  $dS_d$  through QFT.

A foundational argument strongly supports this necessity. Expanding beyond the familiar flat Minkowski spacetime to  $dS_d$  introduces a fundamental challenge to the QFT description of elementary systems. The challenge lies in the absence of a well-defined spectral condition [8]. Regardless of the chosen QFT approach to field quantization on  $dS_d$  spacetime, formulating requirements for locality (microcausality) and covariance is a relatively straightforward task. However, establishing any condition on the spectrum of the “energy” operator proves to be a formidable challenge. Strictly speaking, in  $dS_d$  spacetime, granted that no globally timelike Killing vector exists, neither time nor energy can be globally defined. This inherent ambiguity results in the emergence of numerous inequivalent QFTs (vacuum states) for a single field model on  $dS_d$  spacetime. Each of these theories is typically linked to a specific choice of the time coordinate, leading to associated frequency splitting.

Interestingly, within the Minkowskian framework, a notable observation highlights a direct link between the characteristics of analytic continuation in complexified spacetime and the spectral property of the model under consideration. Specifically, in the context of  $(1+d)$ -dimensional Minkowski spacetime  $\mathbb{R}^{1+d}$ , the spectral property can be expressed as follows: for each  $n > 1$ , the Wightman  $n$ -point function corresponds to the boundary value, in the distributional sense, of a function that is holomorphic within the tube  $T^{+(n)}$  defined as:

$$T^{+(n)} = \left\{ (z_1, \dots, z_n) \in \mathbb{C}^{n(1+d)} ; \text{Im}(z_{j+1} - z_j) \in V^+ \right\}, \quad (4)$$

where  $1 \leq j \leq n-1$  and  $V^\pm = \{y \in \mathbb{R}^{1+d} ; (y)^2 > 0, y^0 \geq 0\}$ .

In the context of  $dS_d$  spacetime (embedded in  $\mathbb{R}^{1+d}$ ), a natural alternative to this property is to propose that the Wightman  $n$ -point function  $\mathcal{W}_n(x_1, \dots, x_n)$  acts as the boundary value, in the distributional sense, for a function that is holomorphic within:

$$\mathcal{T}^{+(n)} = T^{+(n)} \cap [dS_d^{(C)}]^n. \quad (5)$$

As a matter of fact, studies by Bros et al. [9, 10] have shown that  $\mathcal{T}^{+(n)}$  forms a domain and a tuboid — bounded by reals — in a manner that maintains the relevance of the concept of the “distribution boundary value of a holomorphic function from this domain”. Consequently, it becomes feasible to impose:

- *Weak spectral condition.* For each  $n > 1$ , the Wightman  $n$ -point function  $\mathcal{W}_n(x_1, \dots, x_n)$  is derived from the distributional boundary value of a function  $W_n(z_1, \dots, z_n)$ , which is holomorphic within the domain  $\mathcal{T}^{+(n)}$ .

Focusing on a generalized free  $dS_d$  field, which represents the simplest scenario in this context, the assertion of normal analyticity can be explicitly expressed as follows:

- The corresponding two-point function  $\mathcal{W}_2(x_1, x_2)$  arises as the distributional boundary value of a function  $W_2(z_1, z_2)$  that is analytically defined within the tuboid domain  $\mathcal{T}^{+(2)}$  given by:

$$\mathcal{T}^{+(2)} = \left\{ (z_1, z_2) ; z_1 \in \mathcal{T}^-, z_2 \in \mathcal{T}^+ \right\}, \quad (6)$$

where  $\mathcal{T}^\pm = \{\mathbb{R}^{d+1} + iV^\pm\} \cap dS_d^{(C)}$ . Note that  $\mathcal{T}^\pm$  are referred to as the forward and backward tubes of  $dS_d^{(C)}$ , respectively.

