

Parking duration optimization for electric vehicles in a loss queueing system

PGMO days 2022

Alix Dupont¹² Yezekael Hayel¹ Tania Jiménez¹ Cheng Wan²
Raphaël Payen² Olivier Beaude²

¹LIA (Laboratoire d'Informatique d'Avignon), Avignon university

²EDF (Electricité de France) lab, Paris-Saclay

November 30, 2022

Motivation

- Huge and quick development of electric mobility
- Increasing demand of parking spaces equipped with a charging point in the Charging Stations (CS)

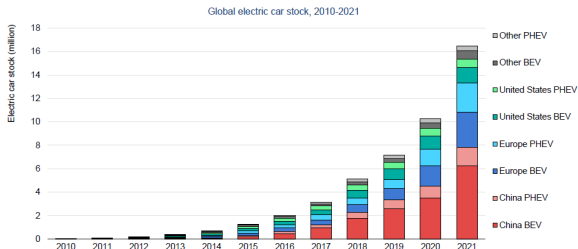


Figure: Source: Global EV Outlook 2022

- **Objectives:** Improving the Quality of Charging Service at public Charging Stations
- **Approach:** Mathematical modeling of Electric Vehicles (EV) arrivals/departures at a given site composed with one CS based on queueing theory [A.Aveklouris et al. (2019)]

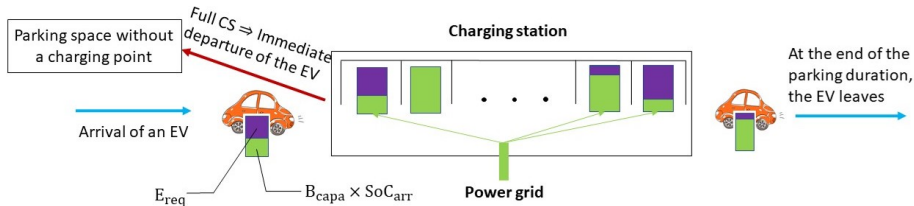
Table of Contents

- 1 Stochastic modeling using queueing theory
- 2 Optimal average parking duration
- 3 Numerical simulation
- 4 Conclusion

Table of Contents

- 1 Stochastic modeling using queueing theory
- 2 Optimal average parking duration
- 3 Numerical simulation
- 4 Conclusion

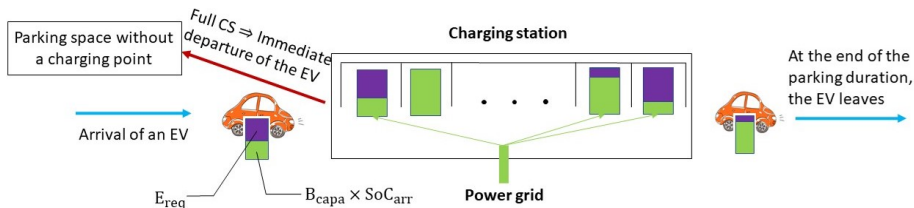
Stochastic model of EV arrivals/departures at a CS



Characteristics of Electric Vehicles

- Random arriving times: Poisson process of parameter λ
- Random need in energy E_{req} at arrival (exponential distribution with parameter μ)
 - B_{capa} : energy storage capacity
 - SoC_{arr} : % of B_{capa} at arrival
 - $E_{req} := B_{capa} \times (1 - SoC_{arr})$
- Random parking duration T_{park} ($\perp\!\!\!\perp E_{req}$) (exponential distribution with parameter ν)

Charging Station characteristics



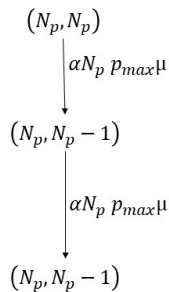
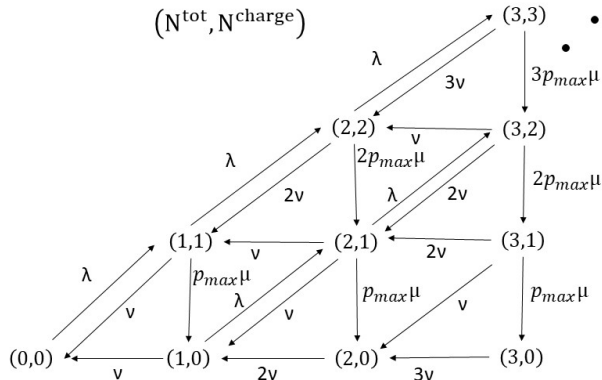
- N_p : nb of parking spots (all equipped with one Charging Point (CP))
- p_{max} : maximum power delivered for an EV
- α : proportion of the nb of CP that can deliver p_{max} simultaneously

Power scheduling: *smoothing*

- Each arriving EVs immediately starts to charge with power:

$$p(\text{nb_EV_charging}) = p_{max} \times \frac{\min(\alpha N_p, \text{nb_EV_charging})}{\text{nb_EV_charging}}$$

2D Markov chain



Notation

- E : random energy received
- A : event "the arriving EV is accepted"
- N_t^{charge} : number of charging EV at time t

