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TOWARDS MAXIMUM POWER CONVERSION FROM REALISTIC VIBRATIONS: LIMITS FOR SIZE-CONSTRAINED INERTIAL KINETIC ENERGY HARVESTERS UNDER BI-CHROMATIC VIBRATION INPUTS

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ABSTRACT

The topic of power conversion limits for size-constrained kinetic energy harvesting systems, and the associated optimal design, has mainly been explored for monochromatic and stochastic excitations. In this work, we present a first extension where we evaluate the absolute power conversion bound in the case of bi-chromatic vibration inputs. The resulting maximum power limit is shown to depend on both the relative amplitude and the phase shift of the input harmonics. We then evaluate this bound numerically for a near-optimally matched velocity-damped resonant generator. We show that for some relative amplitudes and phase shifts of the input vibration harmonics, the converted power drops to 45% of the absolute power bound, well below the well-known 79% limit that holds for monochromatic inputs.

KEYWORDS

Kinetic energy harvesting, power limits, velocity-damped resonant generator, bi-chromatic excitation

INTRODUCTION

Various models of small-size, inertial lineic kinetic energy harvesters (KEH) have been reported over the last two decades. These models describe harvester architectures that rely on different transduction mechanisms (e.g., electrostatic, or piezoelectric), different mechanical designs (e.g., beyond-quadratic potentials [1]), or different electrical interfaces (e.g., synchronized switching circuits [2] or charge-pumps [3]), combined in various configurations. Several limits on the maximum convertible power are obtained from each model: more specific models trade wide-range applicability for tighter bounds on converted power, and vice-versa.

Besides the harvester structural model, the other piece of information necessary to derive power bounds is the input vibration. Each power bound is valid for a specific model of input vibration. Many studies in the open literature report on the analysis of various harvester models under monochromatic harmonic inputs. Other works study power bounds for inputs modeled as wideband stochastic processes [4]. In all, these two canonical types of excitations constitute most of the reported analyses of inertial KEH models, with a few exceptions [5, 6].

However, it is reasonable to expect that in some applications, the harvester can be submitted to other inputs. For instance, the vibrations experienced by the KEH are often due to the response of a larger host structure to external mechanical excitations. These structures can exhibit different vibration modes, submitting the harvester to multi-chromatic inputs corresponding to each mode [7].

Non-monochromatic but regular vibration profiles are also encountered in the case of human motion [8]. While in such cases, the input vibration to the harvester remains periodic, the response to single-chromatic harmonic inputs does not define the behavior for multi-chromatic inputs. This is fundamentally due to the non-linear nature of inertial KEHs. A universal source of nonlinearity for small-scale inertial harvesters is the limited displacement allowed for the harvester's mass. Many reported architectures feature other sources of nonlinearity, deliberately or not.

Among all pairs of harvester model and input excitation, the displacement-limited velocity-damped resonant generator (VDRG) under monochromatic vibration was one of the first studied [9]. Since then, the VDRG has often served as a benchmark for KEHs. Indeed, the VDRG is deemed to be more realistic to implement compared to a KEH targeting the input power limit, because the VDRG can describe piezoelectric and electromagnetic harvesters with good accuracy under the right hypotheses. Besides, the maximum power limit of the VDRG model is a constant factor $\pi/4$ less than the input power limit [10].

In this work, we explore the power limits of a VDRG with displacement limitation for the inertial mass, submitted to bi-chromatic input vibrations. The inputs are characterized in terms of their harmonics' relative amplitudes and phase. We first highlight the dependence of the input power bound on the phase. Using a numerical approach, we study the dependence of the VDRG maximum converted power and optimal design parameters (stiffness, damping) on the input parameters. We show that KEHs designed to target the input power bound [11] show increasing relative performance with respect to the VDRG in the case of multi-chromatic inputs, compared to the mono-chromatic case. Indeed, our analysis in the remainder of the paper shows that the shortfall in converted power of a VDRG architecture drops from $\pi/4$ ($\approx 79\%$) of the input power limit for monochromatic inputs, to as low as about 45% of the maximum input power with bi-chromatic inputs.