This elegant resolution effectively circumvents all ambiguities in  $dS_d$  QFTs, which arise from the absence of a genuine spectral condition in  $dS_d$  spacetime [9, 10]. It leads to the derivation of “vacua” that, despite exhibiting thermal properties, serve as precise analogs of Minkowski vacuum representations. The latter emerges as the limit of the former when the curvature tends to zero.

For additional arguments supporting the necessity of analyticity in the complexified  $dS_d$  manifold, readers are referred to Refs. [6, 11] and the references therein.

### III. The $dS_d$ static patch and the analyticity requirement

Following the approach outlined in Ref. [12], let us adopt the viewpoint of a local observer (on  $dS_d$ ) in motion along the geodesic  $g(x_\bullet)$  that traverses through the point  $x_\bullet = (0, \dots, 0, x^d = H^{-1})$ :

$$g(x_\bullet) = \left\{ x = x(t); x^0 = H^{-1} \sinh Ht, \mathbf{x} = (x^1, \dots, x^{d-1}) = 0, x^d = H^{-1} \cosh Ht, t \in \mathbb{R} \right\}. \quad (7)$$

Note that the chosen point  $x_\bullet$  is located in the  $(x^0, x^d)$ -plane. It is also worth noting that the choice of this point is entirely arbitrary, given the  $SO_0(1, d)$  symmetry of  $dS_d$ .

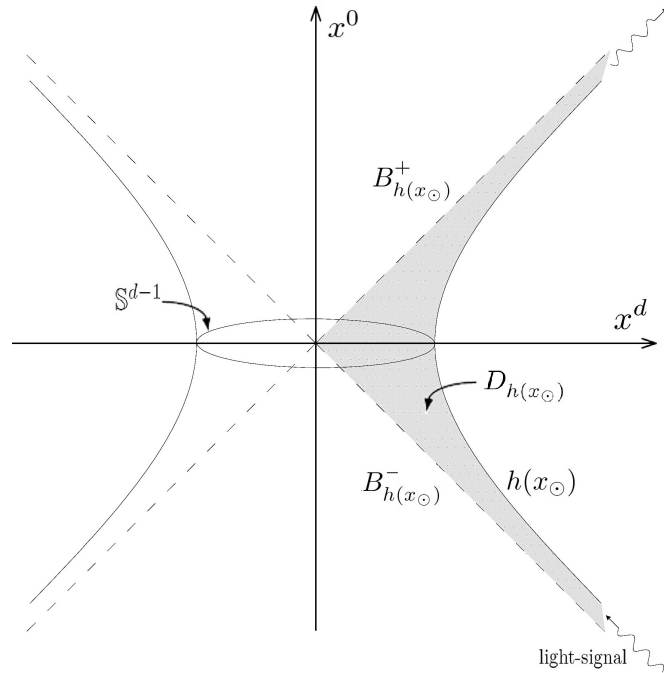


FIG. 1: The static patch of  $dS_d$ , linked with an observer traveling along the geodesic  $g(x_\bullet)$ .

The set of all “events” in  $dS_d$  that can be linked to the observer via the reception and emission of light signals is given by:

$$D_{g(x_\bullet)} = \left\{ x \in dS_d; x^d > |x^0| \right\}. \quad (8)$$

This domain is delineated by two distinct boundaries:

$$B_{g(x_\bullet)}^\pm = \left\{ x \in dS_d; x^0 = \pm x^d, x^d > 0 \right\}. \quad (9)$$

These boundaries are respectively termed the “future horizon” and “past horizon” of the observer following the geodesic  $g(x_\bullet)$ .