- Energy received for a given EV:

$$\begin{aligned} E &= 1_A \times E + 1_{\bar{A}} \times 0 \\ &= 1_A \times \int_{t_{\text{start}}}^{t_{\text{end}}} p(N_t^{\text{charge}}) dt \end{aligned}$$

with

- $t_{\text{end}} = t_{\text{start}} + T_{\text{charge}} = t_{\text{start}} + \min(T_{\text{park}}, T_{\text{full}})$
- T_{full} the solution in t to $\int_{t_{\text{start}}}^t p(N_t^{\text{charge}}) dt = E_{\text{req}}$
- Difficult to obtain the exact expected energy received

Table of Contents

- 1 Stochastic modeling using queueing theory
- 2 Optimal average parking duration**
- 3 Numerical simulation
- 4 Conclusion

Motivation

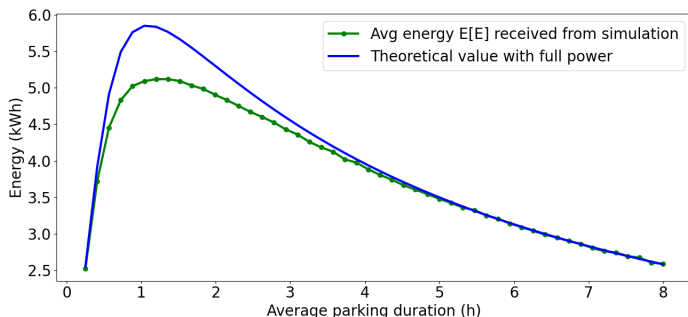
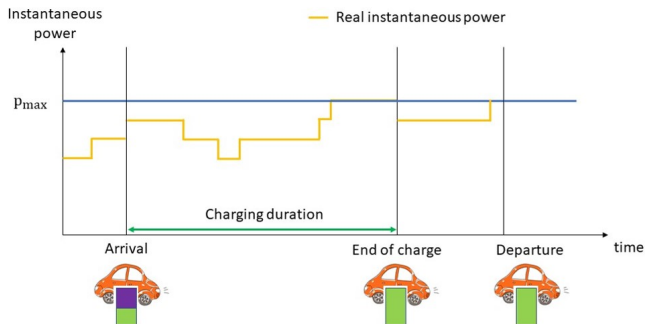


Figure: Expected energy received with respect to the average parking duration

- Model simulated using Simpy (discrete event simulation framework);
- Low avg parking duration \Rightarrow Little time for receiving energy;
- High avg parking duration \Rightarrow High blocking probability;
- Can we find the optimal avg parking duration ?

Approximation of the expected energy received per EV



$$E[E] \approx P(A) \times E[T_{charging}] \times P_{max}$$

- \tilde{E}_{num} : approximated expected energy received per EV with the exact $P(A)$ at steady states (*Erlang B formula*)
- \tilde{E}_{ana} : approximated expected energy received per EV with an approximation on $P(A)$

Notation

- \bar{e}_{req} : avg need in energy
- λ : avg frequency of arrival
- \bar{t}_{park} : avg parking duration

$$(P_1) \quad \min_{\bar{t}_{\text{park}} > 0} \tilde{E}_{\text{num}} = \left(1 - \frac{\lambda^{N_p} (\bar{t}_{\text{park}})^{N_p}}{N_p!}\right) \times \frac{\rho_{\text{max}} \bar{e}_{\text{req}} \bar{t}_{\text{park}}}{\bar{e}_{\text{req}} + \rho_{\text{max}} \bar{t}_{\text{park}}}$$

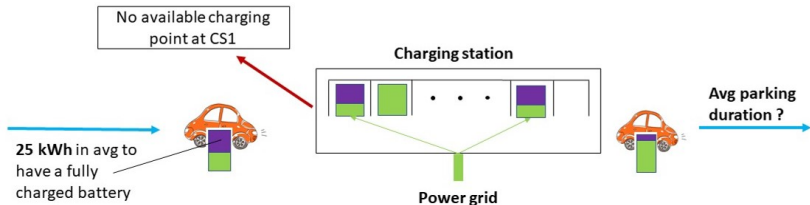
Prop.: unique solution $\bar{t}_{\text{park}}^{\text{opt,num}}$ to (P_1) , but no close-form expression

$$(P_2) \quad \min_{\bar{t}_{\text{park}} > 0} \tilde{E}_{\text{ana}} = \min\left(1, \frac{N_p}{\lambda \bar{t}_{\text{park}}}\right) \times \frac{\rho_{\text{max}} \bar{e}_{\text{req}} \bar{t}_{\text{park}}}{\bar{e}_{\text{req}} + \rho_{\text{max}} \bar{t}_{\text{park}}}$$

Prop.: $\bar{t}_{\text{park}}^{\text{opt,ana}} = \frac{N_p}{\lambda}$ is the unique solution to (P_2)

