Universal scaling laws in turbulent Rayleigh-Bénard convection with and without roughness

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Experimental measurements of heat-flux and velocity in turbulent Rayleigh-Bénard convection are reported on a wide range of operating conditions, with either smooth or rough boundaries, in water or fluorocarbon, and aspect ratios 1 or 2. The heat-flux, expressed as the dimensionless parameter Nusselt number is found in agreement with Grossmann-Lohse prediction in smooth cells. The new measurements in the rough cell with fluorocarbon, which reaches Rayleigh numbers, Ra, up to 2.5×10^{12} at $Pr = 11$, the Nusselt number show a clear $Ra^{1/2}$ scaling, hinting a fully turbulent regime. The Nu *vs* Ra relationship is not unique, in that it depends on the cell geometry, plate roughness, Prandtl numbers, as well as details in the experimental setup. The Reynolds number essentially scales like $Re \propto Ra^{1/2}Pr^{-0.7}$ for all experiments, but show deviations with the various flow configurations. We find that the dimensionless heat-flux, expressed as $RaNu$, has a universal scaling with Reynolds number, collapsing all data, rough or smooth, even those in the literature where the Nu *vs* Ra scaling is in apparent contradiction.

Turbulent Rayleigh-Bénard Convection (RBC) is a model system where a layer of fluid is heated from below and cooled from above. It is controlled by three dimensionless parameters: the Rayleigh number Ra,

$$Ra = \frac{g\alpha\Delta T H^3}{\nu\kappa}, \tag{1}$$

the Prandtl number Pr,

$$Pr = \frac{\nu}{\kappa}, \tag{2}$$

and the aspect ratio $\Gamma = L/H$, where g is the acceleration of gravity, α the thermal expansion coefficient, ΔT the temperature difference across the fluid layer, ν the kinematic viscosity, κ the thermal diffusivity, H the height of the cell, and L the width of the cell. A long standing endeavour consists in finding universal scaling laws for the heat transfer, expressed as the dimensionless parameter Nusselt number Nu [1, 2],

$$Nu = \frac{\dot{q}H}{\lambda\Delta T}, \tag{3}$$

where $\dot{q}$ is the heat-flux, and λ the thermal conductivity. This would allow extrapolating results from laboratory experiments to natural systems where highly turbulent natural convection plays a role. These include in particular convection in the ocean at the pole or in subglacial lakes [3], resulting in Rayleigh numbers beyond 10^{14}, and to the "ultimate regime" of convection where $Nu \sim Ra^{1/2}$ (with logarithmic corrections) [4]. However, experimental heat-transfer measurements published in the literature for $Ra > 10^{11}$ seem to be contradictory, some showing a transition at $Ra \approx 10^{11}$ [5, 6], some showing a delayed transition at higher Ra [7], or no transition at all [8]. These experiments are within Oberbeck-Boussinesq conditions [9] and share similar geometries, but differ in Prandtl numbers and details in the setup. It now seems clear that the transition to the ultimate regime may be a subcritical process, and may depend on aspect ratio and Prandtl numbers [10–12].

In this letter, we evidence a way to recover universality. We report new velocity data obtained from shadowgraph imaging in parallelepiped Rayleigh-Bénard cells with or without roughness and using either deionised water or Fluorocarbon FC-770 as the working fluid. The cell dimensions are $41.5\,\mathrm{cm}\times41.5\,\mathrm{cm}\times10.5\,\mathrm{cm}$ for the aspect ratio 1 cell, and $20\,\mathrm{cm}\times41.5\,\mathrm{cm}\times10.5\,\mathrm{cm}$ for the aspect ratio 2 cell. The details of the cell are described in [13]. The Rayleigh number spans a wide range between 10^9 and 2.5×10^{12}, the Prandtl number between 4.3 and 7.0 in water, and 11 and 14 in FC-770, and the aspect ratio 1 or 2. In these conditions, the flow can be quite different, the scaling exponent of Nu *versus* Ra also quite different, and in apparent disagreement with data from Grenoble, Oregon and Göttingen cited above. And yet, all the data can be collapsed using the appropriate scaling of Reynolds and Prandtl numbers.

