Optimal Pricing for Electric Vehicle Parking Duration under Uncertainty

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***Abstract*—The growing adoption of Electric Vehicles (EVs) has led to a rise in demand for public Charging Points (CPs), resulting in congestion in terms of availability. This study suggests implementing a pricing strategy with the objective of optimizing the utilization of CPs, in order to tackle the aforementioned issue. The policy entails assessing a fixed overstay fee for the EV users whose parking duration exceeds a pre-set threshold, thereby encouraging the EV users with a lower parking duration to charge which enables to free up CPs for other EV users. The study incorporates the uncertainty of parking duration upon arrival into the decision to use a CP, applying the expected utility hypothesis. The expected profit is derived using a multi-class queueing model, which captures the heterogeneity in the charging need and parking duration estimated by the EV users. Numerical analysis is conducted to determine the optimal pricing values, enabling a Charging Point Operator (CPO) to maximize the expected profit. The results show, on a realistic use-case, that an overstay fee improves the CPO's expected profit by over 10% compared to an simple admission fee, that is when all the EV users joining the CS pay a fee, both optimally chosen by the CPO.**

***Index Terms*—Charging station, electric vehicle, queueing theory, charging behavior, pricing**

I. Introduction

The exponentially increasing penetration of Electric Vehicles worldwide creates congestion in terms of availability at the public Charging Points (CPs) [1]. This requires reducing the parking duration of the EVs at a parking space equipped with a CP, thereby allowing a greater number of EV users to share the existing CPs. Moreover, increasing the availability of the CPs can alleviate the well-known range anxiety of EV users, i.e. the fear of running out of energy [2]. In order to assess the impact of pricing into EV users' charging decision, considering a utility function for EV users quality of charging service allows to better understand how they make and quantify their choices, i.e. whether to charge or not, and which type of CP. In [3], a web-based survey is conducted in order to capture EV users charging mode choices, i.e. fast or slow charging, as well as their sensitivity to the charging price, among other factors. In contrast to [3], the authors in [4] consider a nonlinear utility function to model the charging behavior and their travel habits, in particular with respect to the State of Charge (SoC) upon arrival. However, the issue of CP availability is not discussed. In [5], the Charging Station (CS) optimal pricing is investigated in a competitive environment with two Charging Point Operators (CPOs). EV users choices consider many factors such as the pricing, i.e. a parking admission and energy consumption fee, or the convenience to find an available CP. In [6], a CS manager suggests several charging offers in order to customize EV users charging demand and to lower their parking duration in order to free up the CPs. However, in this model all factors such as the parking duration is supposed known by EV users before making their choices, which is not necessarily the case in practice.

Queueing theory can be used in order to capture the stochastic nature of arrival and departure times of EV users. For example, in [7] a queueing game analysis in which EV users chose between two CSs at a given site is presented. In [8], a M/G/s queue is used to develop a pricing policy in order to lower the EV charging demand during peak hours. The CS sizing problem is investigated in [9] using a M/M/s/N queue, but the charging decision only depends on the SoC upon arrival, and no pricing policy is proposed. In [10], arriving EV users choose between two charging modes, and a M/M/N queue is formed for each of these modes. The probabilistic choice of each charging mode is assumed to be linearly decreasing with respect to the service fee, and does not depend on any other factor.

In our work, the problem of CP availability is investigated by considering a pricing policy on the parking duration. The main contributions are summarized as follows: (i) a charging behavior model at a public CS is proposed, with unknown parking duration and heterogeneity in the charging demand; (ii) The optimal parking pricing policy in terms of the CPO's expected profit is investigated in a multi-class M/G/N/N queueing framework.

The rest of the paper is organized as follows. In Section II, the model and scenario of EV users arrival and behavior is described. In Section III, the main metrics such as the CPO's expected profit are studied analytically. In Section IV, a use-case with realistic parameter values is considered and the associated numerical results are presented and discussed. Finally, perspectives are given in conclusive Section V.

II. Model description

A. Scenario with EV users heterogeneity

Upon arriving at a given commercial site equipped with a CS, the EV users decide whether or not they want to join the CS, taking into consideration factors such as their SoC at arrival, their estimated parking duration, and the pricing. The willingness to charge is made without knowledge of the CS occupancy. Considering this, the EV users who decide not to join the CS are called the *balking EV users*. Some of the EV users willing to charge might be blocked because all the CPs at the CS are currently in use. These EV users are called the *blocked EV users*. In this case, and similarly for the balking EV users, these EV users go to a parking space without a CP. The other EV users, i.e. the ones willing to charge and who find an available CP, are called the *joining EV users*. The proportion of balking, blocked, and joining EV users are respectively denoted α^{balk}, α^{block}, and α^{join}. The scenario is illustrated in Fig. 1 These proportions are formally determined in the following section III, depending on the model's parameters.