HARVESTER MODEL

We consider in this paper a harvester following the velocity-damped resonant generator model (VDRG). To review, this is a lineic harvester built around a linear mechanical resonator of natural angular frequency ω_0 , comprised of an inertial mass m allowed to move along the (Ox) axis, and of a linear mechanical stiffness k . Two exogenous forces act upon the inertial mass: the *input force* $F = mA(t)$, i.e., the inertial force from the acceleration of the harvester defined as $-A(t)$, and the *harvester force*,

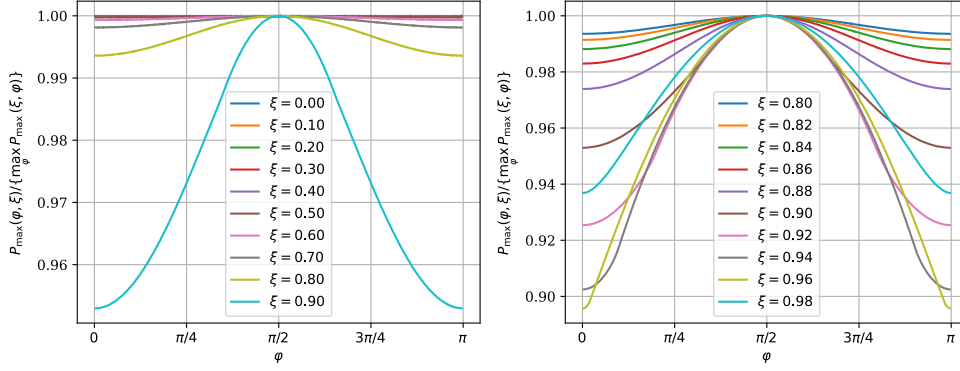


Figure 1: Input power limit for bi-chromatic inputs of varying relative amplitudes ξ and phases φ . Each curve is normalized by the maximum power reached for the specific amplitude ratio. The plot on the right focuses on the region of ξ where the variation is the most pronounced.

which, for the VDRG harvester, is supposed to be a linear dashpot. We quantify its effect in terms of a finite quality factor Q for the resonator. It is assumed that the work associated with this linear dashpot represents the energy converted from the mechanical into the electrical domain. The position of the mass $x(t)$ is referenced to be zero at the minimum of the quadratic potential. In addition, the model features a displacement limit L , beyond which the dynamics are not defined within this model: all trajectories must satisfy this constraint. In all, the model takes the form:

$$\begin{cases} \ddot{x} + \omega_0^2 x + \frac{\omega_0}{Q} \dot{x} = A(t) \\ |x(t)| \leq L. \end{cases}$$

For the rest of this study, we consider general bichromatic vibrations:

$$A(t) = A_0(\xi \cos(\omega t) + \sqrt{1 - \xi^2} \cos(2\omega t + \varphi))$$

with $0 \leq \xi \leq 1$. Note that it is customary to include an additional damping term to model air friction. We neglect this effect in the following to focus on the effect of the displacement limitation. This damping merely adds multiples regimes of operation, some of which are not displacement constrained. Furthermore, the assumption of negligible damping can be considered realistic in the cases of cautious harvester packaging to reduce the impact of friction, or when the input levels are sufficiently large (to wit, when the ratio $L\omega^2/A_0$ is sufficiently small).

The analysis for the remainder of the paper is done on a non-dimensional form of the model:

$$\begin{cases} \ddot{y} + \kappa^2 y + \frac{\kappa}{Q} \dot{y} = f(t), \\ |y(t)| \leq \lambda \end{cases}$$

where we have defined $y(t) \triangleq x(t/\omega) \times \omega^2/A_0$, $\kappa \triangleq \omega_0/\omega$, $\lambda \triangleq L\omega^2/A_0$, and $f(t) \triangleq A(t)/A_0$.

INPUT POWER LIMIT

There is an upper bound on the input energy that holds for any linear, inertial KEH with displacement limitation, and for arbitrary input vibration. It is reached when the mass follows the toggling trajectory described in [12]. When the transducer force is set to implement this trajectory, the converted energy is maximized in the

absence of damping phenomena. The corresponding energy is equal, up to constant terms, to the sum of local maxima, minus the local minima of the input force [11]. Thus, the upper bound on the average power for periodic forcing of period $T = 1/f$ is:

$$P_{\max} = 2mfL \sum_i G_i - g_i$$

where the $(G_i)_i$ are all local maxima of $A(t)$ in one period and $(g_i)_i$ are the local minima. Applied to the case of harmonic $A(t)$ of amplitude A_0 and frequency $f = \omega/2\pi$, this limit evaluates to $P_{\max}^{\text{1h}} = P_0 \triangleq 4mfLA_0$.

