Beyond space and blocks: Generating networks with arbitrary structure
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Random graph models are ubiquitous in network science. Often used as null models, they require to preserve the size of the observed networks, i.e., the number of nodes and edges. We focus on models in which the probability to observe an edge between each pair of node is independent, as in an ER or expected degree preserving (Chung-Lu) model. Such models are widespread for block structures (SBM) and spatial structure (e.g., Waxman, gravity model), but there is no simple solution to create random graphs with arbitrary —defined by the analyst— structures.

The probability function controls the expected number of edges:

\[ P(n) = \exp(bm) \]

where \( P(n) \) is the expected number of edges, \( n \) is the number of nodes, \( b \) is the probability function which assigns to each node pair a probability to be connected by an edge, and \( m \) is the expected number of edges.

To go from a ranking of node pairs to a random network generator, we use a rank probability function. The model requires:

- The number of nodes \( n \)
- The expected number of edges \( m \)
- A cost function describing the generated network’s structure

Let’s take a simple example: a possible cost function for a spatial graph is simply the distance between the nodes. This distance can be attributed randomly, or kept from observed data.

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Beyond space and blocks: Generating networks with arbitrary structure

**CONTEXT**

**CONTRIBUTION**

We propose a model allowing to generate graphs of a chosen number of nodes and expected number of edges, following an arbitrary structure, and with a controllable amount of randomness.

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- The number of nodes \( n \)
- The expected number of edges \( m \)
- A cost function describing the generated network’s structure

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**PROCESS OVERVIEW**

**PROBABILITY FUNCTION**

We use a function allowing to tune the amount of randomness of the structure, \( c \in [0, 1] \). When \( c = 0 \), the structure is deterministically determined by the cost function. When \( c = 1 \), the network is an ER random graph. The function interpolates between the two using a Bezier function, as illustrated in Fig. 2.

**EXAMPLE: FRACTAL STRUCTURES**

(\( \epsilon = 0.5 \), \( m = m \)) for an example. Such a structure can be interpreted as a fuzzy version of a non-assortative small-world network. (We use a normalized short path index increasing with \( c \).)

**SMALL WORLD EXPERIMENT**

To illustrate the flexibility of this approach, we first reproduce the famous Watts and Strogatz experiment, on which a regular circular network is progressively randomized, leading to the appearance of a small-world structure. (We use a normalized short path index increasing with shortest paths, in order to compare different network structures.)

**SMALL WORLDNESS OF OTHER STRUCTURES**

Following the same experimental process, we can observe how different structures behave in term of clustering coefficient and shortest paths.

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**STRUCTURE ZOO**

**EXAMPLE: FRACTAL STRUCTURES**

- Fractal forest tree embedding
- Fractal leaf tree embedding

**SMALL WORLDNESS OF OTHER STRUCTURES**

- Three small-world profiles of networks generated by generators of the structure zoo. Blue is clustering, orange is the short path index (lower values correspond to longer paths). We observe super-small-world, e.g., fractal hierarchy, anti-small-world, e.g., nestedness, profiles similar to the Watts-Strogatz network, e.g., spatial or blocks assortative.