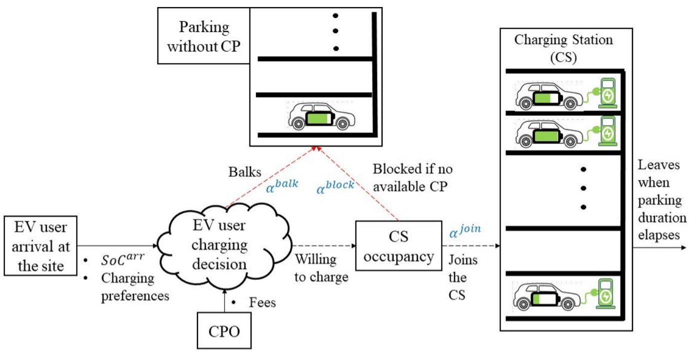

Fig. 1: Schematic diagram of the model.

1) Stochastic model for parking duration: The arrival times of the EV users at the site follow a Poisson process with rate λ. EV users are divided into I classes, as in [8]. The classes differ by their parking duration distribution. For each EV user in class i, his/her parking duration is independently determined from other EV users and follows a random variable with a continuous Probability Density Function (PDF) $f_i^{park}(.)$ with mean $\bar{D}_i$. Upon arrival at the CS, EV users do not know their parking duration, but they are aware of their PDF $f_i^{park}(.)$. An arriving EV user belongs to class $i \in \{1, ..., I\}$ with a probability θ_i. Given this, the arrival times of the class-i EV users follow a Poisson process with parameter $\lambda_i := \theta_i \lambda$, and the mixed PDF $f^{park}(.)$ of an EV user parking duration is then $f^{park}(.) = \sum_{i=1}^{N} \theta_i f_i^{park}(.)$, with mean $\bar{D} = \sum_{i=1}^{N} \theta_i \bar{D}_i$.

2) Energy demand distribution: The SoC at the time of arrival, denoted $S^{arr} \in [0,1]$, is continuously distributed among the population of EVs, with a cumulative density function $F_{S^{arr}}(.)$. Unlike the parking duration, EV users are aware of their SoC at arrival. This value mainly depends on the distance traveled in order to reach the site since the last charging session, and many other exogeneous parameters [11]. EV users leave the CS only when their parking duration is elapsed, independently of the current SoC, as the primary purpose of their visit to the site is for other activities such as dining, shopping, etc.

B. CS description

1) Charging Power: The CS is composed with N_{cp} parking spaces, each one of them equipped with a single CP offering a maximum nominal charging power $p^{\max}$. This power depends on the physical characteristics of the chargers, as explained in [12]. Due to battery constraints, when the SoC S reaches a transition SoC $\hat{S}^{trs} \in]0,1[$, the charging power $p(S)$ drawn by each EV reduces, which is formalized by the following two-step function as in [13]:

$$p(S) = \begin{cases} p^{\max} & \text{if } S \leq \hat{S}^{trs}, \\ \frac{1-S}{1-\hat{S}^{trs}}\, p^{\max} & \text{if } S > \hat{S}^{trs}. \end{cases} \quad (1)$$

2) Parking duration pricing policy: Upon leaving the CS, EV users pay a fee based on their parking duration. The *pricing policy* $C(.)$ is the following piece-wise constant function with two pieces with respect to the parking duration:

$$C(d) = a\, \mathbb{1}_{d \geq d_{thr}},$$

where a and d_{thr} are non-negative parameters which fully determine the pricing policy. This type of pricing is nowadays more and more deployed in public CPs, see for example [14]. The parking duration of an EV user is exogeneous: it depends only on his/her activities at the site. This is a reasonable assumption because the parking demand is very inelastic [15]. This form of pricing is expected to motivate the EV users with a short estimated parking duration to join the CS.

