



**HAL**  
open science

# Multi-echelon inventory optimization in closed-loop supply chain

Rodrigue Fokouop, Z Jemai, E Sahin, Y Dallery

► **To cite this version:**

Rodrigue Fokouop, Z Jemai, E Sahin, Y Dallery. Multi-echelon inventory optimization in closed-loop supply chain. MIM 2022, Jun 2022, Nantes, France. pp.2992-2997, 10.1016/j.ifacol.2022.10.187 . hal-04430397

**HAL Id: hal-04430397**

**<https://hal.science/hal-04430397v1>**

Submitted on 31 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## Multi-echelon inventory optimization in closed-loop supply chain

R. Fokouop<sup>\*,\*\*</sup>, Z. Jemai<sup>\*</sup>, E. Sahin<sup>\*</sup>, Y. Dallery<sup>\*</sup>

*\* LGI, CentraleSupélec, Université Paris-Saclay, 3 Rue Joliot Curie, 91190 Gif-sur-Yvette, France*

*(rodrigue-fokouop.wafo@centralesupelec.fr, zied.jemai@centralesupelec.fr, evren.sahin@centralesupelec.fr, yves.dallery@centralesupelec.fr)*

*\*\* Computational and data science Lab, Paris Innovation Campus, Air Liquide R&D*

**Abstract:** In this paper, we are interested in the stock control in the closed-loop supply chain and its application to the cylinder packaged gas supply chains of Air Liquide company. Air Liquide is a multinational company producing and distributing industrial and medical gases. Firstly, we introduce a new classification of demand profile and perform a comprehensive study of the goodness of fit of demands sizes and time between successive demands based on real data. As well known in the literature (Boylan and Syntetos (2021)), we find that negative binomial distribution fits better demand sizes and intervals. Secondly, we explain how to use a single echelon model that is easy to implement on the field, to control inventory and compute the stock target in the observable part of the closed-loop supply chain. It is modeled as a multi-echelon serial inventory system. Finally, we perform numerical analysis and comparisons of methods to compute the stock target of the observable part of the closed-loop supply chain with real data. Those methods are derivated from the up-to-level periodic inventory review policy where we vary the statistical distribution of the demand and the measure of the service level. Using the Item Fill Rate (IFR) as a service level measure in the stock target calculation, reduce the stock target by more than 10% compared to the stock target obtained using the Cycle Service Level (CSL). We also investigate methods using compound Poisson distribution for demand models.

Copyright © 2022 The Authors. This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0/>)

*Keywords:* Closed-loop supply chain, Multi-echelon inventory control, working stock target

### 1. INTRODUCTION

Air Liquide specialty is gas production for different purposes such as Large Industry, Health or Private Individuals. Air Liquide uses different kinds of units to supply gases to their customers. ASUs (Air Separation Units) use cryogenic distillation to extract oxygen, nitrogen, and rare gases from the air. Other gases not present in natural resources such as hydrogen or helium are produced. For example, HYCO Units (Hydrogen and Carbon Monoxide) use SMR technology (Steam Methane Reformer) to extract hydrogen and carbon monoxide from methane. These gases can be delivered to customers by pipelines, bulk (using cryogenic tank trucks), or cylinder packaged gas. The Air Liquide packaged gas cylinders supply chain is described in the figure 1. It is a supply chain, from repairing empty cylinders in testing centers to delivering finished products to different customers. It consists of an upstream part in which the filling centers are the main link where the cylinders are sorted, filled, and customers' orders are prepared. These filling centers have the vital function of storing cylinders. It stores products filled on-site and products from other plants necessary to meet customers' demands. This distribution network also has

hubs with similar functions to filling centers except for the filling function. We also have distributors that are either supplied by hubs or by filling centers. End customers can be delivered through a filling center, hub, or distributor.

