



Beyond space and blocks: Generating networks with arbitrary structure

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CONTEXT

Random graph models are ubiquitous in network science. Often used as null models, they require to preserve the size of the observed networks, i.e., the number of nodes and edges. We focus on models in which the probability to observe an edge between each pair of node is independent, as in an ER or expected degree preserving (Chung-Lu) model. Such models are widespread for block structures (SBM) and spatial structure (e.g., Waxman, gravity model), but there is no simple solution to create random graphs with arbitrary —defined by the analyst— structures.

CONTRIBUTION

We propose a model allowing to generate graphs of a chosen number of nodes and expected number of edges, following an arbitrary structure, and with a controllable amount of randomness.

The model requires:

- The number of nodes n
- The expected number of edges m
- A *cost function* describing the generated network’s structure

Let’s take a simple example: a possible cost function for a spatial graph is simply the distance between the nodes —This distance can be attributed randomly, or kept from observed data.

PROCESS OVERVIEW

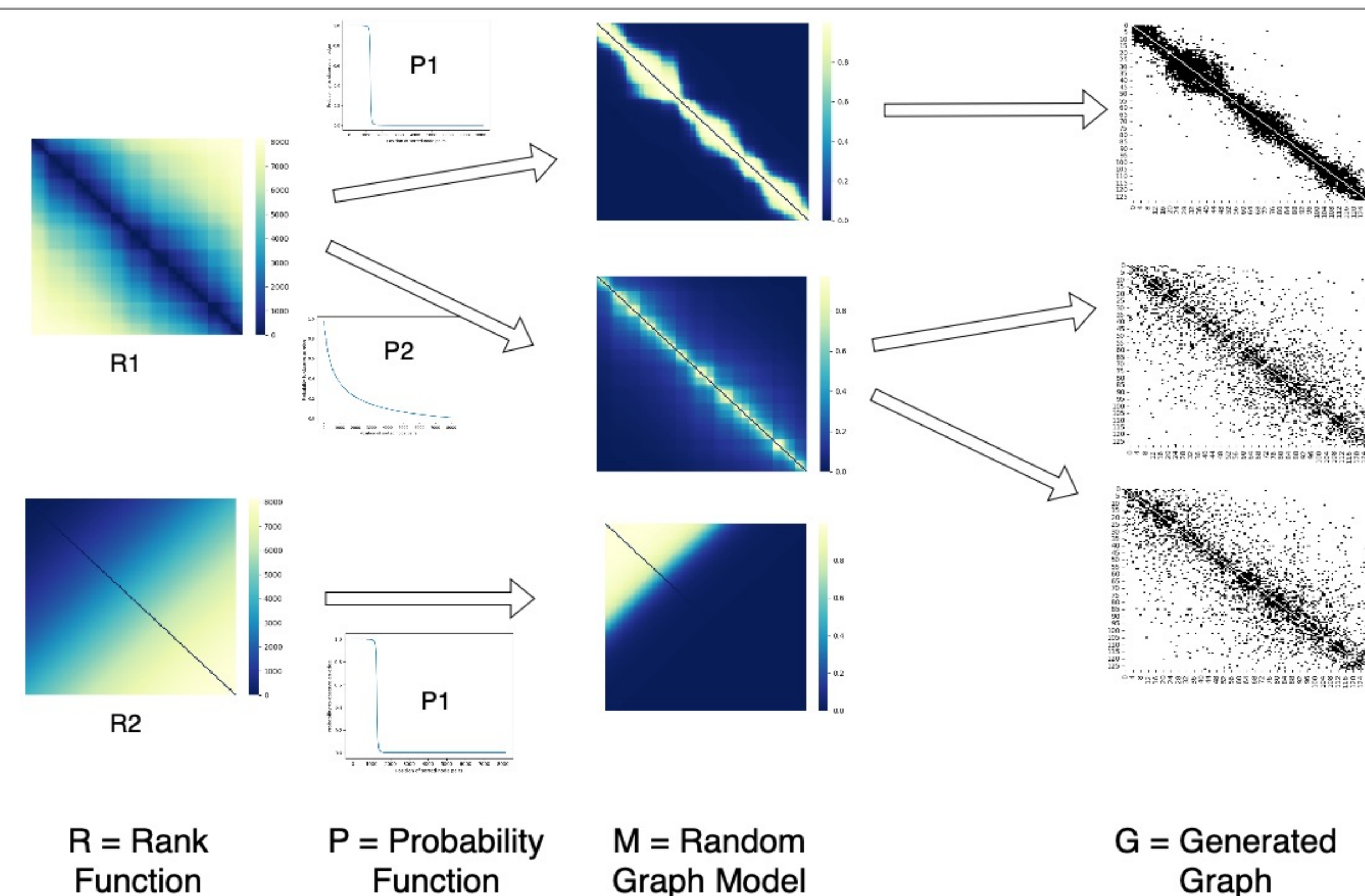
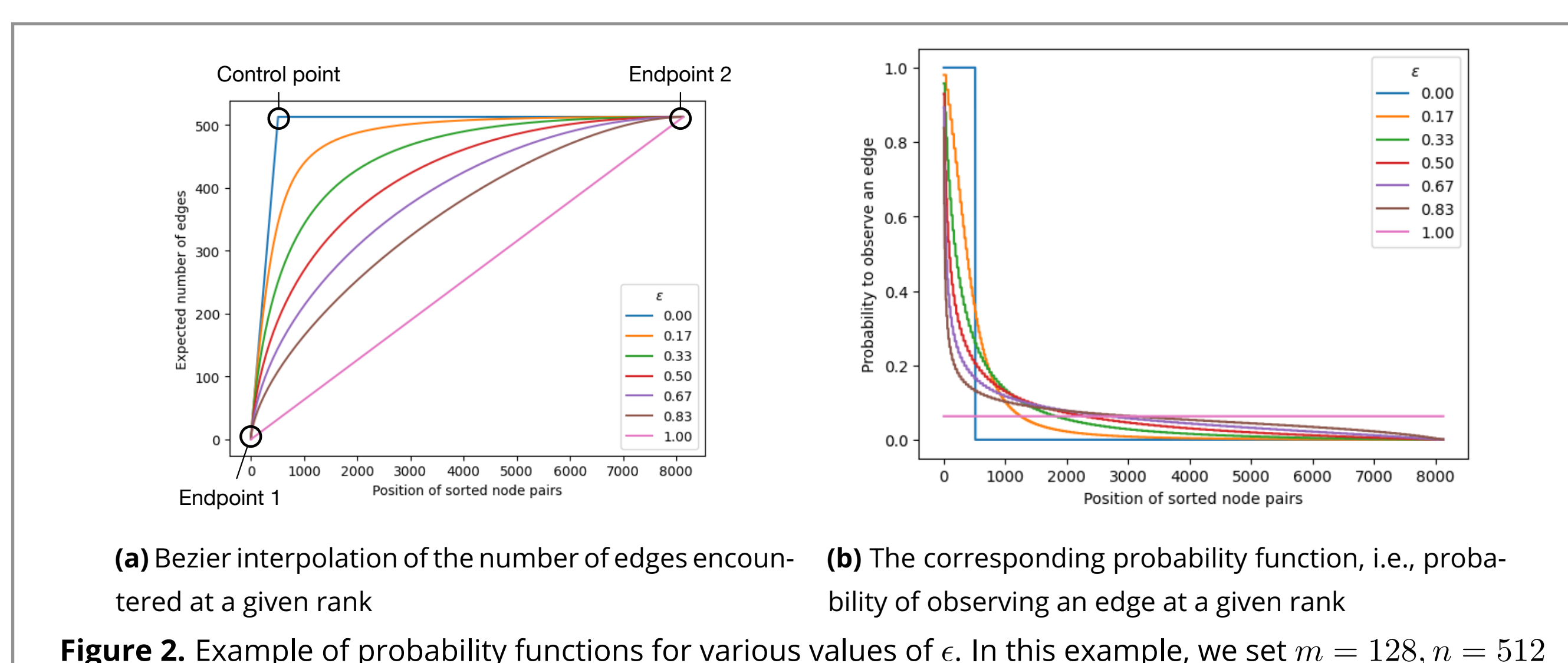


Figure 1. Generating networks using the Structify-Net approach. A rank function defines an ordering between node pairs. A probability function is used to assign an edge probability to each node pair based on their rank in the ordering. This gives a Random graph model, that can be used to generate graph instances. Note how the same Rank function $R1$ can give 2 Random Graph Models using different Probability function $P1$ and $P2$, how the same Probability function $P1$ is used for two different Rank functions $R1$ and $R2$, and how multiple graphs can be generated from a single Random Graph Model.

