

Optimal Price Menu Design of Electric Vehicle Charging Stations

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Motivation

Context:

- Huge and quick development of Electric Vehicles (EVs)
- Increasing demand in power consumption for charging the EVs

Motivation: Help a public Charging Station Operator (CSO) to:

- maximize profit / social welfare
- provide Demand Response¹(DR)
- ensure a guaranteed quality of charging service to the EV users

¹Ex: french main distribution network operator Enedis yearly *call for tenders* for flexibility

Figure 1.1 Global electric car stock in selected regions, 2010-2022

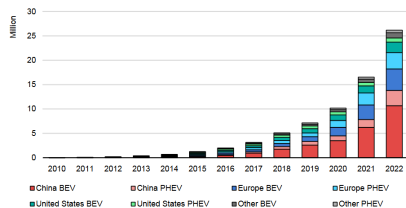


Figure: Source: Global EV Outlook 2023

Approach:

- Offer several charging power levels options to the EV users
- Dynamically adjust the charging prices considering electricity cost and the DR

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Scenario: a bi-level approach EV / CSO

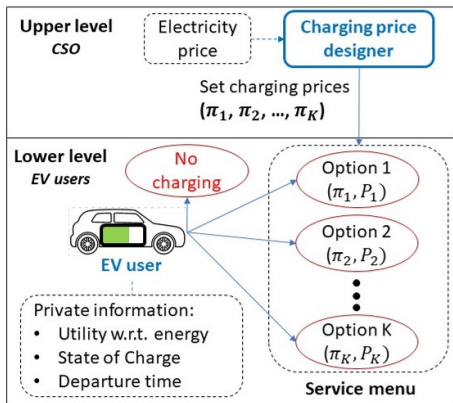


Figure: Bi-level scheme of the model

- Charging menu $\mathcal{K} := \{1, \dots, K\}$: set of K charging options
- Each charging option:
 - ▶ a constant (over time) charging power
 - ▶ a per kWh charging price
- Charging power (fixed parameter) ordering:
 $0 = P_0 < P_1 < P_2 < \dots < P_K$
- Dynamic charging prices, chosen by the CSO, \nearrow w.r.t. power:
 $0 = \pi_0 \leq \pi_1 \leq \pi_2 \leq \dots \leq \pi_K$ ²

²In practice, higher charging rate incurs higher price, e.g. at *Charge Place Scotland Tariffs*

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EV characteristics and charging choice model

- Exogenous parking duration d (depends on the on-site activities).
- Utility function $U(P)$ for charging at (constant) power P :

$$\frac{\partial U}{\partial P} > 0 \quad \leftarrow \text{Increasing}$$
$$\frac{\partial^2 U}{\partial^2 P} < 0 \quad \leftarrow \text{Concave} \quad [\text{Samadi et al. (2010)}]$$

- EVs welfare for option k : $W_k^{EV}(\pi_k) := \underbrace{U(P_k)}_{\text{Utility for charging with option } k} - \underbrace{\pi_k P_k d}_{\text{total charging price}}$

Deterministic choice model: maximizing the individual welfare

EVs choose the option k^* which maximize their individual welfare:

$$k^* := \arg \max_k W_k^{EV}(\pi_k).$$

Charging choice illustration

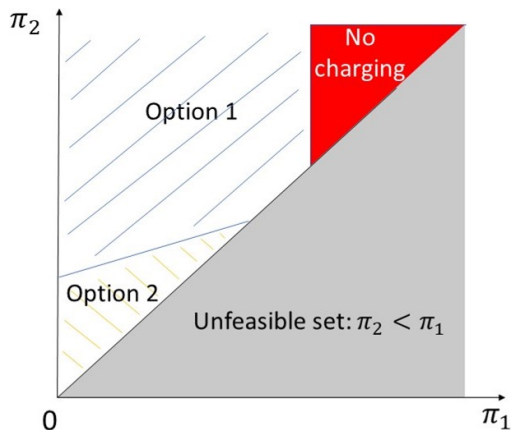


Figure: Illustration of the choice option *w.r.t.* the charging price menu in the case of 2 options