Upstream of this cylinder's distribution network is a supply chain for the production and repair of empty cylinders essential to guarantee the right level of packaging needed to meet the demand. The main link in this upstream supply chain is the testing centers, which recovers defective cylinders and provide new cylinders to filling centers. In that supply chain, the packaging corresponding to the empty cylinders is considered a vehicle for transporting the products. Their cost is about 65% of the cost of the supply chain. Customers return these packages during the deliveries of new products. Cylinders are then refilled to meet future demands. Therefore, the downstream part of this supply chain is in a closed-loop. The sizing of packaged gas capacity is essential since the packaging is not generic and cannot be replaced by temporary packaging, as is often the case in other sectors like automotive.

Our interest is the multi-echelon inventory optimization in a closed-loop supply chain of reusable articles. Gallego (2010) propose three classes of reusable articles: Reusable Transport Items (RTI) used as packaging without direct contact with the product; Reusable Product Material (RPM) in direct contact with the final product; Reusable

\* Sponsor and financial support acknowledgment to partners of the centraleSupélec supply chain 2019 edition: Air Liquide, L'Oreal and Safran

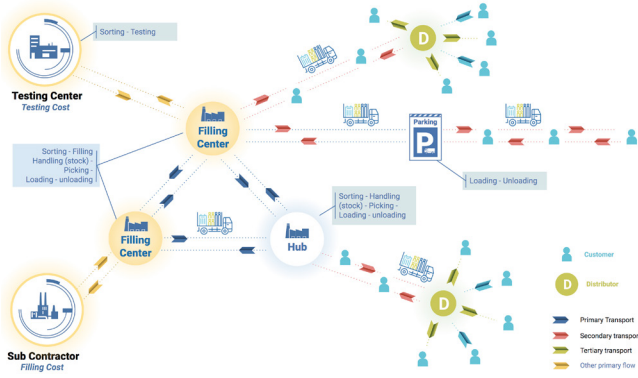


Fig. 1. Air Liquide packaged gas supply chain.

Product (RP). We are specifically interested in RPMs with an application to the supply chain of Air Liquide packaged gas.

The Clark and Scarf (1960) seminal study on a multi-echelon serial system with demand at the last echelon extended by DeCroix (2006) to the case with negative demands representing the product’s return. They prove that the echelon base stock policies are optimal in those cases. Many studies in the literature are interested in the stock’s control in the closed-loop supply chain Fleischmann M. (2004). Comprehensive studies are proposed for the two levels system of recovery and serviceable stocks separated by the recovery process with a possible external supply of the products in serviceable stock. Mitra (2012) interested in the two-echelon system where demands and returns are correlated with the main focus on managerial implications. Despite the important numbers of studies proposed in the literature focused on the stock’s control in the closed-loop supply chain, studies with real data are still limited. Many literature reviews on multi-echelon optimization are proposed in the literature. de Kok et al. (2018) proposed a typology framework to classify the stochastic multi-echelon inventory model and an extended literature review. Despite many theoretical works proposed in the literature to determine optimal stock optimization policies in a multi-echelon system, their implementation in practice remains difficult because required parameters between echelons are challenging to capture. This paper has three research contributions. Firstly, we introduce a new classification of demand profile and perform a comprehensive study of the goodness of fit of demands sizes and time between successive demands based on real data. As well known in the literature (Boylan and Syntetos (2021)), we find that negative binomial distribution fits better demand sizes and intervals.

Secondly, we explain how to use a single echelon model that is easy to implement on the field to control inventory and compute the stock target in the observable part of the closed-loop supply chain, which is modeled as a multi-echelon serial inventory system.

Finally, we perform numerical analysis and comparisons of methods to compute the stock target of the observable part of the closed-loop supply chain real data.

In section 2, after a description of the Air Liquide closed-loop supply chain, we are interested in demand’s profile classification and demand’s goodness of fit of different

statistical distributions. We consider different demand patterns ( Regular: Normal distribution, frequent: poison distribution, very rare (slow movers): empirical distribution). Section 3 describes our methodology to compare different methods to determine the stock’s target needed to guarantee a given service level. Section 4 focuses on local stock inventory control policies(full cylinder inventory). We are interested in determining the stock’s target by taking into account two different measures of service levels. We explain using a single echelon model to determine the stock target in the observable part of the Air Liquide multi-echelon closed-loop supply chain. Section 5 performs a numerical computation using real data and compares different methods for the stock’s target computation.