C. Charging behavior under parking duration uncertainty

The perceived satisfaction $R(S)$ associated with the *current* SoC S is a quadratic function, as in [16]:

$$\forall S \in [0,1],\ R(S) = \omega\, B^{capa}\, (S - \frac{1}{2} S^2), \quad (2)$$

where B^{capa} is the battery storage capacity of the EVs, and ω is a weight coefficient (in €/kWh) that describes the value given to the energy received. The marginal satisfaction given to the energy received decreases linearly in the current SoC value. For a given SoC at arrival S^{arr} and parking duration d, the utility function $U_i(S^{arr}, d)$ for any class-i EV users depends on the the variation in SoC $\Delta^S(S^{arr}, d)$ due to the energy received during the parking duration, and the pricing policy $C(d)$ as follows:

$$U_i(S^{arr}, d) = R(S^{arr} + \Delta^S(S^{arr}, d)) - R(S^{arr}) - C(d),$$

where $R(S^{arr} + \Delta^S(S^{arr}, d)) - R(S^{arr})$ represents the perceived benefit of increasing the SoC from S^{arr} to $S^{arr} +$

$\Delta^S(S^{arr}, d)$. The formula of $\Delta^S(S^{arr}, d)$ can be obtained using the linear differential equation (1), and is given in [13]. EV users' utility function is random, due to the uncertainty of their parking duration d at the time they arrive. For this reason, any class-i EV user consider his/her *expected utility* $\bar{U}_i(S^{arr})$ with respect to the parking duration distribution. In particular, a class-i EV user joins the CS if $\bar{U}_i(S^{arr}) > 0$, with:

$$\bar{U}_i(S^{arr}) = \int_0^{+\infty} R(S^{arr} + \Delta^S(S^{arr}, x))\, f_i^{park}(x)\,\mathrm{dx} - R(S^{arr}) - \bar{C}_i, \quad (3)$$

where $\bar{C}_i = a\,(1 - \int_0^{d_{thr}} f_i^{park}(x)\mathrm{dx})$ is the expected parking cost for the class-i EV users. Given this, the characteristics of EV users joining the CS are analyzed in next Section III.

III. Charging behavior analysis and CS profit

In this section, the influence of the SoC at arrival S^{arr} and the pricing policy parameters (a, d_{thr}) on the charging decision of each class of EV users is investigated.

Proposition III.1. *There exists a unique SoC threshold S_i^{thr}, defined by:*

$$S_i^{thr} = \begin{cases} \textit{Unique solution of } \bar{U}_i(S^{arr}) = 0 & \textit{if } \bar{U}_i(0) > \bar{C}_i, \\ 0 & \textit{otherwise,} \end{cases} \quad (4)$$

and such that any class-i EV user is willing to join the CS iff $S^{arr} < S_i^{thr}$.

Up to a SoC S_i^{thr} at arrival, any class i EV user needs enough energy so that they are willing to pay the expected parking duration cost in order to get charged. Otherwise, they prefer to balk from the CS. Given this, $F_{S^{arr}}(S_i^{thr})$ corresponds to the proportion of EV users willing to charge, i.e. willing to pay the expected parking fee in order to get charged, among class-i EV users. By applying the *implicit function theorem*, it is straightforward to shown that, for each class i, the SoC threshold S_i^{thr} strictly increases with respect to the expected cost $\bar{C}_i$, for $\bar{C}_i \in]0, \bar{U}_i(0)[$. In other words, the proportion of balking EV users increases when the parking fee a increases, or when the parking duration limit d_{thr} decreases.

By *Bayes formula*, the probability θ_i^{join} that a parked EV at the CS is from class i is:

$$\theta_i^{join} = \frac{\theta_i\, F_{S^{arr}}(S_i^{thr})}{\sum_{i=1}^{I} \theta_i\, F_{S^{arr}}(S_i^{thr})}. \quad (5)$$

It is expected that in average the parking duration at the CS is lower than among the whole population of EV users at the site, because the pricing discourage EV users with a high expected parking duration to park at the CS.

Proposition III.2. *The proportion α^{block} (resp. α^{join}) of blocked (resp. joining) EV users are:*

$$\alpha^{block} = (\textstyle\sum_{i=1}^{I} \theta_i\, F_{S^{arr}}(S_i^{thr})) \times \frac{\frac{\rho^{N_{cp}}}{N_{cp}!}}{\sum_{j=0}^{N_{cp}} \frac{\rho^j}{j!}},$$

$$\alpha^{join} = \textstyle\sum_{i=1}^{I} \theta_i\, F_{S^{arr}}(S_i^{thr})) \times (1 - \frac{\frac{\rho^{N_{cp}}}{N_{cp}!}}{\sum_{j=0}^{N_{cp}} \frac{\rho^j}{j!}}),$$

with $\rho = \left(\lambda\,(\sum_{i=1}^{I} \theta_i\, F_{S^{arr}}(S_i^{thr}))\right) \times \left(\sum_{i=1}^{I} \theta_i^{join}\, \bar{D}_i\right)$ the CS utilization rate.