CSO's profit

- Revenue generated for the charging an EV:

$$\text{Revenue} := \underbrace{\pi_{k^*}}_{\text{charging price per kWh}} \times \underbrace{P_{k^*} d}_{\text{energy delivered}}$$

- Cost incurred for charging an EV:

$$\text{Cost} := \int_{t^{\text{start}}}^{t^{\text{start}}+d} \text{elec_cost}_t P_{k^*} dt = \underbrace{\bar{c}}_{\text{Avg. elec. cost per kWh}} \times \underbrace{P_{k^*} d}_{\text{energy delivered}}$$

CSO's profit per EV formulation

The profit per EV Π is difference between the revenue and the cost:

$$\Pi := \underbrace{(\pi_{k^*} - \bar{c})}_{\text{profit per kWh}} \times \underbrace{P_{k^*} d}_{\text{energy sold}} .$$

Optimal CSO's profit

Profit: $\Pi := (\pi_{k^*} - \bar{c}) \times P_{k^*} d$.

- The profit is piece-wise linear (with $K + 1$ pieces) w.r.t. the charging price vector π ;
- The optimal CSO's profit is:

$$\Pi_i^{opt} := \left(\frac{U_{k_1} - U_{k_2}}{(P_{k_1} - P_{k_2}) d} - \bar{c} \right) P_{k^{opt}} d,$$

with

$$k^{opt} = \underbrace{\arg \max_k (\pi_{k-1,k} - c) P_k d}_{\text{option yielding to the maximal CSO's profit at maximum}}$$

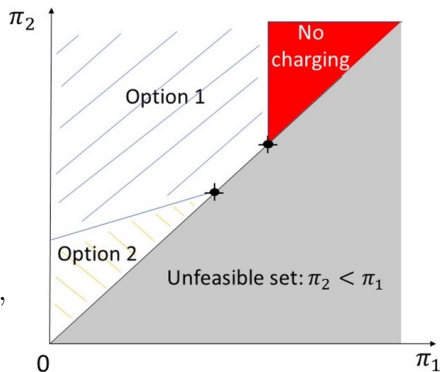


Figure: Illustration of the choice option w.r.t. the charging price menu in the case of 2 options

Social welfare

Social welfare expression

The *social welfare* is the sum between the CSO's profit and the EV welfare:

$$W^S := \underbrace{U_{k^*} - \pi_{k^*} P_{k^*} d}_{\text{EV welfare}} + \underbrace{(\pi_{k^*} - \bar{c}) P_{k^*} d}_{\text{CSO's profit}} = \underbrace{U_{k^*}}_{\text{EV utility}} - \underbrace{\bar{c} P_{k^*} d}_{\text{CSO's cost}}.$$

- The social welfare is piece-wise constant (with $K + 1$ pieces) *w.r.t.* the charging price vector π ;
- The optimal social welfare is:

$$\Gamma^{opt} := U_{k^{opt}} - P_{k^{opt}} d,$$

$$\text{with } k^{opt} = \arg \max_k U_k - P_k d.$$

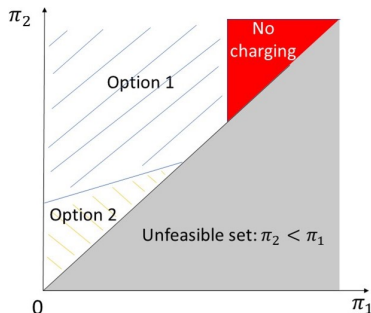


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EVs model with heterogeneity

EVs classes

EVs are clustered in a set of classes $\mathcal{I} = 1, \dots, I$, where each EV class $i \in \mathcal{I}$ have a weight θ_i share the same:

