

# Stochastic Modeling of the Service Quality for Electric Vehicles in Competition at two Charging Stations

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## Executive Summary

With the rise of Electric Vehicles (EVs), the demand for parking spots equipped with plugging devices in Charging Stations (CSs) is tremendously increasing. To face this demand, a Charging Station Operator (CSO) deal with limited resources, both in terms of number of parking spaces equipped with a charging point (CP) as well as available power. This leads to congestion at the CSs and lower the Quality of Charging Service (QoS). This motivates us to study how to improve the QoS of EVs at the CSs. We study, for a mid-term analysis, the best ways a CSO responsible for two CSs divides power between them in terms of QoS. In order to realistically model the demand in power, we consider the decision-making problem of EV users of which CS is optimal in terms of QoS.

*Keywords: EV (Electric Vehicle), Optimization, Charging, User behaviour, Utility*

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## 1 Introduction

Government and political authorities have the objective that, by 2030, the EV fleet should account for 7% of the global vehicle fleet [1]. To facilitate the transition toward electric mobility, it is therefore essential to improve the QoS for EV users. Several definitions of the QoS can be used [2]. The limited resources, both in number of parking spaces and in amount of power, has to be shared between EV users. The stochastic nature of arrival/departure of EVs at public parking spaces equipped with a CP and the behavior of EV who consider the resource sharing can be mathematically modeled using queuing games [3], where rational behaviors are considered. A study of the QoS was made in [4] but did not consider the congestion in terms of free parking spaces in the decisions of EV users. In our study, we aim at studying how to improve the QoS at CSs at a shopping center by allocating power resources at the CSs. During the shopping time, EV users plug their EV at a parking space equipped with a CP and want to leave with a fully charged battery.

## 2 Model

### 2.1 Charging stations infrastructure

We model the arrival of EVs at a shopping center, composed with two CSs. At each CS, there is a limited number of parking spaces, each one of them equipped with one CP. The CSs have to deal with many power limitations, which can come from electrical contracts between the CSO and the electricity supplier, or can be a physical limitation of the transformers, cables or CPs. In our model, we consider simple power constraints. (i) The power arriving at the shopping center is limited, so that the two CSs have to share this limited power between them. This can happen if the two CSs share the same electricity contract or the same electricity delivery point. (ii) Moreover, each CS  $i$  cannot receive more than a certain maximal power  $P_M^i$ . (iii) Finally, at CS  $i$ , the power delivered by a single CP cannot exceed a maximum value  $p_{max}^i$ . This latter value can be different for each CS, but is the same for each CPs at a CS.

### 2.2 Power scheduling: processor sharing

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Since each CS must deal with a limited power, a CSO has to decide how to share the power between the EVs at a CS. The power scheduling policy considered in our model is called processor sharing, or sometimes referred as smoothing, which is already used by some CSOs. All arriving EVs immediately start to charge (hence there is no waiting time), and all charging EVs charge at the same level of power  $p^i$  given by Equation (1). At each CS, when the number of charging EVs  $N_{CEV}^i$  is small enough, all EVs benefit with the maximum power available at the CPs. Otherwise, they equally share the available power at the CSs. At CS  $i$ , denoting  $P_{available}^i (\leq P_M^i)$  the available power, the power given to each EV is:

$$p^i(N_{CEV}^i) = \frac{\min(P_{available}^i, p_{max}^i \times N_{CEV}^i)}{N_{CEV}^i}. \quad (1)$$

Note that the maximum number  $\frac{P_T^i}{p_{max}^i}$  of EVs that can simultaneously recharge with the maximum power  $p_{max}^i$  depends on the available power of the CS.

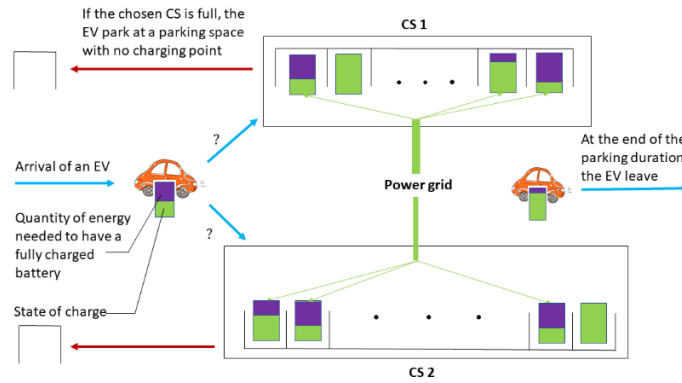


Figure 1: Schematic diagram of the model.