PROBABILITY FUNCTION

We use a function allowing to tune the amount of randomness of the structure, $\epsilon \in [0,1]$. When $\epsilon = 0$, the structure is deterministically determined by the cost function. When $\epsilon = 1$, the network is an ER random graph. The function interpolates between the two using a Bezier function, as illustrated in Fig.2



STRUCTURE ZOO

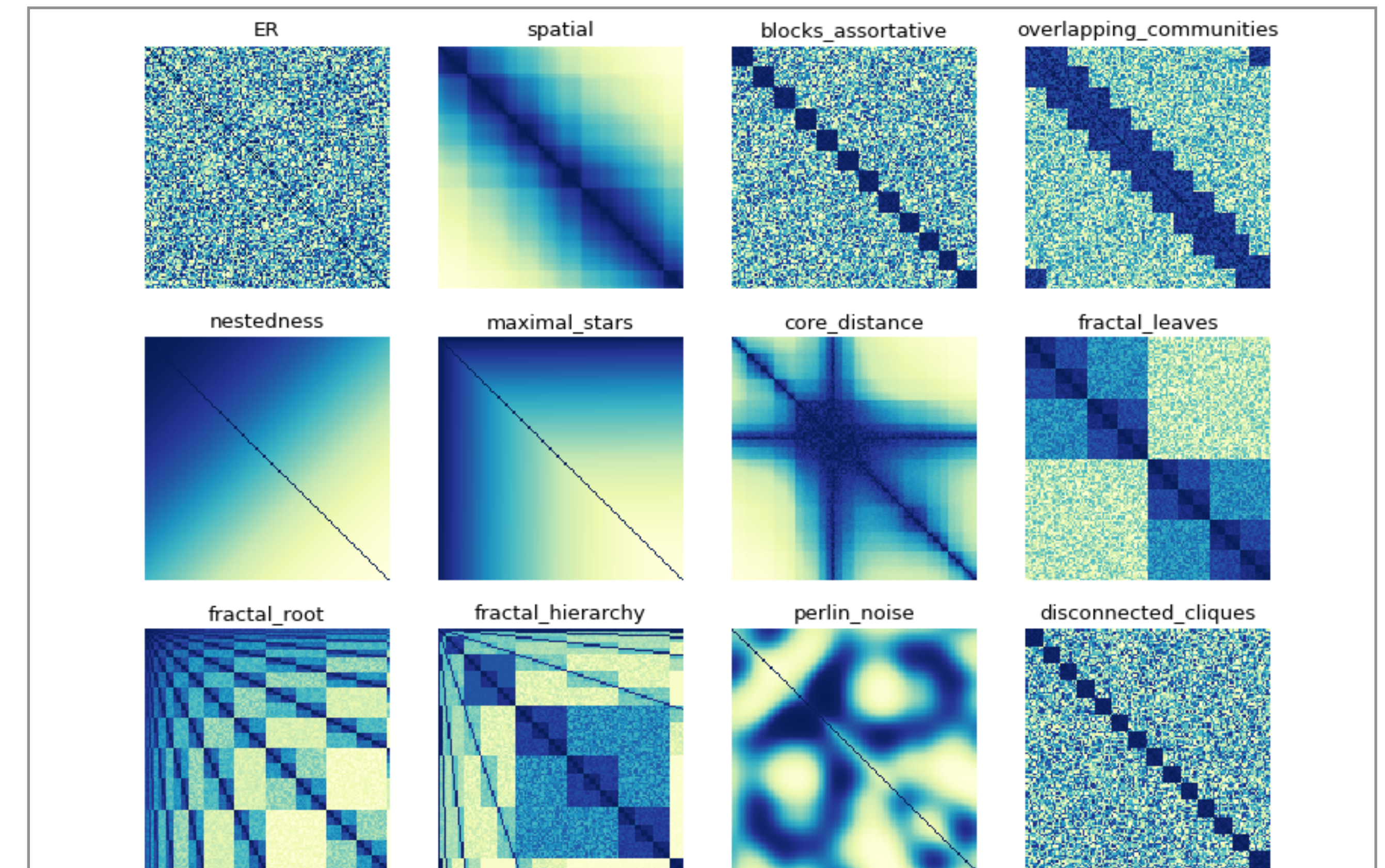


Figure 3. The Structure Zoo. Matrix of node-pairs ranks for networks with 128 nodes. Darker colors correspond to lower ranks. For *Disconnected cliques*, we set $m = 128 * 8$. When involving spatial or clique positions, nodes are ordered according to this value.