The cells can be set up with roughness on the bottom plate, consisting in square roughness elements of height $h_0 = 2\,\mathrm{mm}$, machined in the plate, as described in [14]. In that case, the cell (later referred to as "RS"), is asymmetrical, with a rough bottom plate, and a smooth top plate. As shown in our previous works [15, 16], this allows to define Rayleigh and Nusselt numbers of the rough and smooth halves. The smooth cell (later referred to as "SS") has entirely smooth boundaries, as described in [13]. We have now operated the rough configuration (RS) with Fluorocarbon FC-770 thus significantly extending the previous heat-transfer results in water [17], see Figure 1. In the Fluorocarbon cell, the thermal boundary layer, estimated as $\delta_{th} = H/(2Nu)$, ranges between 250 µm and 500 µm, much smaller than the roughness height h_0. The kinetic boundary layer, estimated as $\delta_v \approx \delta_{th}Pr^{1/3}$ ranges between 600 µm and 1100 µm,

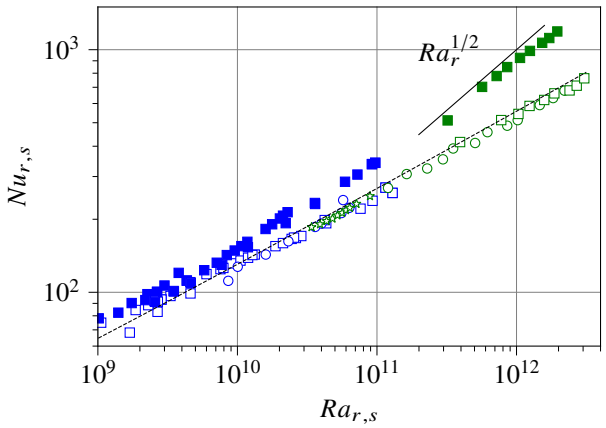

FIG. 1. Heat-transfer measurements. Blue: deionized water. Green: Fluorocarbon FC-770. Full squares: rough half-cell of RS cell. Open squares: smooth half-cell of RS cell. Circles: SS cell. Stars: SS cell with $\Gamma = 2$. RS cell in water from [17]. SS cells in FC-770 from [13]. SS cell in water and RS cell in FC-770: new data. Solid black line: $Ra^{1/2}$. Dashed black line: Grossmann-Lohse model [2]

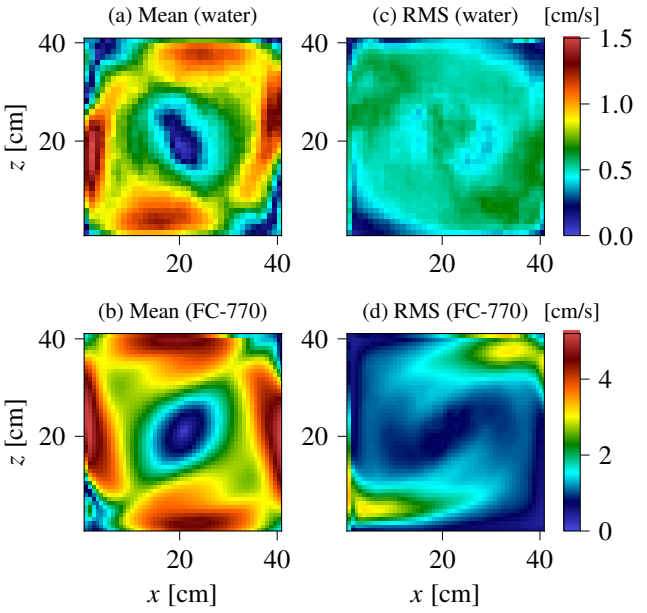

FIG. 2. Mean (a, b) and RMS (c, d) velocity maps computed from the Shadowgraph recording. (a, c) Water $Ra = 2.3 \times 10^{10}$, $Re = 5.4 \times 10^3$, $v_{\max} = 1.5\,\mathrm{cm/s}$; (b, d) FC-770 $Ra = 1.7 \times 10^{12}$, $Re = 3.3 \times 10^4$, $v_{\max} = 5.2\,\mathrm{cm/s}$.

smaller than the roughness height h_0. Therefore, the rough cell lies in Regime III as defined by Xie & Xia [18]. While the rough half-cell has a $Ra^{1/2}$ scaling, the smooth half-cell is in good agreement, over more than 3 decades of Rayleigh numbers, with heat-transfer measurements in SS cell, as well as with the Grossmann-Lohse model [2]. This shows that the top and bottom plates remain independant, even very far from the roughness-triggered enhancement threshold, and holds even when the Nusselt number of the bottom half is nearly twice as large as that of the top half. This is a major result that shows how the boundary layers adapt themselves to maintain the heat-flux constant.

We computed the velocity fields using *Correlation Image Velocimetry* on shadowgraph recordings for all of them, yielding consistent estimates for both the mean and fluctuation velocities, as well as consistent Reynolds number estimates in all configurations. The method has previously been validated against *Particle Image Velocimetry* in experimental conditions where both were possible [13]. While we find that the mean flow has the same structure at $Ra = 10^9$ and $Ra = 10^{12}$, the velocity fluctuations differ, see Figure 2. At higher Rayleigh numbers, and in this range of Prandtl where we do not see evidence for a transition to the ultimate regime, we find that the velocity fluctuations are globally much larger, and also more focused near the impinging jets than in the lower Rayleigh number case.