- utility for each option: $\{U_{i,1}, U_{i,2}, \dots, U_{i,K}\}$;
- parking duration d_i .
- choice set $\mathcal{K}_i := \{1, \dots, K_i\}$ (with K_i the highest option s.t. the battery is not fully charged before departure)

dimensionality problem: The number of “combination of choices” is equal to $(K + 1)^I$ ($= \text{nb_options}^{\text{nb_classes}}$) \Rightarrow previous analytical method not usable

EV choice formulation

The choice of the EV class i is represented by a vector $\{a_{i,k}\}_k$, where:

$$a_{i,k} = \begin{cases} 1 & \text{if EV class } i \text{ choose option } k, \\ 0 & \text{otherwise.} \end{cases}$$

Mathematical formulation

Mathematically, $\forall i, k, a_{i,k} \in \{0, 1\}$, with [Lu et al. (2023)]:

$$\sum_k a_{i,k} = 1 \quad \forall i \in \mathcal{I} \quad (1)$$

$$W_{i,k}(\pi_k) + M \sum_{m \neq k} a_{i,m} \geq W_{i,k}(\pi_l) \quad \forall i \in \mathcal{I}, k, l \in \mathcal{K}_i, k \neq l, \quad (2)$$

- (1): EVs choose one, and only one, option;
- (2): EVs choose the option maximizing their own welfare, using the *big M method*.

CSO's expected profit

CSO's profit maximization problem formulation: the **Base Case**

$$\underset{\mathbf{a}, \boldsymbol{\pi}}{\text{maximize}} \quad \mathbf{E}_{\theta}[\Pi] := \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \theta_i \mathbf{a}_{i,k} (\pi_k - \bar{c}_i) P_k d_i \quad (3)$$

such that:

$$\sum_k \mathbf{a}_{i,k} = 1 \quad \forall i \in \mathcal{I}$$

$$W_i(\pi_k) + M \sum_{m \neq k} \mathbf{a}_{i,m} \geq W_i(\pi_l) \quad \forall i \in \mathcal{I}, \forall k, l \in \mathcal{K}_i, k \neq l$$

$$\pi_{k-1} \leq \pi_k \quad \forall k \in \mathcal{K} \quad \leftarrow \text{Charging price} \nearrow \text{w.r.t. power}$$

- (3) can be solved independently of each time slots;
- (3) can be linearized from a *MIQP* into a *MILP* by introducing a new continuous variable $b_{i,k} := \mathbf{a}_{i,k} \pi_k$;
- *MILP* can be solved with commercial solvers like CPLEX.

Social welfare

Social welfare maximization problem formulation: the **Benchmark**

$$\underset{\mathbf{a}, \boldsymbol{\pi}}{\text{maximize}} \quad E_{\theta}[W^S] := \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \theta_i a_{i,k} (U_{i,k} - \bar{c}_i P_k d_i) \quad (4)$$

such that:

$$\sum_k a_{i,k} = 1 \quad \forall i \in \mathcal{I}$$

$$W_i(\pi_k) + M \sum_{m \neq k} a_{i,m} \geq W_i(\pi_l) \quad \forall i \in \mathcal{I}, \forall k, l \in \mathcal{K}_i, k \neq l$$

$$\pi_{k-1} \leq \pi_k \quad \forall k \in \mathcal{K} \quad \leftarrow \text{Charging price} \nearrow \text{w.r.t. power}$$

$$\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \theta_i a_{i,k} (\pi_k - \bar{c}_i) P_k d_i \geq 0 \quad \leftarrow \text{CSO's profit} \geq 0$$

- (4) can be solved independently of each time slots;

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Demand response

Mechanism:

- Reducing the aggregate power consumption at the CS compared to the *Base Case*, for a given period of time;

Consequences:

- The optimization problem is not decentralized in time anymore: the choice of EVs have an impact on the consumption in the future.