The parameter  $t$  in expression (7) signifies the proper time of the observer moving along the geodesic  $g(x_\bullet)$ . This enables us to designate the “time-translation group linked with  $g(x_\bullet)$ ” as the one-parameter subgroup  $T_{g(x_\bullet)}$  (isomorphic to  $SO_0(1, 1)$ ) of the  $dS_d$  group  $SO_0(1, d)$ . The  $T_{g(x_\bullet)}$ ’s transformations involve hyperbolic rotations parallel to the  $(x^0, x^d)$ -plane. To characterize how these transformations operate on the domain  $D_{g(x_\bullet)}$ , let  $x = x(t, \tilde{\mathbf{x}})$  represent an arbitrary point within this domain:

$$x(t, \tilde{\mathbf{x}}) = \begin{cases} x^0 = \sqrt{H^{-2} - (\tilde{\mathbf{x}})^2} \sinh Ht, \\ \tilde{\mathbf{x}} = (x^1, \dots, x^{d-1}), \\ x^d = \sqrt{H^{-2} - (\tilde{\mathbf{x}})^2} \cosh Ht, \quad t \in \mathbb{R}, \end{cases} \quad (10)$$

while  $(\tilde{\mathbf{x}})^2 = (x^1)^2 + \dots + (x^{d-1})^2 < H^{-2}$ . The operation of  $T_{g(x_\bullet)}$  on  $x(t, \tilde{\mathbf{x}})$  is expressed as follows:

$$T_{g(x_\bullet)}(\tau) \diamond x(t, \tilde{\mathbf{x}}) = x(t + \tau, \tilde{\mathbf{x}}) = x^\tau, \quad \tau \in \mathbb{R}. \quad (11)$$

It defines a group of isometric automorphisms within the domain  $D_{g(x_\bullet)}$ . The associated orbits  $g_{\tilde{\mathbf{x}}}(x_\bullet)$  depict separate hyperbolic branches in  $D_{g(x_\bullet)}$ . These hyperbolas lie in two-dimensional plane sections that are parallel to the  $(x^0, x^d)$ -plane. Within these orbits, the singular one that defines a geodesic of  $dS_d$  is  $g(x_\bullet) = g_{\tilde{\mathbf{x}}=0}(x_\bullet)$ . Hence, understanding the group  $T_{g(x_\bullet)}$  as time translation is particularly applicable to observers in motion on or near  $g(x_\bullet)$ , with the assumption that the proximity is significantly smaller than the  $dS_d$  radius of curvature.

Now, let us evaluate the depicted scenario by acknowledging the necessity of analyticity within the complexified  $dS_d$  manifold ( $dS_d^{(C)}$ ). The complex orbits of  $T_{g(x_\bullet)}$  are denoted as:

$$g_{\tilde{\mathbf{x}}}^{(C)}(x_\bullet) = \left\{ z^\tau = z(t + \tau, \tilde{\mathbf{x}}), \tau \in \mathbb{C} \right\}. \quad (12)$$

Note that all nonreal points linked to the complex orbits  $g_{\tilde{\mathbf{x}}}^{(C)}(x_\bullet)$  reside in  $\mathcal{T}^\pm$ , the very domains of analyticity required for the QFT description of elementary systems in the global structure of  $dS_d$  spacetime (see Eq. (6)). In this context, due to the analytic nature of  $dS_d$  QFTs, a notable connection emerges between the domain  $D_{g(x_\bullet)}$  and its corresponding mirror region, as shown in FIG. 1:

$$D_{g(-x_\bullet)} = \left\{ x = (x^0, \tilde{\mathbf{x}}, x^d) \in dS_d; (-x^0, \tilde{\mathbf{x}}, -x^d) \in D_{g(x_\bullet)} \right\}, \quad (13)$$

To see the point, with the coordinate system (10) in consideration, setting  $\text{Im}(\tau) = \pi$  in (12) suffices. For a given  $dS_d$  elementary system, specifically a free quantum field in the global structure of  $dS_d$  spacetime, this connection inherently establishes a significant and highly nontrivial link between the physical interpretation of the elementary system in  $D_{g(x_\bullet)}$  and the mirror region  $D_{g(-x_\bullet)}$ . Refer to Ref. [10] for a comprehensive exploration of this connection.