EXAMPLE: FRACTAL STRUCTURES

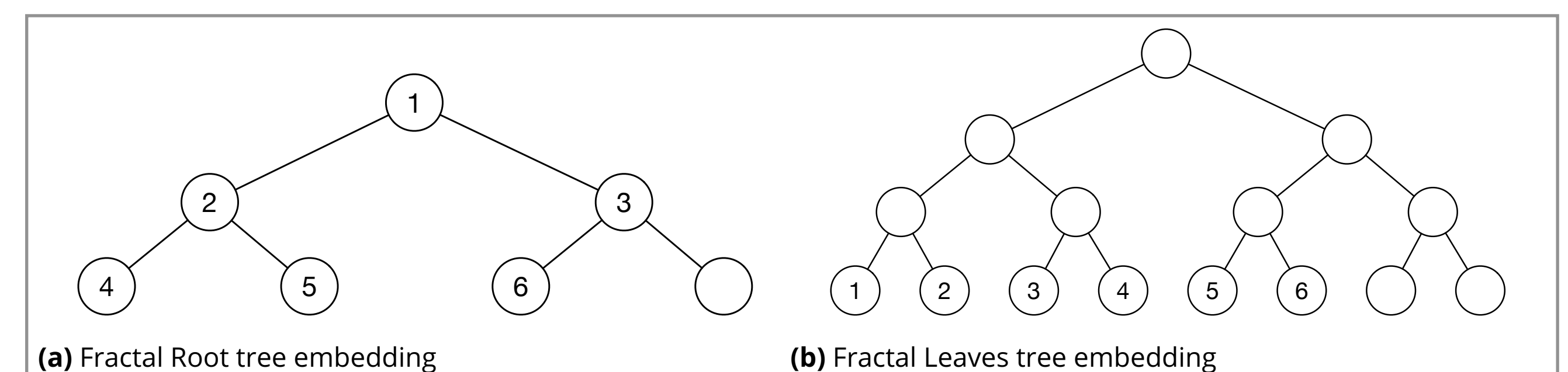


Figure 4. Two methods to create fractal structures by embedding nodes into complete binary trees. In the example, we embed 6 nodes. In the simpler case, the probability to observe a graph in the resulting graph is proportional to the distance between the nodes in the tree.

SMALL WORLD EXPERIMENT

To illustrate the flexibility of this approach, we first reproduce the famous Watts and Strogatz experiment, on which a regular circular network is progressively randomized, leading to the apparition of a *small-world* structure. (We use a normalized short path index increasing with shortest paths, in order to compare different network structures)

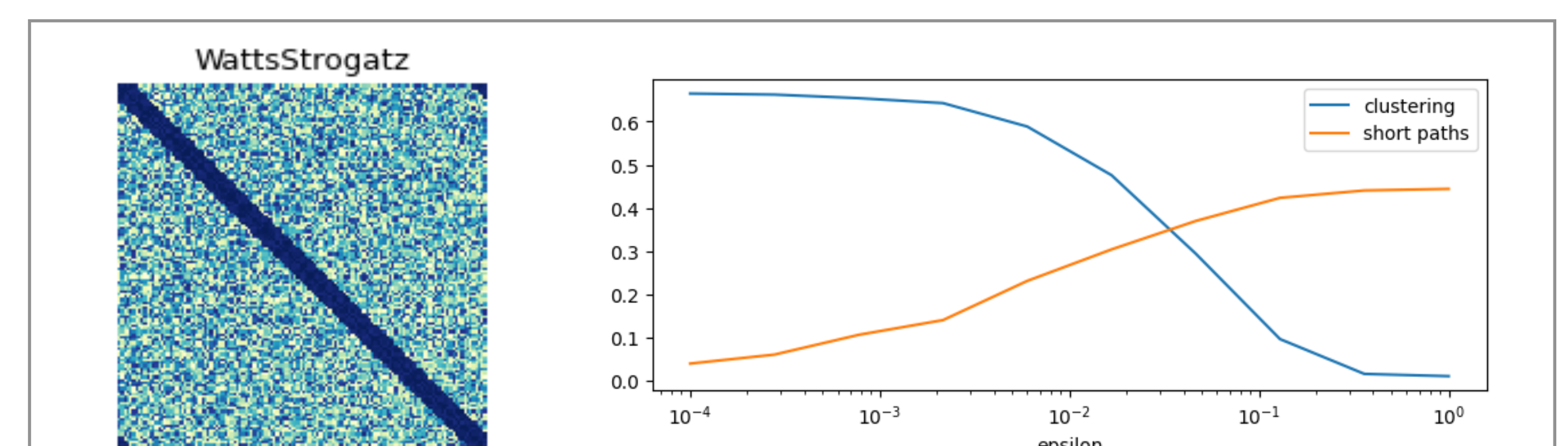


Figure 5. Replicating the Watts-Strogatz experiment

SMALL WORLDNESS OF OTHER STRUCTURES

Following the same experimental process, we can observe how different structures behaves in term of clustering coefficient and short paths