### 2.3 Modeling of arriving EVs

From the point of view of a CSO, it is possible to anticipate the frequency of arrivals of EVs, but the arrival time of EVs is unknown. Therefore, we suppose that they are random, according to a Poisson process, which has the property that the time between two arrivals is independent. This hypothesis is commonly used in queuing games, see [3] [5] [7]. The batteries capacity ( $Capa$ ) of EVs is the total amount of energy that can be stored, and the State of Charge (SoC) is the amount of energy already stored in the battery when arriving at a CS. The latter quantity depends on the distance travelled since the last recharge, and on some exogenous parameters like ambient temperature and traffic conditions. These two quantities determine the amount of energy  $E = Capa - SoC$  for a full recharge. In our model, for each EV, this quantity is random and follow an exponential distribution. We suppose that the shopping duration of EV users is random according to an exponential distribution<sup>1</sup> as well and is not influenced by the SoC of their EV's battery, see [7].

### 2.4 Decision of EV users

The arriving EV users at the shopping center (and which aim to recharge their battery) choose in which CS there are going to park and recharge, cf Figure 1. We suppose here that the criterion of satisfaction is the same for all EV users and is the probability to leave with a fully charged battery, which given by Equation 2:

$$QoS = \mathbb{P}(CS \text{ not full}) \times \mathbb{P}(charging \text{ duration} \leq parking \text{ duration}). \quad (2)$$

EV users choose the CS which gives them the highest QoS with perfect rationality<sup>2</sup>. This choice must consider two forms of congestion: (i) The first one is the congestion in terms of the number of parking spaces. Indeed, each CS having a limited number of parking spaces, an EV can face a full CS. For simplification of the model, it is supposed that a rejected EV at the chosen CS will not go to the other CS and will simply park at a parking space with no CP, cf Figure 1. In this case, the EV will not leave with a fully charged battery. (ii) The other congestion is in terms of power. Within a CS, beyond a certain number of charging EVs, they have to share the available power equally between them, according to Equation (1). Hence, the more charging EVs there are simultaneously, the less power will be delivered for each one of them. These two forms of congestion imply that the decision of an EV user depends on

<sup>1</sup> The energy  $E$  and the parking duration is different for all arriving EV but follow the same probability distribution.

<sup>2</sup> The QoS is the only decision variable considered for the choice of the CS by the EV users.

the decision of the other EVs. Because of the very large number of arriving EVs, we suppose mathematically that infinitely many EVs arrive at the CSs. Therefore, only the decision of a non negligible proportion of EV can have an impact on the state of the system, i.e. the number of charging and fully charged EVs at each CSs, and then change the decision of other EV users.

## 2.5 Comparison between centralized and decentralized systems

In the decentralized system, all EV users individually choose the best CS in order to maximize their QoS, without considering the externalities of their actions. This can lead to equilibria, which is a notion commonly used in queueing games, and more generally in game theory. An equilibrium is a profile of strategies used by the EV users such that no one has an incentive to change their behavior. In our case, a strategy is the choice of a CS. In the centralized system, the CSO decides the proportion of EVs going to each CS, in order to maximize the common QoS, which leads to the social optimality. A mathematical metric to compare the performance of these two points of views is the price of anarchy [6].

## 3 Results

### 3.1 Theoretical results

It can be shown that there exists a unique repartition of EVs at each CS which leads to an equilibrium. This important result implies that it is possible to anticipate the “natural” repartition of EVs between the two CSs. In particular, it enables to create some incentives toward a more socially optimal repartition, if it exists. Moreover, the computation of the probability to leave with a fully charged battery was done, based on [7]. Finally, the queueing game studied here belong to the family of games called potential game<sup>3</sup> [8].

### 3.2 Numerical analysis

This section gives numerical illustrations corresponding to a particular use case. Table 1 gives the resource limitations at each CS in terms of the number of parking spaces and available power. In addition, the CSO responsible for the two CSs have control on the allocation of power to each CS, such that the total power doesn't exceed a total capacity of 250kW between the two CSs. This corresponds to a mid-term sizing decision, which cannot be change at short-term. Note that 3 over 10 CPs can deliver simultaneously maximum power for CS 1, and 15 over 30 for CS 2, which is lower than the typical sizing decisions of CSOs. There are fewer CPs at CS 1 (than at CS 2), but they allow at maximum capacity a faster charging duration for the EVs. Table 2 gives the parameter values of the characteristics of the EVs.