A few notations

- $n_{i,t}$: expected nb of EV class i arriving at time slot t ;
- λ_t^R (€/kW): remuneration for power reduction;
- \mathcal{T} : period considered of the day

Expected aggregated power consumption

- Exp. aggregated power \hat{P}_t at time t and with DR:

$$\hat{P}_t = \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \sum_{t'=1}^t \underbrace{n_{i,t'}}_{\text{Expected nb of EV class } i} \underbrace{a_{i,k} P_k \delta_{i,t',t}}_{\text{Power consumption at } t \text{ for EV class } i \text{ arriving at } t'},$$

with $\delta_{i,t',t}$ indicating that the EV class i arriving at time t' is still charging at time t .

- Exp. aggregated power P_t^Π at time t and for the Base Case:

$$P_t^\Pi = \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} \sum_{t'=1}^t \underbrace{n_{i,t'}}_{\text{Expected nb of EV class } i} \underbrace{a_{i,k}^{\Pi^{CSO}} P_k \delta_{i,t',t}}_{\text{Power consumption at } t \text{ for EV class } i \text{ arriving at } t'}.$$

Maximization problem

Formulation

$$\begin{aligned} \max_{\substack{\mathbf{a}, \boldsymbol{\pi}, \\ \mathbf{p}^R}} E[\Pi] &:= \sum_{t \in \mathcal{T}} \left(\underbrace{\sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}_i} n_{i,t} \mathbf{a}_{i,k} (\pi_k - \bar{c}_i) P_k d_i}_{\text{CSO's expected profit}} + \underbrace{\lambda_t^R \mathbf{p}_t^R}_{\text{Revenue for demand response}} \right) \\ \text{s.t.} \quad & \sum_k \mathbf{a}_{i,k} = 1, \quad \forall i \in \mathcal{I}, \\ & W_i(\pi_k) + M \sum_{m \neq k} \mathbf{a}_{i,m} \geq W_i(\pi_l), \quad \forall i \in \mathcal{I}, \forall k, l \in \mathcal{K}_i, k \neq l, \\ & \pi_{k-1,t} \leq \pi_{k,t}, \quad \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad \leftarrow \text{Price } \nearrow \text{ w.r.t. power} \\ & \mathbf{p}_t^R = \mathbf{P}_t^\Pi - \hat{\mathbf{P}}_t, \quad \forall t \in \mathcal{T} \quad \leftarrow \text{Power reduction for DR} \end{aligned}$$

- Linearized from *MIQP* to *MILP*

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Test-case

- 4 charging options (low charging rates): $P_1 = 2.5$ kW; $P_2 = 5$ kW; $P_3 = 7.5$ kW; $P_4 = 10$ kW;
- Battery capacity: 50 kWh and $\text{SoC} \in [20\%, 80\%]$;
- Quadratic utility function:

$$U_i(P) = \alpha_i \left(P d_i - \frac{1}{2} \beta_i (P_i d_i)^2 \right).$$

Table: Parameters of EV Classes

$\text{SoC}_i \setminus d_i$	Class i : K_i				Utility Parameters	
	1 h	2 h	3 h	4 h	α_i	β_i
10 kWh	1 : 4	2 : 4	3 : 4	4 : 3	0.425	0.017
20 kWh	5 : 4	6 : 4	7 : 2	8 : 2	0.35	0.021
30 kWh	9 : 4	10 : 2	11 : 1	12 : 1	0.275	0.027

- Demand response between **16:00** and **20:00** and time slots of 1 hour

Base Case and Benchmark: optimal charging price menu

Notations

- π_{opt}^{CSO} : optimal charging price menu for profit maximization;
- π_{opt}^{WS} : optimal charging price menu for social welfare maximization;
- λ_t : electricity cost

- $\pi_{opt}^{CSO} > \pi_{opt}^{WS}$
- Base Case: π_{opt}^{CSO} not sensitive to small changes in λ_t
- Benchmark: profit positive while $\pi_{opt}^{WS} < \lambda_t$ for some charging options but profit