Based on Proposition III.2, the proportion α^{balk} of balking EV users is:

$$\alpha^{balk} = 1 - \alpha^{block} - \alpha^{join} = 1 - \sum_{i=1}^{I} \theta_i\, F_{S^{arr}}(S_i^{thr}).$$

The expected profit per time unit Γ of the CPO is defined as the expected rate of EV users joining the CS times the expected cost per EV user, i.e.:

$$\Gamma = \alpha^{join} \times \lambda \times \sum_{i=1}^{I} \theta_i^{join}\, \bar{C}_i, \quad (6)$$

where θ_i^{join} and α^{join} and respectively given by (5) and Proposition III.2. The pricing policy parameters (a, d_{thr}) can then be adjusted in order to optimize the expected profit of the CPO. This optimization problem is investigated numerically in next Section IV.

IV. Numerical illustrations and discussion

The main metrics of the model described are numerically illustrated in this section, and the optimal pricing is investigated.

A. Use case description at a commercial site

In this illustrative use-case scenario, EV users are divided into $N_c = 5$ classes equally distributed, i.e. $\theta_i = \frac{1}{5}$ for all $i \in \{1, \ldots, 5\}$. The random parking duration of the different classes follows a gamma distribution respectively among the classes with means 0.5, 1, 2, 3, 5 hours, and standard deviation $\frac{1}{6}$, $\frac{1}{3}$, $\frac{2}{3}$, 1, and $\frac{5}{3}$ hours: the parking duration uncertainty proportionally increases with the mean parking duration, as stated in [6]. The SoC at arrival is distributed among the whole population of EV at the site, i.e. all classes together, according to a beta distribution, as in [8] and [17], with mean 0.5 and standard deviation 0.2. The other parameters of the system are the same for all EV classes and are shown in Table I.

TABLE I: Parameters description and values. *Starting from an empty battery, it takes about 3.5 hours to reach a SoC of $\hat{S}^{trs} = 0.8$, after which the power starts to decrease.*

Definition	Notation	Value
Exp. frequency of EV arrivals (h^{-1})	λ	12
Value given to energy (€/kWh)	ω	0.2
Battery storage capacity (kWh)	B^{capa}	50
SoC power transition	$\hat{S}^{trs}$	0.8
Static max. charging power of the CPs (kW)	p^{max}	11
Number of CPs	N_{cp}	10

B. Sensitivity analysis of the EV users willingness to charge: a trade-off between the energy received and the parking cost

Fig. 2 shows the expected utility with respect to the SoC at arrival. As stated in the proof of Proposition III.1 in Section VI, the incentive to charge decreases with the SoC at arrival, as the expected perceived benefit of using a CP decreases.

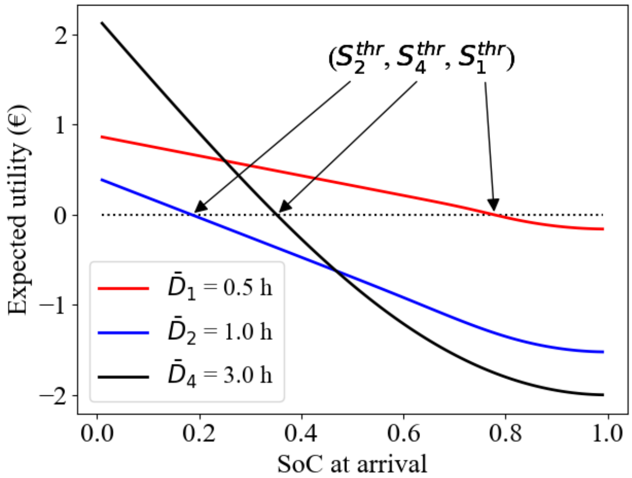

Fig. 2: Expected utility, defined in (3), for different classes, for a pricing policy $a = 3$€ and d_{thr}=45min. *EV users of class 1 (resp. 2 and 4) decide to charge if their SoC at arrival is lower than 78% (resp. 20% and 37%). Given that the parking duration has a positive impact on energy received but a negative one regarding the probability to pay the parking fee, the relation between S_i^{thr} and $\bar{D}_i$ is not trivial, here not monotonic.*