## 2. PROBLEM STATEMENT

### 2.1 Closed-loop supply chain of a filling center

The figure 2 corresponds to the closed-loop supply chain of products filled in filling centers. The stock of full cylinders representing here the well-known ‘serviceable stock’ Fleischmann M. (2004) used to satisfy the demand of the customers. Many steps constitute the recovery process; we returned and unloaded the cylinders then sorted and filled them. The stock of empty cylinders recovered before sorting can be considered as the ‘recovery stock’Fleischmann M. (2004). Unlike conventional systems, the recovery process contains more than one step, and the inventory of empty cylinders to be filled may be supplied by another source, namely the testing center. Furthermore, we assume that another site cannot supply the stock of filled cylinders. Therefore, the stock of finished products has one source. The stock of empty cylinders to fill comes from two separate sources: the returned cylinders from customers and the supplied new cylinders by the testing centers.

The principle of exchanging full cylinders against empty cylinders is recommended during deliveries to motivate customers to return cylinders. We will then assume that daily delivered cylinders are equal to the daily returned cylinders. The problem is to determine the stock target of empty and full cylinders in the observable part of this closed-loop supply chain to guarantee a given level of service. Only the stock of full cylinders is controlled regularly. The overall stock target of the observable part of the loop is controlled monthly or quarterly. In this work, we assume that all the cylinders received from customers are compliant to be filled, and there is no external supply of empty cylinders.

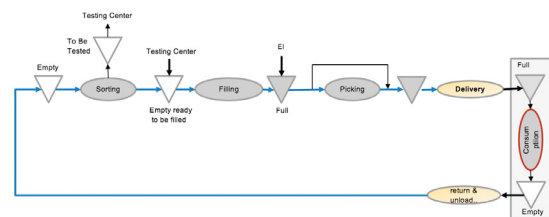


Fig. 2. Focus on the closed-loop supply chain in a filling center.

Table 1. Descriptive statistics of the times between orders and demand’s sizes of all the products of the considered dataset

	mean time between orders	std time between orders	Mean demand size	std demand size
count	482	482	482	482
mean	25,91	16,37	7,59	4,49
std	46,30	25,86	24,67	10,78
min	1	0	1	0
25%	2,15	1,38	1,33	0,51
50%	5,86	4,83	2,33	1,49
75%	26,90	20,83	5,57	4,27
max	395	202,50	400,59	129,36

2.2 Demand modeling: classification and goodness of fit distribution

In this section, we focus on demand modeling. The question is to determine the statistical distribution which fits better the product’s demand. Given the large product (sku), they are classified into categories according to their demand’s profile, and the question becomes which statistical distributions fit better all the products of each category? Several classifications are proposed in the literature. The classification of Syntetos et al. (2005) may be considered as a reference. It proposes to classify products according to two axes, the coefficient of variation of the demand’s size (*CVD*) and the mean duration between two successive demands (*MTBO*). Products are classified between classes smooth ( $CVD < 0.49$  and  $MTBO < 1.31$ ), intermittent ( $CVD < 0.49$  and  $MTBO < 1.31$ ), Erratic ( $CVD < 0.49$  and  $MTBO < 1.31$ ), Lumpy. ( $CVD < 0.49$  and  $MTBO < 1.31$ ).

The actual used classification considers three axes. The coefficient of variation of the demand’s size is used. Rather than considering the mean duration between two successive demands as in the Syntetos et al. (2005) classification, we consider over a given time horizon the proportion of periods with demands (*PWDO*). The monthly average demand (*MAD*) is used as the third axis to define the classes of products rarely ordered. We deduce a new classification where the product’s categories are regular ( $CVD < 0.49$  and  $PWDO < 0.5$ ), irregular ( $CVD < 0.49$  and  $PWDO < 0.5$ ), sporadic ( $PWDO < 0.5$  and  $MAD < 2$ ) and slow mover ( $PWDO < 0.5$  and  $MAD < 2$ ).