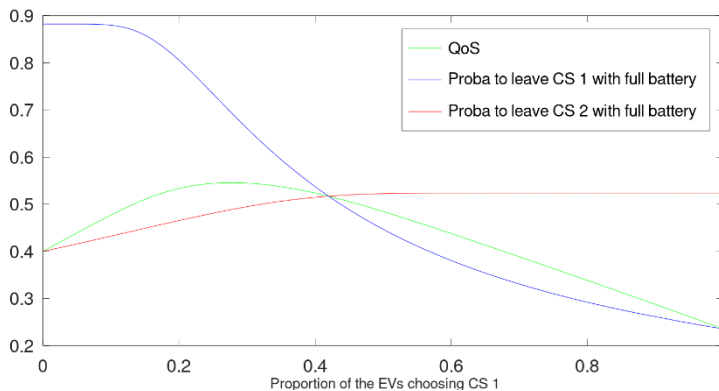


Figure 2: There are respectively  $P_{available}^1 = 85kW$  and  $P_{available}^2 = 165kW$  at CS 1 and CS 2, so that both the coupling power constraint of 250kW is and the maximum power  $P_M^i$  at each CS are met. At optimum in the centralized system (maximum of the green curve), about 25% of the EV users go to CS 1, whereas at the equilibrium in the decentralized system (intersection of the blue and the red curves) about 40% go to CS 1. The centralized system is more optimal than the decentralized system because at equilibrium EVs overuse CS 1 as the power available for each EV at CS 1 is higher than at CS 2.

Figure 2 shows that the social optimum can be strictly more efficient than the equilibrium in the decentralized system. When EVs individually choose the best CS in terms of QoS, more EVs go to CS 1 than it is at social optimality. Figure 2 highlights the fact that selfish behaviors can lead to inefficiency for the common good. However, the centralized system can be considered as unfair because at social optimum, the EVs which choose CS 2 receives a lower QoS.

Table 1: Parameter values characterizing the two CSs

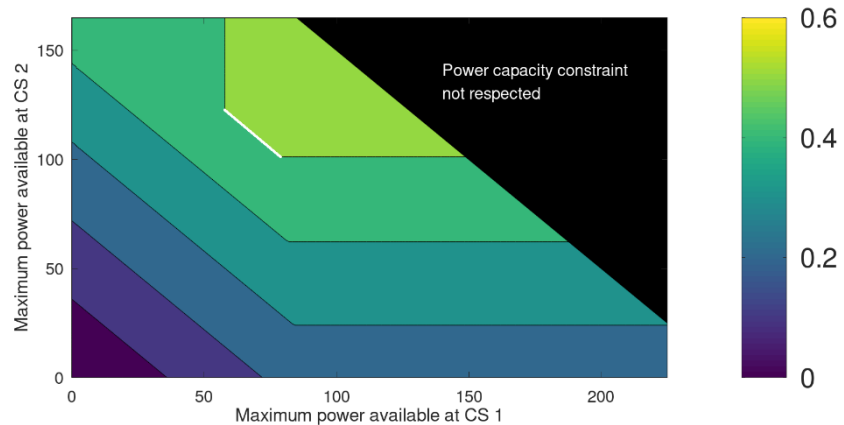
	CS 1	CS 2
Number of parking spaces	10	30
Maximum power $p_{max}^i$ at each CPs	75kW	11kW
Maximum power $P_M^i$	225kW	165kW

<sup>3</sup> This property enables to find the equilibria by maximizing a differentiable function, called the potential function.

Table 2: Parameter values characterizing the EVs
The EVs arrive in average about every 3 minutes
The EVs need in average 20kWh of energy to have a fully charged battery
The EVs stay parked in average 2h at the CSs

Figure 3: QoS at equilibrium of the decentralized system as a function of the available power given at each CS. The yellow area represents the optimal ways of sharing power between the two CSs in terms of QoS. The white line represents the economically optimal way to share the power. At the white line, since less power is required, the investment cost will be smaller.

Figure 3 gives, in the decentralized system, the optimal ways of sharing power between the EVs in terms of QoS. It shows that a relatively fair power sharing between the two CSs gives a higher QoS for the EVs. Also, centralizing the power to one CS is not efficient because it will cause more congestion at this CS.



### 3.3 Perspectives

For future works, it would be more realistic to consider a redirection of the EV users to the other CS in case the chosen CS is full. Moreover, other metrics for the QoS can be studied, such as the expected amount of energy stored during the charging process.

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## Presenter Biography



**Alix Dupont** received an engineering degree from ENSTA-Paris and a M.Sc. degree from Paris-Saclay University in 2021, both in applied mathematics. He is currently pursuing a PhD thesis in applied mathematics with EDF Lab R&D and Avignon University. His research includes queueing systems and game theory applied to electric mobility.