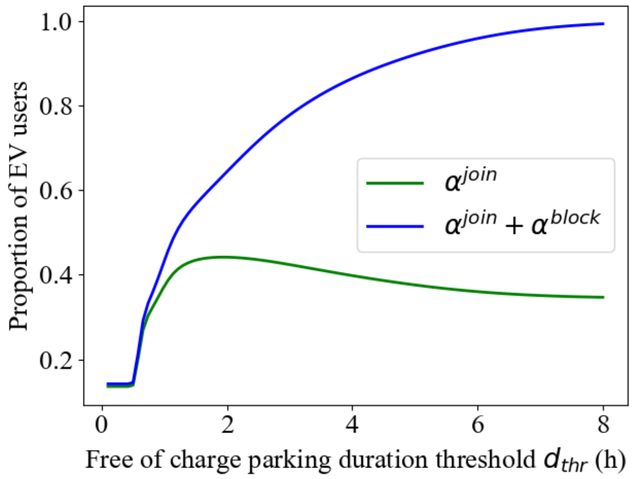

Fig. 3: Proportion of EV users willing to charge and joining, see Proposition III.2, with respect to the parking duration limit d_{thr}, for a parking fee $a = 3$€. *The proportion of blocked EV users, i.e. the difference between the blue curve and the green curve, increases with d_{thr} because it creates more congestion at the CS. As a consequence, the proportion of EV users actually joining the CS can decrease in d_{thr}.*

The decision to charge can be analyzed by two factors: (i) The EV users willingness to pay in order to get charged, which depends on the fee a, the energy received, and the values accorded to the energy parameterized by ω and; (ii) the probability to pay the fee a, which depends on the EV users parking duration, and on the parking duration limit d_{thr}. Fig. 3 shows the influence of d_{thr} on the proportion $\alpha^{join} + \alpha^{block}$ of EV users willing to charge, and the proportion α^{join} of EV users actually joining the CS. When $d_{thr} \gtrsim 30$min, some class-1 EV users are willing to charge because the probability that they pay is significantly decreased.

C. CPO's expected profit

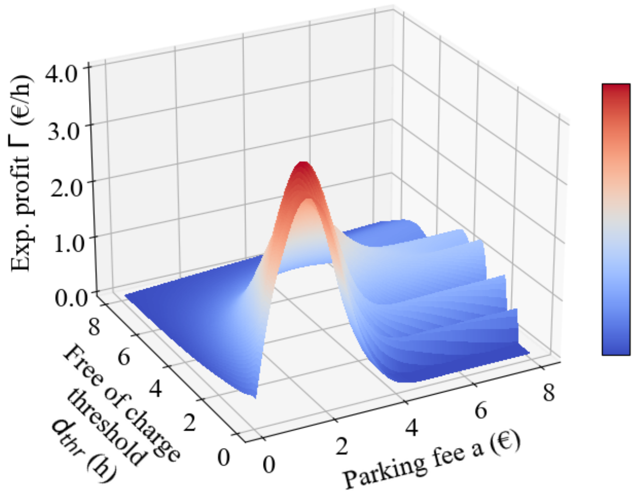

Fig. 4: CPO's expected profit Γ with respect to the pricing. *When the parking fee a is high, the profit increases each time d_{thr} approaches the mean arriving time of a new class of EV users as some of this class are willing to charge.*

Fig. 4 displays the CPO's expected profit per hour with respect to the pricing (a, d_{thr}). The influence of d_{thr} on the CPO's expected profit is coupled with a. When a is low, a high proportion of EV users is willing to charge, although they may pay the fee a. On the opposite, when a is high, the EV users want to join the CS only when they are sure to have a sufficiently low parking duration so that they will not pay the fee a. Although not optimal in terms of profit, choosing a high value for a can significantly lower the congestion at the CS because only the EV users with a short estimated parking duration compared to d_{thr} will join the CS.

A CPO may aim at adjusting the pricing policy parameters (a, d_{thr}) in order to maximize its expected profit Γ, defined by Equation 6. In order to highlight the effectiveness of introducing the free of charge parking duration threshold, two maximization problems are considered:

$$\begin{aligned} \Gamma^{opt} &= \max_{d_{thr} \geq 0, a \geq 0} \Gamma, \\ \tilde{\Gamma}^{opt} &= \max_{d_{thr} = 0, a \geq 0} \Gamma, \end{aligned} \tag{7}$$

where Γ^{opt} (resp. $\tilde{\Gamma}^{opt}$) is the optimal expected profit with (resp. without) the parking duration threshold d_{thr}. The solutions of (7), denoted (a^{opt}, d_{thr}^{opt}) for Γ^{opt} and $\tilde{a}^{opt}$ for $\tilde{\Gamma}^{opt}$, are computed using the heuristic *simulated annealing* algorithm[1] [18] because the expected CS profit Γ, as seen in Fig. 4, is

[1]the algorithm is stopped after 500 iterations, and the temperature parameter is initialized to 1, with a discount factor at every iteration of 0.98.