The table 1 presents descriptive statistics of the demand profiles of the different products of the dataset under consideration. In this dataset, we have 482 products with significant variability of the size of the Daily demand and the duration between two successive requests.

The table 2 compares the two product’ classifications by applying them to the dataset. We can observe a correspondence between the smooth and regular classes. The irregular products class corresponds to the erratic and lumpy product classes. The sporadic products correspond mainly to the lumpy products and a small part to the intermittent products. The products of the slow mover class are mainly intermittent, and some of these products are erratic products.

This new classification is inspired by current practice in the field for several years. This classification and that of Syntetos et al. (2005) are very close. This encourages us

Table 2. Classification of products of the dataset

Category	Regular	Irregular	Sporadic	Slow mover	All
Smooth	14				14
Erratic	1	42			43
Lumpy		51	177	28	256
Intermittent	1		59	108	168
<b>All</b>	<b>16</b>	<b>93</b>	<b>236</b>	<b>136</b>	<b>481</b>

Table 3. Goodness of fit of demand’s sizes of all the products of the considered dataset. (a) is the number of products in each category and (b) is the number of products with fitted distributions

Category	(a)	(b)	Exp	Gamma	Lnorm	Logis
Regular	16	14	1	10	2	9
Irregular	94	68	26	28	1	18
Sporadic	229	172	138	8	3	19
Slow mover	17	17	17			
<b>All</b>	<b>356</b>	<b>271</b>	<b>182</b>	<b>46</b>	<b>6</b>	<b>46</b>

Category	Norm	Weib	Geom	Nbinom	Pois
Regular	10	13	1	13	
Irregular	6	45	9	62	15
Sporadic	4	16	95	161	121
Slow mover			17	17	17
<b>All</b>	<b>20</b>	<b>74</b>	<b>122</b>	<b>253</b>	<b>153</b>

to continue to use the new classification, which is better understood in the field.

To determine the distributions to be assigned to the different product categories, we made several statistical estimations of the parameters of the different families of statistical distributions, including exponential, gamma, lognormal, normal, logistic, Weibull, geometric, binomial negative, and Poisson distributions. For each dataset’s product, we used the Chi2 statistical test with a p-value < 0.005 to test the acceptance or rejection of the estimated distributions. Tables 3 and 4 represents the number of products in each category for which the statistical test accepted the distribution’ family. The table 3 corresponds to the demand’s size. The negative binomial distribution fitted most products in the different categories. The normal distribution is often used to model the demand for regular and irregular classes. This hypothesis is plausible for products of the regular class where the normal distribution is accepted for 10 products over 16.

The table 4 presents the distributions of time between two successive demands. The negative binomial distribution also fits well with most of the products. The exponential distribution is accepted for a large majority of products then, the assumption commonly made in the literature of a demand arrival following a Poisson distribution is plausible. In the following section, we will be particularly interested in modeling the demand with compound Poisson distribution where demand’s size follows a negative binomial distribution.

3. METHODOLOGY

The objective is to determine the best method of calculating the target stock to guarantee a given level of service by considering the demand profiles of products. Therefore, we will compare the four methods of calculating the stock’s target.