Here, it is crucial to highlight that the intrinsic time variable pertinent to an observer moving along the geodesic  $g(-x_\bullet)$  (within the mirror region  $D_{g(-x_\bullet)}$ ) is equivalent to  $-t$ . This arises from the fact that the associated time-translation group  $T_{g(-x_\bullet)}$  validates the relationship  $T_{g(-x_\bullet)}(\tau) = T_{g(x_\bullet)}(-\tau)$ ; it is noteworthy that  $T_{g(-x_\bullet)}$  is derived from  $T_{g(x_\bullet)}$ , for instance, through conjugation in the form  $T_{g(-x_\bullet)} = R T_{g(x_\bullet)} R^{-1}$ , where  $R$  represents a rotation of angle  $\pi$  in a plane orthogonal to the  $x^0$ -axis. This characteristic serves as yet another indication of the fact that  $dS_d$  spacetime lacks a globally timelike Killing vector.

This pivotal property forms the cornerstone of our reasoning, as elaborated further in the subsequent sections.

#### IV. Discussion: matter-antimatter asymmetry?

So far, in the initial two steps, we have articulated the foundational assumptions guiding our reasoning. Firstly, we assumed that our Universe presently reflects a phase akin to  $dS_{d=4}$ , converging toward a pure  $dS_{d=4}$  spacetime — a proposition bolstered by observational data [1–4]. Secondly, we emphasized the imperative of analyticity within the complexified  $dS_d$  manifold ( $dS_d^{(C)}$ ), underscored by the inherent requirement for a natural substitution of the spectral condition. This necessity, in turn, facilitates a coherent QFT depiction of elementary systems within the global structure of  $dS_d$  spacetime [6, 8–11]. Building on these foundational assumptions, we demonstrated in the third step that for a given  $dS_d$  elementary system, specifically a free quantum field in the global structure of  $dS_d$  spacetime, the physical interpretation in a region observable by a local observer ( $D_{g(x_\bullet)}$ , with proper time  $t$ ) is intricately linked to its interpretation in the corresponding mirror region ( $D_{g(-x_\bullet)}$ , with proper time  $-t$ ).

With these steps established, we now move on to the discussion:

1. Consider an elementary system, defined as a free quantum field, within the global structure of  $dS_d$  spacetime. Due to the absence of a preferred direction of time in  $dS_d$  spacetime, *in the sense given by Feynman-Stueckelberg*<sup>1</sup>, there is no inherent preference for interpreting this elementary system as matter over antimatter, or vice versa, within the global structure of  $dS_d$  spacetime.

---

<sup>1</sup> The Feynman-Stueckelberg interpretation proposes that antimatter and antiparticles behave like ordinary particles but travel backward in time.

2. Before observation, therefore, the elementary system exhibits a dual nature, existing simultaneously as both matter and antimatter. However, once a local observer fixes a time direction (proper time  $t$ ), this act breaks the time-reversal symmetry within the observable region of  $dS_d$  spacetime (i.e.,  $D_{g(x_\bullet)}$ ). Consequently, within this causal domain accessible to the observer, the elementary system is identified as matter. In this context, the origin of the current observable small amount of antimatter can be attributed to specific exceptions in Standard Model interactions where baryon number conservation is broken (see Ref. [13]).
3. This choice, however, does not disrupt the inherent symmetry between matter and antimatter in the global structure of  $dS_d$  spacetime. Indeed, according to the Feynman-Stueckelberg interpretation and the intrinsic connection established in the third step, in the mirror region of the observable segment (i.e.,  $D_{g(-x_\bullet)}$ , with the reversed direction of proper time  $-t$ ), the very same elementary system is interpreted as antimatter by the very same observer, thus preserving the system's dual nature as both matter and antimatter simultaneously. Note that the two mirror regions ( $D_{g(x_\bullet)}$  and  $D_{g(-x_\bullet)}$ ) are causally disconnected but for infinitesimally small curvature radius values ( $H \rightarrow \infty$ ), characteristic of the inflationary epoch, quantum tunneling effects might occur between these regions.