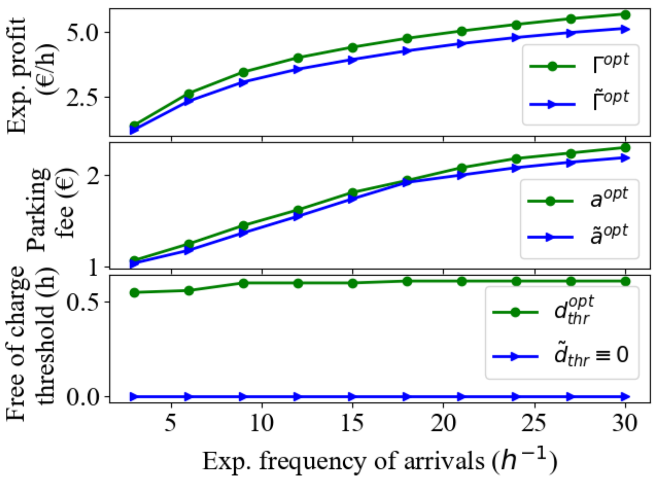

Fig. 5: Optimal pricing policy in (7). *The CPO should increase the parking fee when the demand grows. $d_{thr}^{opt} \approx 0.6$ is set to incentivize the class-1 EV users to join the CS because their low parking duration does not create much congestion.*

not convex and possesses many local optima. The optimal pricing and CPO's expected profit are shown with respect to the expected frequency of arrivals at the site, that is the potential demand, in Fig. 5. Introducing a parking duration threshold $d_{thr} > 0$ enables to increase the number of EV users willing join the CS while maintaining low congestion, and then increases CPO's expected profit Γ^{opt} by over 10% compared to $\tilde{\Gamma}^{opt}$.

V. Conclusion

In order to effectively manage a Charging Station (CS), this paper suggests a threshold-based parking duration pricing strategy. The impact of this pricing strategy on EV users' willingness to charge at the CS is analyzed. According to the proposed model, the EV users are willing to charge only if their SoC at arrival is low enough, as this allows them to receive more energy. Additionally, the pricing policy is shown to reduce the average parking duration among EV users who are willing to use a CP. This mechanism increases the availability of public charging points (CPs) and leads to an increase in the CPO's expected profit by over 10% compared to a fixed admission fee, which encourages further investigations of such pricing policies. For future research, the proposed model could be extended to include different types of CPs with different charging power capacities and pricing policies, while also integrating an energy-based pricing scheme.

VI. Appendix

Proof of Proposition III.1

Due to the strict convexity of the reward function (2), the expected utility $\bar{U}_i(S^{arr})$ of each class i is strictly decreasing with respect to the SoC at arrival S^{arr}, so that the solution of $\bar{U}_i(S^{arr}) = 0$ is unique. Moreover, $\bar{U}_i(S = 1) \leq 0$ because $\bar{C}_i \geq 0$, there exists a solution if $\bar{U}_i(0) > \bar{C}_i$. ∎

Proof of Proposition III.2

The probability that all the N_{cp} CPs are in use is known as the *Erlang loss formula* $\frac{\frac{\rho^{N_{cp}}}{N_{cp}!}}{\sum_{j=0}^{N_{cp}} \frac{\rho^j}{j!}}$. Due to the PASTA property [19], it is also the proportion of blocked EV users among the EV users willing to charge, i.e.: $\frac{\alpha^{block}}{\alpha^{block}+\alpha^{join}} = \frac{\frac{\rho^{N_{cp}}}{N_{cp}!}}{\sum_{j=0}^{N_{cp}} \frac{\rho^j}{j!}}$. Combined with the fact that the proportion of EV users willing to charge is: $\alpha^{block} + \alpha^{join} = \sum_{i=1}^{N} \theta_i F_{S^{arr}}(S_i^{thr})$, it is sufficient to determine the values of α^{block} and α^{join}, which concludes the proof. ∎

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