Table 4. Goodness of fit of times between orders of all the products of the considered dataset. (a) is the number of products in each category and (b) is the number of products with fitted distributions

Category	(a)	(b)	Exp	Lnorm	Logis	Norm
Regular	16	16	16			
Irregular	91	91	91			
Sporadic	206	185	137	47	54	13
Slow mover	21	17	10	12	12	6
<b>All</b>	<b>334</b>	<b>309</b>	<b>254</b>	<b>59</b>	<b>66</b>	<b>19</b>

Category	Weib	Geom	Nbinom	Pois
Regular		15	16	16
Irregular	1	77	91	91
Sporadic	85	67	154	38
Slow mover	12	12	12	1
<b>All</b>	<b>98</b>	<b>171</b>	<b>273</b>	<b>146</b>

- Method 1: In this method, we assume that the demand follows a normal distribution for products of the regular and irregular categories and empirical distribution for sporadic and slow mover’s products. In this formulation, we use a Cycle Service Level to measure the service level.
- Method 2: In this method, the demand’s distribution is identical to the demand’s distribution of method 1. The difference with method 1 is on the measure of the level of service. We use Item Fill Rate as a measure of the level of service.
- Method 3: The product’s demand follows a compound Poisson distribution, and the measure of service level considered is the Cycle Service Level.
- Method 4: The product’s demand follows a compound Poisson law, and the level of service considered is the Item Fill Rate.

The methodology to evaluate stock calculation’s methods is following:

- Modeling of demand over the protection period: we consider the one-year history of quantities delivered for each product per day.
- Calculation of the target stock of cylinders with the four methods: For each method, the target service level is 98% for each product.
- Evaluation of methods by simulation: we simulate the stock’s dynamics by replaying six months of deliveries history in the case of a periodic stock control policy where the stock’s target is calculated for each method. We assess the quantity service rate for each method over 6 months.

For each method, we analyze the distribution of simulated fill-rate service levels for all products and differences between the target stocks of each method with the stock target provided by method 1 considered as a reference’s method.

#### 4. PERIODIC REVIEW AND ORDER UP-TO-LEVEL INVENTORY’S CONTROL POLICY

We consider a mono-product and single-echelon inventory system with stationary demand. The replenishment lead-time  $L$  is assumed to be fixed. It can be a few days if

the product is filled on another site or a few hours if the product is filled on site.

We focus on the periodic review and up-to-level inventory control policy. At the beginning of a period of length  $T$  if the stock position is below the order up to level  $S$ , a variable quantity is ordered to bring the stock position back to  $S$ . The placed order is delivered after a lead time  $L$ .

We consider the following notation:

- $D_T$  and  $D_{T+L}$  are random variables corresponding to demand’s product during the  $T$  and  $T + L$  periods respectively.
- $m_{D_P}$  and  $\sigma_{D_P}$  are the mean and standard deviation of the demand’s product  $D_P$ , with  $P = T$  or  $P = T + L$
- $\phi_{D_P}(\cdot)$  is the density profile of the demand’s product during a period  $P$ , with  $P = T$  or  $P = T + L$
- $\Phi_{D_P}(\cdot)$  is the cumulative density function of the demand’s product during a period  $P$ , with  $P = T$  or  $P = T + L$
- $P(x)$  is the probability of a given event  $x$

Optimizing inventory with a periodic review up to level control policy  $(T, S)$  is about determining the optimal  $T^*$  and  $S^*$  parameters to minimize inventory costs while ensuring a given level of service. We consider the two following formulations depending on the measure of the selected service level. In the first (respectively second) formulation, we want to minimize the inventory cost under the constraint of satisfying a cycle service level rate  $csl_0$  (respectively a quantity service level commonly called Item Fill Rate).

In practice, the parameter  $T$  is often fixed, and the question is limited to the determination of the parameter  $S$  with a safety stock part allowing to cover against disruptions risk on the protection interval  $(T + L)$ . We assume in the following the parameter  $T$  is known.

##### 4.1 First formulation: Cycle Service Level

Note  $csl_0$  the target service level. Determining the optimal up-to-level stock to guarantee a cycle service level’s target is equivalent to find  $S$  such as  $P(\text{Demand over } T+L \leq S) = csl_0$  which is equivalent to  $S^* = \Phi_{D_{T+L}}^{-1}(csl_0)$  (1). Generally  $S$  can be determined by numerically solving the equation (1).  $\phi_{D_{T+L}}(S)$  being a growing function of  $S$ , it is sufficient to increment  $S$  from  $m_{D_{T+L}}$  to reach  $csl_0$ .