Table of Contents

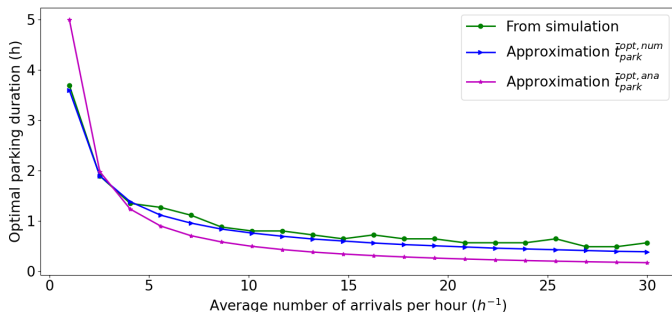
- 1 Stochastic modeling using queueing theory
- 2 Optimal average parking duration
- 3 Numerical simulation**
- 4 Conclusion



Charging station characteristics

- Power sizing coefficient $\alpha = 0.6$
- Maximum power per charging point $p_{\max} = 11 \text{ kW}$

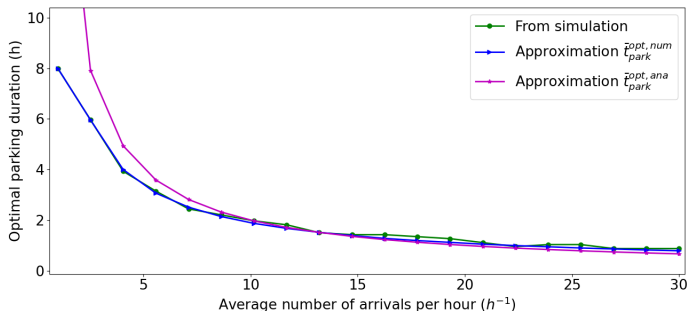
Sensitivity analysis of the parameters



5 parking spaces

- Decreasing optimal parking duration wrt λ
- For $\bar{t}_{park}^{opt,num}$: more accurate (Error $\approx 5\%$)
- For $\bar{t}_{park}^{opt,ana}$: less accurate (Error $\approx 50\%$)

Sensitivity analysis of the parameters



20 parking spaces: more accurate approximations

- For $\bar{t}_{park}^{opt,num}$: error $\in [0\%, 10\%]$
- For $\bar{t}_{park}^{opt,ana}$: error $\in [0\%, 25\%]$

Table of Contents

- 1 Stochastic modeling using queueing theory
- 2 Optimal average parking duration
- 3 Numerical simulation
- 4 Conclusion**

Main ideas

- Analysis of the parking duration effect on the expected energy received per EV
- Optimal average parking duration: trade-off between
 - enough time to charge
 - not overuse a parking space

Perspectives

- Find a pricing mechanism to reduce parking duration when needed
- Deepen the analysis of the exact energy received per EV
- Introduce several types of charging modes (with \neq power capacities)

Thank you for your attention

- A.Aveklouris, M.Vlasiou & B.Zwart (2019), 'Bounds and limit theorems for a layered queueing model in electric vehicle charging', *Queueing Systems* **93**, 83–137.
- Feldman, R. M. & Valdez-Flores, C. (2010), *Applied Probability and Stochastic Processes*, Springer.

Model: Number of charging EVs

2 forms of congestion : on the power and on the limited number of parking spaces

Notation

\bar{z} = Average number of charging EVs in stationary regime.

Fixed point equation for \bar{z} given by [A.Aveklouris et al. (2019)]

$$\bar{z} = \min(\lambda, \nu N_p) \mathbb{E} \left[\min \left(D, \frac{B}{\rho_{\max}} \max \left(1, \frac{\bar{z}}{\alpha N_p} \right) \right) \right]. \quad (1)$$

Proposition [Avg nb of charging EVs]:

In stationary regime, the average number of charging EVs is:

$$\bar{z} = \begin{cases} \frac{\min(\lambda, \nu N_p)}{\nu + \rho_{\max} \mu} & \text{if } \frac{\nu}{\nu + \rho_{\max} \mu} \leq \alpha \text{ or } \frac{\lambda}{\nu + \rho_{\max} \mu} \leq \alpha N_p, \\ \frac{\min(\lambda, \nu N_p) - \rho_{\max} \mu \alpha N_p}{\nu} & \text{else.} \end{cases} \quad (2)$$

Idea of the proof : Resolution of the fixed point equation 1.

Model: Number of charging EVs

Markov process Z_t

- Z_t if a Markov process [Feldman & Valdez-Flores (2010)].
- Arrivals of Z_t : Poisson process of parameter λ .
- Departure process: Poisson process of parameter $\nu \times Z_t + \mu \times \min(Z_t, \alpha N_p)$.

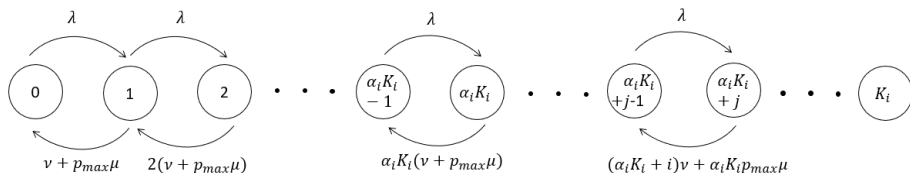


Figure: Transition rates of the Markov process Z_t