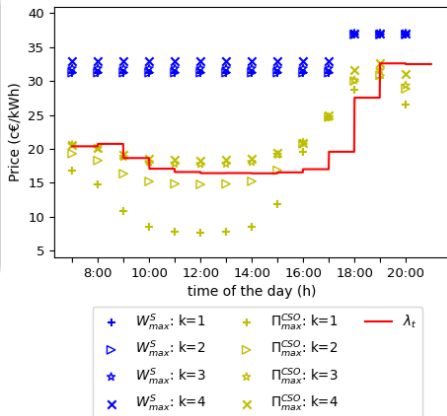


Figure: Hourly optimal price menu, Base Case and Benchmark

Demand response: charging price and aggregated power

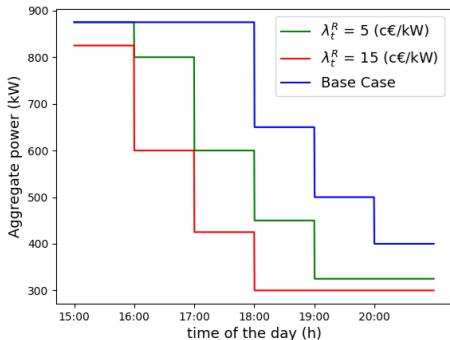
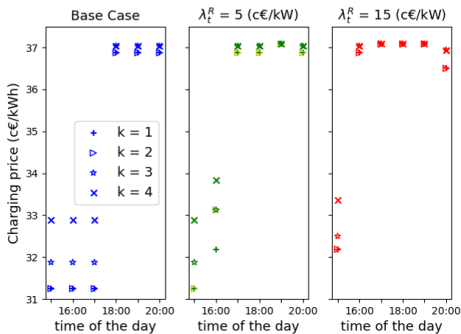
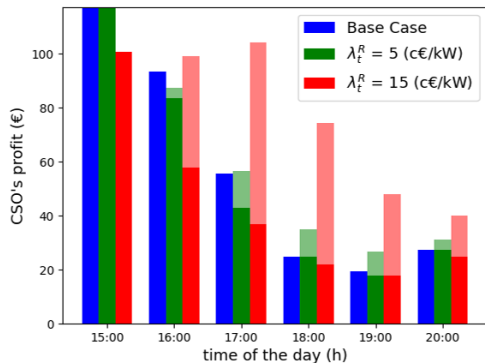


Figure: Hourly optimal charging price for different DR remuneration

Figure: Hourly agg. power at optimum for different DR remuneration

- Charging prices \nearrow & agg. power \searrow w.r.t. λ_t^R .
- Charging prices \nearrow before the DR period;

Demand response: CSO's profit



	Profit (€)	\nearrow (%)
<i>Base Case</i>	1323.45	
$\lambda_t^R = 5$	1339.49	+ 1.21
$\lambda_t^R = 15$	1451.86	+ 9.7

Table: Aggregate CSO's expected profit over the day

Figure: Hourly profit (lower part: profit for energy; upper part: revenue from DR)

- Aggregated profit over time \nearrow w.r.t. the DR remuneration λ_t^R
- Lower profit at 15:00 (before the DR period) for $\lambda_t^R = 15$ c€/kW

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Contributions

- Price menu design problem formulation at a public charging station that differentiates the options in the charging power rate;
- Analysis and computation of the optimal price menu for profit or social welfare maximization, with the provision of demand response to the electricity grid operator.

Perspectives

- Consider a bounded rationality model in the charging option choice by the EVs;
- Integrate robust optimization for the demand response;
- Take into account the limitation in the number of charging points.
- ...

Thank you for your attention

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References

- Lu, C., Wu, J., Cui, J., Xu, Y., Wu, C. & Gonzalez, M. C. (2023), 'Deadline differentiated dynamic ev charging price menu design', *IEEE Transactions on Smart Grid* **14**(1), 502–516.
- Samadi, P., Mohsenian-Rad, A.-H., Schober, R., Wong, V. W. & Jatskevich, J. (2010), Optimal real-time pricing algorithm based on utility maximization for smart grid, *in* 'IEEE International Conference on Smart Grid Communications', pp. 415–420.