In the case of demand’s following normal distribution,

$$S = m_{D_{T+L}} + \Phi_s^{-1}(csl_0) \sigma_{D_{T+L}}$$

where  $\Phi_s^{-1}(\cdot)$  is the inverse of the cumulative distribution profile of the standard normal distribution.

##### 4.2 Second formulation: service level in quantity

Note  $fr_0$  the service level’s target and  $n(S)$  the average number of stock-out during the protection interval  $T + L$ . The demand’s mean over a cycle is  $m_{D_T}$  and the quantity service level (Item Fill Rate) is  $Fr(S) = 1 - \frac{n(S)}{m_{D_T}}$  where  $n(S) = \int_{x=S}^{+\infty} (x - S)\phi_{D_{T+L}}(x)dx$ . Determining the optimal up to level’s stock to guarantee a targeted service level

$fr_0$  is equivalent to find  $S$  such as  $Fr(S) = fr_0$  (2). As a general rule  $S$  can be determined by numerically solving the equation (2).  $Fr(S)$  being a growing function of  $S$ , it's sufficient to increment  $S$  from  $m_{D_{T+L}}$  to reach  $fr_0$ . Suppose the demand follows a normal distribution, then  $n(S) = \sigma_{D_{T+L}} \text{Loss} \left( \frac{S - m_{D_{T+L}}}{\sigma_{D_{T+L}}} \right)$  where  $\text{Loss}(z) = -z(1 - \Phi_s(z)) + \phi_s(z)$  is the loss function of the standard normal distribution. We deduce that,

$$S^* = m_{D_{T+L}} + \text{Loss}^{-1} \left( \frac{m_{D_T}(1 - fr_0)}{\sigma_{D_{T+L}}} \right) \sigma_{D_{T+L}}$$

### 4.3 Application to cylinder's inventory control at Air Liquide: closed-loop multi-echelon inventory control

The closed-loop has several stock points. Only empty cylinders and full cylinders can be controlled. The observable part of the closed-loop can therefore be reduced to a multi-echelon system with two levels, the stock before filling (empty cylinders) and the stock after filling (full cylinders). Vercraene and Gayon (2013) interested in the optimal policy in a closed-loop two-level multi-echelon system using queuing theory. In their system, the return of products from customers is not the main source of replenishment of the upstream stock essential for the constitution of the final product. Their main question is to consider the return of products to avoid overstocking. The return of empty cylinders from customers is the only supply of upstream stock (empty cylinders). To determine the inventories of cylinders, we will use a two-step heuristic. Firstly, we will evaluate the full cylinders' stock target by assuming there are always enough empty cylinders to fill. In this first step, the system is reduced to a single level with a replenishment lead time equal to the duration of filling.

Secondly, we will determine the overall target stock of the observable part of the supply chain. In the case where the quantities of empty cylinders returned are equivalent to the quantities delivered full cylinders, the determination of the overall target stock in the observable part of the closed-loop supply chain can be reduced to the determination of the target stock in a mono level system with a replenishment lead time equal to the sum of all the lead times of the activities of the recovery process.

$$L = L_{picking} + L_{delivery} + L_{return\&unload} + L_{sorting} + L_{filling}$$

where  $L_{picking}$ ,  $L_{delivery}$ ,  $L_{return\&unload}$ ,  $L_{sorting}$  and  $L_{filling}$  are respectively the lead time of the picking, delivery, return, sorting and filling activities. We then deduce the stock target of empty cylinders by making the difference between the two previously determined stock targets.

In the following section, we suppose that the replenishment lead-time  $L$  is one day. So to compare the different stock target's calculation methods, we focus on the stock target of the observable part of the closed-loop supply chain cylinders needed to guarantee a given service level.