When roughness is added to the bottom plate, mean velocity is increased between 5 % and 20 % in the range of parameters explored in this work, but the overall structure of the mean flow is not significantly changed, see Figure 3. We focus on the velocity of the upwelling and downwelling jets to allow comparison with the literature where velocity is estimated with pairs of thermometers at mid-height [19–21]. In symmetrical cells, we average the velocity of the downwelling and upwelling jets. In RS cells, we associate the downwelling jet with the top plate, and the upwelling jet with the bottom plate. Indeed, we use the following definition for the Reynolds numbers, $Re_{r,s}$ of the rough bottom and the smooth top,

$$Re_{r,s} = \frac{U_{\mathrm{up,down}} H}{\nu}, \tag{4}$$

where ν is the kinematic viscosity and $U_{\mathrm{up,down}}$ is the maximum of the profile along x of vertical velocity in the plane at mid-height.

At first glance, the Reynolds number is quite similar across various experiments in the literature, regardless of discrepencies in the Nusselt number. Rough or smooth, ultimate regime or not, Rayleigh number ranging from 10^{10} to 10^{13}, Prandtl number between 0.8 and 14, their Reynolds number seems to collapse with a scaling $Re \sim Ra^{1/2} Pr^{-0.7}$, see Figure 4. However, there are some small differences, and these may become more significant for quantites that scale with a power of the Reynolds number. In particular, the Reynolds number in the rough cell using fluorocarbon is larger than in the smooth case, and additionnally is asymmetrical (velocity is larger near the rough plate). This Reynolds number enhancement triggered by roughness was not visible in our previous measurements using PIV in water

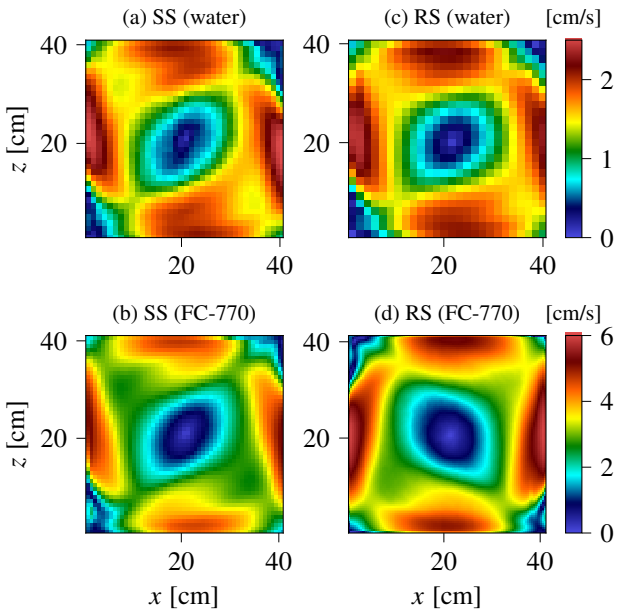

FIG. 3. Mean velocity maps computed from the Shadowgraph recording in SS (a, b) and RS (c, d). (a) SS Water $Ra = 6.3 \times 10^{10}$, $Re = 1.6 \times 10^4$, $v_{\max} = 2.5\,\mathrm{cm/s}$; (b) SS FC-770 $Ra = 1.7 \times 10^{12}$, $Re = 3.3 \times 10^4$, $v_{\max} = 5.2\,\mathrm{cm/s}$; (c) RS Water $Ra = 5.6 \times 10^{10}$, $Re = 1.4 \times 10^4$, $v_{\max} = 2.4\,\mathrm{cm/s}$; (d) RS FC-770 $Ra = 1.6 \times 10^{12}$, $Re = 4.0 \times 10^4$, $v_{\max} = 5.9\,\mathrm{cm/s}$.

at lower Rayleigh numbers [22]. Indeed, the new data obtained in fluorocarbon is further from the threshold where roughness-enhanced heat-transfer is triggered than the rough cell using water. While the previous measurements in water was in Regime II (as defined by Xie & Xia [18]), where the kinematic boundary layer remains larger than the roughness elements, the fluorocarbon cell is in Regime III, where both thermal and kinetic boundary layers are smaller than the roughness height.