This discussion underscores how even a small, nonzero cosmological constant can have significant implications for our understanding of elementary systems. However, these observations do not apply in flat Minkowski spacetime, where the cosmological constant is zero.

In conclusion, it is important to recognize that the cosmological model considered in this letter, specifically the  $dS_{d=4}$  Universe, represents an idealized scenario for real-world cosmology. While the cosmological constant appears consistent with supernova data, indicating a  $dS_{d=4}$ -like expansion, potential evidence of dynamical dark energy (possibly suggested by DESI data) could challenge this exact  $dS_{d=4}$  behavior. Moreover, although our Universe may have undergone accelerated expansion during its early stages (such as during inflation), this expansion likely does not precisely match the  $dS_{d=4}$  form.

To address these issues, it is crucial to extend this study to a less idealized cosmological scenario, specifically the Friedmann-Robertson-Walker (FRW) spacetime, which asymptotically approximates a  $dS_{d=4}$  spacetime. This approach will provide more concrete and robust conclusions, ensuring that our findings are applicable within a broader and more realistic context. Despite the mathematical complexities involved, we anticipate that our key results will extend beyond the specific case of  $dS_{d=4}$ .

Furthermore, a deeper investigation into the matter-antimatter (a)symmetry within the context of  $dS_d$ , particularly through the lens of quantum electrodynamics in de Sitter space-time, would be a natural and valuable continuation of this work, building upon previous studies [14, 15].

### Acknowledgment

Hamed Pejhan is supported by the Bulgarian Ministry of Education and Science, Scientific Programme ‘Enhancing the Research Capacity in Mathematical Sciences (PIKOM)’, No. DO1-67/05.05.2022.

- 
- [1] S. Perlmutter et al., *Astrophys. J.*, 483, 565 (1997).
  - [2] B. Schmidt et al., *Astrophys. J.*, 507, 46 (1998).
  - [3] A.J. Riess et al., *Astron. J.*, 116, 1009 (1998).
  - [4] Richard Panek, *The Most Shocking Discovery in Astrophysics Is 25 Years Old*, Scientific American (2023).
  - [5] C. Fronsdal, *Rev. Mod. Phys.* 37, 221 (1965).
  - [6] M. Enayati, J.P. Gazeau, H. Pejhan, and A. Wang, *The de Sitter (dS) Group and its Representations; An Introduction to Elementary Systems and Modeling the Dark Energy Universe* (2nd edition), Springer Nature (2024).
  - [7] M. Enayati, J.P. Gazeau, M.A. del Olmo, and H. Pejhan, arXiv:2307.06690.
  - [8] R.F. Streater and A.S. Wightman, *PCT, Spin and Statistics, and All That*, W.A. Benjamin, New York (1964).
  - [9] J. Bros, J.P. Gazeau, and U. Moschella, *Phys. Rev. Lett.*, 73, 1746 (1994).
  - [10] J. Bros and U. Moschella, *Rev. Math. Phys.*, 08, 327 (1996).
  - [11] J.P. Gazeau, M.A. del Olmo, and H. Pejhan, *Phys. Lett. B* 848, 138402 (2024).
  - [12] G.W. Gibbons and S.W. Hawking, *Phys. Rev. D*, 15, 2738 (1977).
  - [13] D. Tong, *Gauge theory*, Lecture notes, DAMTP Cambridge 10 (2018).
  - [14] P. Bartesaghi, J.P. Gazeau, U. Moschella, and M.V. Takook, *Class. Quant. Grav.* 18, 4373 (2001).
  - [15] T. Garidi, J.P. Gazeau, S. Rouhani, and M. V. Takook, *J. Math. Phys. (N.Y.)* 49, 032501 (2008)