## 5. RESULTS AND DISCUSSION

In this part, we analyze the results of the comparisons of the four methods of calculation of target stock. Method

1 is considered as the reference's method with which the other methods are compared. The table below thus shows the dispersion of the levels of services obtained with this reference method. In these results, we look at the 199 products filled on site. The review period considers  $T$  is one day, and the lead-time is also one day. The service level target is 98%

Table 5. Method 1's service level by product's category: Number of products by category

IFR_M.1	Regular	Irregular	Sporadic	Slow mover	All
[0.7, 0.8[		4			4
[0.8, 0.9[	2	15	1		18
[0.9, 0.98[	8	42	10		60
[0.98, 1[		3	34		37
1	2	15	62	1	80
All	12	79	107	1	199

Table 5 presents the dispersion of Method 1 service rates when inventory levels are determined using the CSL as a measure of the level of service. With this method, 22 products have a level of service in quantity (IFR) less than 90% and 37 products have a level of service higher than or equal to 98% and less than 100% (see tables ref and ref). 80 products at 100% service level, which could indicate over-storage of its products.

Table 6 presents the dispersion of service levels obtained with Method 2. There is a slight improvement in service levels in regular and irregular products. We also note a stock level reduction of 20.1% for products in the regular's category and 15.4% for irregular products compared to the target stock level obtained with the reference method 1 (see table ref). We also see a slight reduction in service levels for sporadic products, followed by a 4.2% reduction in inventory.

Table 6. Method 2's service level by product's category: Number of products by category.

IFR_M.2	Regular	Irregular	Sporadic	Slow mover	All
[0, 0.8[			3		3
[0.8, 0.9[		11	4		15
[0.9, 0.98[	6	48	29		83
[0.98, 1[	4	5	9		18
1	2	15	62	1	80
All	12	79	107	1	199

Table 7. Inventory gaps by review's period by product's category: Difference between Method 2 and Method 1 Inventory Levels.

IFR_M.2	Regular	Irregular	Sporadic	Slow mover	All
[0, 0.8[	0	0	11	0	11
[0.8, 0.9[	0	-33	-2	0	-35
[0.9, 0.98[	-74	-283	-59	0	-416
[0.98, 1[	-158	-17	5	0	-170
1	-12	-14	-5	0	-31
Gap	-244	-347	-50	0	-641
All	1216	2250	1182	3	4651
Gap(%)	-20,1	-15,4	-4,2	0	-13,8

We deduce that for almost equivalent service levels, the use of quantity service levels in the calculation of stocks leads to a reduction in stocks of the order of 13% compared to a method using cycle service levels. This reduction is significant when demand follows a normal distribution and fades slightly when demand follows an empirical distribution.

If the demand is modeled with a compound Poisson distribution with the demand size following a negative binomial distribution, the level of service is obtained using the CSL in the calculation method (method 3) or the Item Fill Rate (method 4) improves considerably for products of regular and irregular categories. For products of the sporadic category, we observe a degradation of the level of service (see table 10) followed by a significant reduction in the stock level using method 4 (see table 11).

Table 8. Method 3's service level by product's category: Number of products by category

IFR_M.3	Regular	Irregular	Sporadic	Slow mover	All
[0,6, 0,7[			1		1
[0,8, 0,9[		5	5		10
[0,9, 0,98[	10	57	53		120
[0,98, 1[		10	22		32
1	2	7	26	1	36
All	12	79	107	1	199

Table 9. Inventory gaps by review's period by product's category: Difference between Method 3 and Method 1 Inventory Levels.

IFR_M.3	Regular	Irregular	Sporadic	Slow mover	All
[0,6, 0,7[	0	0	4	0	4
[0,8, 0,9[	0	9	-31	0	-22
[0,9, 0,98[	91	244	-340	0	-5
[0,98, 1[	0	74	-133	0	-59
1	9	205	-164	-2	48
Gap	100	532	-664	-2	-34
All	1460	2838	1232	3	5533
Gap(%)	6,8	18,7	-53,9	-66,7	-0,6

Table 10. Method 4's service level by product's category: Number of products by category

IFR_M.4	Regular	Irregular	Sporadic	Slow mover	All
[0, 0,8[			21		21
[0,8, 0,9[		7	24		31
[0,9, 0,98[	4	45	32		81
[0,98, 1[	6	20	4		30
1	2	7	26	1	36
All	12	79	107	1	199

Table 11. Inventory gaps by review's period by product's category: Difference between Method 4 and Method 1 Inventory Levels.