One interesting quantity is the friction coefficient,

$$\frac{RaNu}{Re^3 Pr^2}, \tag{5}$$

which was used by Chavanne, *et al.* as an indicator of the transition to turbulence in the boundary layers [19], and can alternatively be interpreted as proportional to the ratio between the kinetic energy dissipation rate ϵ and the bulk energy dissipation rate $\epsilon_{u,bulk}$ [23],

$$\epsilon = \frac{\nu^3}{H^4}(Nu - 1)RaPr^{-2}, \tag{6}$$

$$\epsilon_{u,bulk} \propto \frac{\nu^3}{H^4} Re^3. \tag{7}$$

In the range of parameters explored in this work, we evidence a transition at a critical Reynolds number,

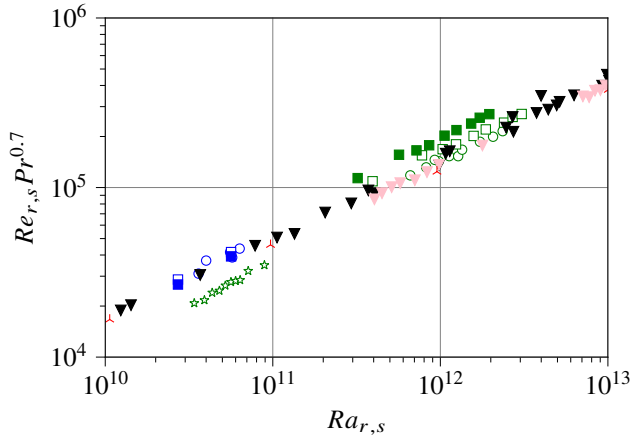

FIG. 4. Reynolds number measurements in water (blue) and FC-770 (green), in RS (squares) and SS (circles) cells. Open squares are smooth half-cell. Full squares are rough half-cell. Green stars are the $\Gamma = 2$ cell. For comparison: Grenoble data [19] (black triangles), Göttingen data [20] (pink triangles), Oregon data [21] (red 3-branch stars).

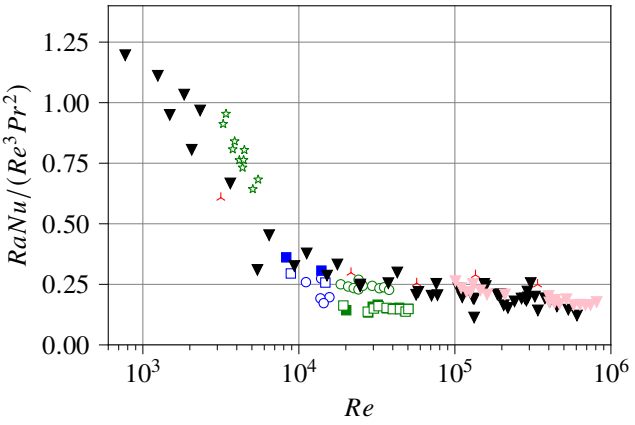

FIG. 5. Friction coefficient for all data points (same colors as Figure 4). The heat flux, $RaNu$ collapses for all data with $RaNu \sim 0.2Re^3Pr^2$ (with this definition of the Reynolds number).

$Re_c \approx 10^4$. Beyond this transition, the friction coefficient no longer depends on the Reynolds number, or only very weakly, see Figure 5. Therefore, we get a relationship between the dimensionless heat-flux, $RaNu$, and the Reynolds and Prandtl numbers, where

$$RaNu = \frac{g\alpha H^4}{\mu\lambda^2}\dot{Q}, \tag{8}$$

where g the acceleration of gravity, α the thermal expansion coefficient, H the height of the cell, μ the dynamical viscosity, λ the thermal conductivity, and $\dot{Q}$ the heat-flux. The phenomenological relationship is,

$$RaNu \approx 0.2Re^3Pr^2. \tag{9}$$

However, this result does not allow to directly infer

whether the kinetic energy dissipation rate is dominated by the bulk energy dissipation rate, or how much dissipation in the boundary layers still contributes, because of prefactors stemming from the non-unique definition for the Reynolds number and of the turbulent dissipation.

It is remarkable that this relationship, Eq. 9, holds for the Grenoble data which evidence a transition to the ultimate regime, as well as data which does not evidence such transition, and also in the case of rough cells in which $Nu \sim Ra^{1/2}$, as well as smooth cells with a classical scaling. This shows that whatever changes are trigged by either the roughness or the transition to the ultimate regime are entirely taken into account in the Reynolds number. While it may seem at first glance that the Reynolds number is only a function of Ra and Pr, it really is not. Multiple flows are possible at a given Rayleigh number [9, 24], and it is therefore elusive to try to determine a threshold in terms of Rayleigh numbers for such transitions. The universal scaling for the heat-flux, Eq. 9, evidenced in this work, has to be taken into account in tentative theoretical models of turbulent thermal convection.

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