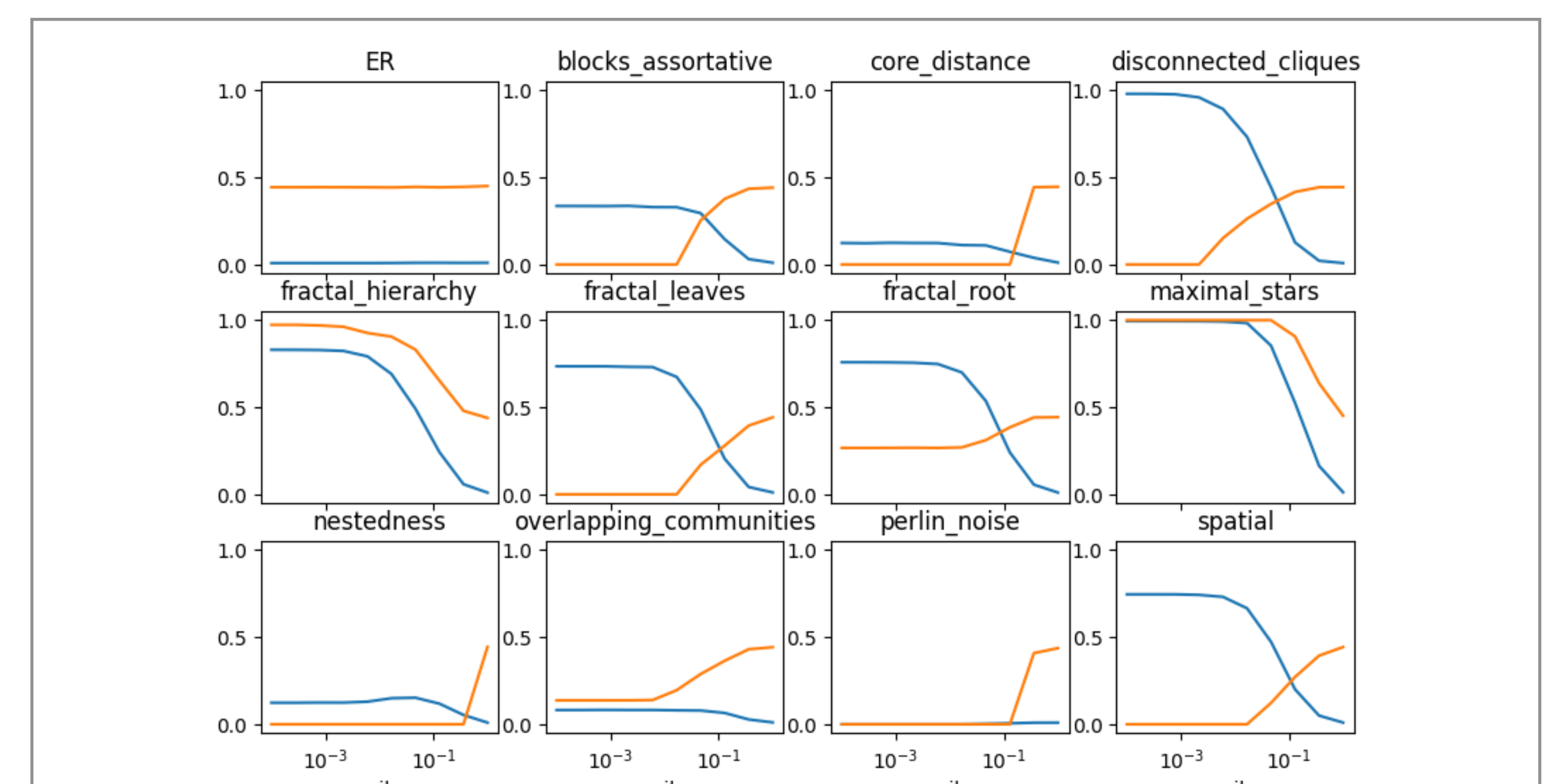


Figure 6. Small-World profiles of networks generated by generators of the *structure zoo*. **Blue** is clustering, **orange** is the short path index (lower values correspond to longer paths). We observe *super-small-world*, e.g., fractal-hierarchy, *anti-small-world*, e.g., nestedness, profiles similar to the Watts-Strogatz network, e.g., spatial or blocks assortative.

Structify-Net python library

arXiv preprint



https://github.com/Yquetzal/structify_net

<https://aps.arxiv.org/abs/2306.05274>