IFR_M.4	Regular	Irregular	Sporadic	Slow mover	All
[0, 0,8[	0	0	-117	0	-117
[0,8, 0,9[	0	10	-116	0	-106
[0,9, 0,98[	-19	47	-140	0	-112
[0,98, 1[	-147	142	-7	0	-12
1	-10	223	-95	-1	117
Gap	-176	422	-475	-1	-230
All	1460	2838	1232	3	5533
Gap(%)	-12,1	14,9	-38,6	-33,3	-4,2

At almost equivalent IFR service level, method 4 proposes target stock level at 4.2% lower than the target stock level of method 1 with a significant reduction in the stock of "sporadic" and "regular" products and a significant increase in the stock of "Irregular" products.

## 6. CONCLUSION

We introduce a new classification of demands profile and compare it to the well-known classification of Syntetos

et al. (2005). Extended goodness of fit statistical analysis allows us to identify the most convenient statistical distribution to model the product's demand according to their category. After explaining how to use a simple mono-echelon model to control the inventory in the observable part of a closed-loop supply chain, we introduce four methods to compute the stock target. The table 12 presents the percentages of reduction of the stock's target of different methods compared to the target stock of method 1.

- At equivalent IFR service level, Method 2 proposes target stock levels at 11.6% lower than the target stock level of Method 1.
- At almost equivalent IFR service level, Method 3 proposes target stock levels equivalent to the target stock level of Method 1.
- At almost equivalent IFR service level, Method 4 proposes target stock levels at 4.2% lower than the target stock level of Method 1.

Table 12. Percentage of reduction of cylinders stocks target relative to the target stock of Method 1 with almost equivalent IFR level of service.

	Regular	Irregular	Sporadic	Slow mover	All
Method 2	-16,7%	-12,2%	-4,1%	0%	-11,6%
Method 3	6,8%	18,7%	-53,9%	-66,7%	-0,6%
Method 4	-12,1%	14,9%	-38,6%	-33,3%	-4,2%

There is often a difference between the full-delivered and the empty returned cylinders. Nevertheless, the incentive of equivalent exchanges during customer deliveries creates a dependency between the full-delivered and empty returned cylinders. We will investigate the unequal exchanges of cylinders in the stock target calculation in future work and compare the results of our heuristic system with the exact solution of our closed-loop multi-echelon system.

## REFERENCES

- Boylan, J. and Syntetos, A. (2021). *Compound Demand Distributions*.
- Clark, A. and Scarf, H. (1960). Optimal policies for a multi-echelon inventory problem. *Management Science*, 50.
- de Kok, T., Grob, C., Laumanns, M., Minner, S., Rambau, J., and Schade, K. (2018). A typology and literature review on stochastic multi-echelon inventory models. *European Journal of Operational Research*, 269(3).
- DeCroix, G.A. (2006). Optimal policy for a multiechelon inventory system with remanufacturing. *Operations Research*, 54(3).
- Fleischmann M., M.S. (2004). *Inventory Management in Closed Loop Supply Chains*. Springer, Berlin, Heidelberg.
- Gallego, R.C. (2010). A management model for closed-loop supply chains of reusable articles.
- Mitra, S. (2012). Inventory management in a two-echelon closed-loop supply chain with correlated demands and returns. *Computers & industrial engineering*, 62(4).
- Syntetos, A.A., Boylan, J.E., and Croston, J.D. (2005). On the categorization of demand patterns. *Journal of the Operational Research Society*, 56(5).
- Vercaene, S. and Gayon, J.P. (2013). Optimal control of a production-inventory system with product returns. *International Journal of Production Economics*, 142(2).