For the bi-chromatic inputs characterized by parameters ξ and φ , this quantity can be computed in closed form, although the expressions are cumbersome as they involve roots of a general fourth-order polynomial. Plots of a normalized $P_{\max}^{\text{2h}}(\xi, \varphi)$ for the bi-chromatic input are depicted in Fig. 1. The plots show, for each value of ξ , the value of the power vs. φ , each normalized by the maximum power for that specific ξ . It is therefore apparent that the phase has an impact on the maximum power that can be converted. The impact depends on the relative amplitudes of the harmonics as set by ξ . The impact is greater roughly in the region $\xi \in [0.8; 1.0]$, as shown in the rightmost part of the figure. There, the phase shift between the input harmonics can induce up to about 10.5% of shortfall in converted power when $\varphi = 0$ or $\varphi = \pi$ compared to the most favorable case where $\varphi = \pi/2$.

An obvious consequence of dependence of the input power limit on the phase φ is that the size and mass of the harvester being given, the power spectral density of the input does not characterize the maximum harvestable power in general.

VDRG POWER LIMIT

We now study the power limits for the VDRG. The problem of interest is, for given (ξ, φ, λ) :

$$\begin{cases} \max_{(\kappa, Q) \in \{1, 2\} \times \mathbb{R}_0^+} p_\lambda(\kappa, Q) \triangleq \frac{\kappa}{\lambda Q} (y_1^2 + 4y_2^2) \\ \text{subject to } \max_{t \in [0; 2\pi)} |y_1 \cos(t) + y_2 \cos(2t + \theta)| \leq \lambda \end{cases}$$

Here, y_1 and y_2 denote the amplitudes of the steady-state response to the first and second harmonic of the input,

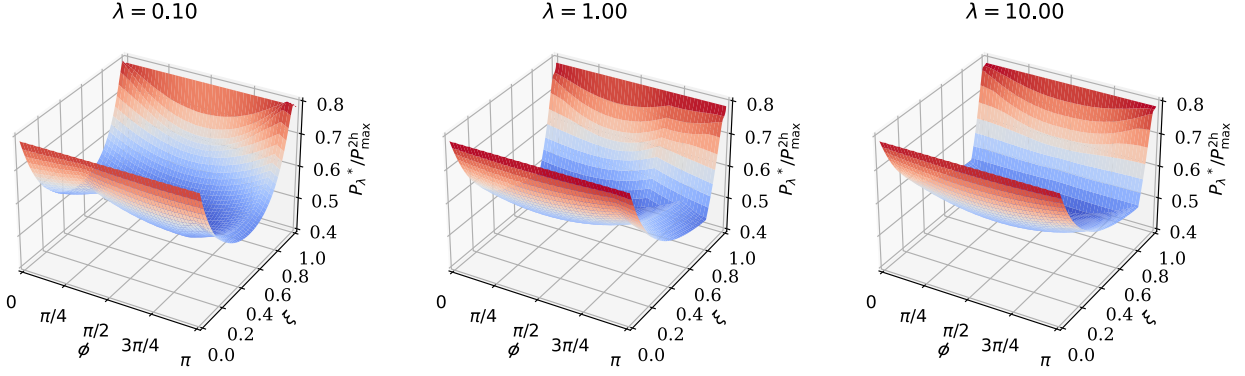


Figure 2: Plots of $P_\lambda^*(\xi, \varphi)/P_{\max}^{2h}(\xi, \varphi)$, for bi-chromatic inputs characterized in terms of relative amplitudes ξ and phase φ , and for different displacement constraints λ .

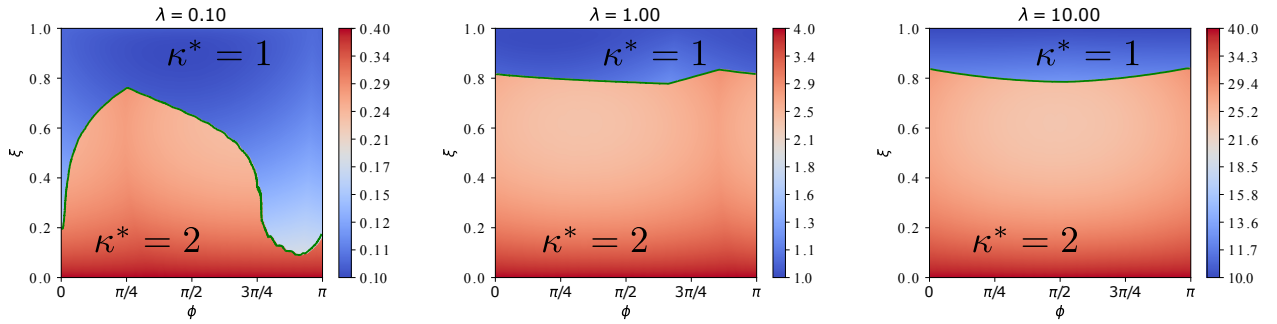


Figure 3: Optimal $(\kappa^*, Q^*) \in \{1, 2\} \times \mathbb{R}_{>0}^+$ for which the VDRG power is maximized, for different displacement constraints λ .

respectively, and θ denotes the phase shift between these two responses. The dependence of the solution upon (ξ, φ) is via these three quantities. The objective function is the non-dimensional power p_λ . It is related to the physical VDRG converted power $P_\lambda = \frac{m\omega\omega_0}{2\pi Q} \int_0^{2\pi/\omega} \dot{x}^2 dt$ by $p_\lambda = P_\lambda / (\frac{\pi}{4} P_0)$. In the following, we consider that the factor P_0 fixed, i.e., that (κ, Q) can be chosen independently of it.

Monochromatic input case

For the monochromatic case at unit frequency (this is the case $\xi = 1$), the limit in converted power under constrained displacement is readily found analytically: the condition reduces to $y_1 \leq \lambda$, and y_1 's expression is standard linear resonator theory. The optimization can then be done, e.g., by augmenting the objective function with a Lagrange multiplier. In all, one finds that the maximum power is reached for optimal parameters of $(\kappa^*, Q^*) = (1, \lambda)$, and evaluates to $p_\lambda^* = 1$. Or, in physical dimensions, $P_\lambda^* = \pi m f A_0 L$. This is the well-known result of [9] for large excitation amplitudes. This maximum amounts to a fraction $\pi/4 \approx 79\%$ of the input power limit P_{\max}^{1h} , as was already noted in previous studies [10].

Bi-chromatic input case

For the case of bi-chromatic inputs, we follow a numerical optimization procedure to find power-maximizing values of the stiffness and dashpot coefficient. For each (ξ, φ, λ) , the solution is numerically evaluated on

one period (using y_1, y_2 and θ 's expressions from linear resonator theory). The solution is discarded if there exists $t \in [0; 2\pi)$ such that $|y(t)| > \lambda$. The maximum p_λ^* across (κ, Q) is then sought among the remaining p_λ 's, by a simple grid search on $Q \in [\frac{\lambda}{10}; 10\lambda]$ of size 10000 (repeated for each value of κ). The results are obtained with a linear grid of size 1000×1000 for (ξ, φ) .

The resulting ratios $P_\lambda^*/P_{\max}^{2h}$ vs. parameters (ξ, φ) are displayed in Fig. 2, for different values of λ . The corresponding values of optimal $\kappa \in \{1, 2\}$ and Q are depicted in Fig. 3.

On Fig. 2, a slight phase dependence can be noted. Perhaps more notably, the results in Fig. 2 show that the ratio of $\pi/4$ is a maximum (it is reached in the plots for the monochromatic limits $\xi = 0$ and $\xi = 1$). Hence, the performance of the VDRG relatively to the input power limit is degraded for any bi-chromatic input compared to the monochromatic case. The most important decrease occurs roughly for $\xi \in [0.4; 0.8]$, where the relative maximum power drops below 50%, for all three values of λ , reaching 45% at the lowest.

Let us insist on the fact that the optimal κ is sought in $\{1, 2\}$, instead of a continuous space. This is a limitation of this work. Further work can explore optimization of κ across a continuous parameter space, especially for small λ 's. This optimization might improve the 45% power drop figure that was reported above.

CONCLUSION

This work is a contribution to the systematic study of various harvesters under realistic models of vibration inputs, beyond those usually studied. It was shown that the limit in input power for a lineic inertial KEH under bi-chromatic vibration is a function of both the relative amplitudes and the phase between the input's harmonics. The power limits of a displacement constrained VDRG harvester under this same excitation were then evaluated. The ratio of the maximum VDRG power to maximum input power is found to decrease from about 79% with monochromatic input, to as low as 45% with bi-chromatic inputs, the exact decrease ratio depending on the parameters on the input.

The example studied in this paper is the size-constrained VDRG (admittedly one of the simplest harvester models) submitted to bi-chromatic inputs (admittedly the simplest multi-chromatic input). This configuration already shows that it is necessary to think of optimal design as that of the pair (harvester, input), and that performance for harmonic inputs is not necessarily a meaningful optimization target depending on the application. Actively controlled KEHs that explicitly aim for the input power limit might be a path worth pursuing to target vibrations occurring in some applications [11-14], especially when the size of the device is limited.

Future studies are required to investigate the effect of allowing for arbitrary mechanical stiffnesses, most importantly for very small displacement constraints. It is also natural to consider extensions of this work to other harvester models, or to inputs with larger number of harmonics.

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