



HAL
open science

Revision of prioritized

εℒ

ontologies

Rim Mohamed, Zied Loukil, Faiez Gargouri, Zied Bouraoui

► **To cite this version:**

Rim Mohamed, Zied Loukil, Faiez Gargouri, Zied Bouraoui. Revision of prioritized

εℒ

ontologies. Applied Intelligence, 2023, 53 (24), pp.30359-30383. 10.1007/s10489-023-05074-6 . hal-04426740

HAL Id: hal-04426740

<https://hal.science/hal-04426740v1>

Submitted on 30 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Revision of Prioritized \mathcal{EL} Ontologies

Rim Mohamed^{1*}, Zied Loukil^{1†}, Faiez Gargouri^{1†} and Zied Bouraoui^{2†}

^{1*}University of Sfax, Miracl Laboratory, Isims, Tunisia.

²CRIL CNRS & Univ Artois, Lens, France.

*Corresponding author(s). E-mail(s):

rymmohammed2@gmail.com;

Contributing authors: zied.loukil@isims.usf.tn;

faiez.gargouri@usf.tn; bouraoui@cril.fr;

†These authors contributed equally to this work.

Abstract

In this paper, we investigated the evolution of prioritized \mathcal{EL} ontologies in the presence of new information that can be certain or uncertain. We propose an extension of \mathcal{EL} description logic, named \mathcal{EL}_{\perp}^{+} , within possibility theory to encode such knowledge. This extension provides a natural way to deal with the ordinal scale and represent knowledge in a way that can handle incomplete information and conflicting data. We provided a polynomial algorithm for computing the possibilistic entailment. Then, we defined the evolution process at the semantic and syntactic levels. Interestingly enough, we show that the syntactical algorithm is done in polynomial time.

Keywords: \mathcal{EL} Description Logic, Semantical Revision, Syntactical Revision, Possibility Theory, Prioritized Ontology

1 Introduction

Description logics (DLs for short) [1] are a family of logic-based knowledge representation languages. It provides a powerful framework for organizing and formally expressing knowledge in various application domains, such as ontology-based data access [2] and information and data integration [3]. The

term “description logic” is induced for two reasons: the first is that the key concepts of the domain are expressed using atomic concepts (unary predicates) and atomic roles (binary predicates) using the concept and role constructors provided by the particular DL. However, DLs are distinct from their predecessors, such as frames and semantic networks [4–6], in that they have formal, logic-based semantics that can be translated into first-order logic [7].

A DL knowledge base (KB) is built upon two major elements: The terminological Box (also called TBox), in which we can specify the characteristics of concepts and roles as well as their relationships. This refers to the schema in a database setting [8]. The assertional Box (also called ABox) contains data. DLs provide the foundations of the Web Ontology Language *OWL2*¹, and its three profiles *OWL2-QL*, *OWL2-EL* and *OWL2-RL*.

In this paper, we consider the second profile, i.e., *OWL2-EL*, which is designed as a subset of *OWL2*. This profile is based on the lightweight family of DL, called \mathcal{EL} description logic [9–13]. The \mathcal{EL} is especially well-suited for applications that use ontologies with a large number of classes and properties (for example the Systematized Nomenclature of Medicine Clinical Terms (SNOMED CT)² and gene ontology (GO)³), the \mathcal{EL} provides the expressivity power used by many such ontologies and guarantees the decidability of the main reasoning tasks, such as ontology consistency [14], concept subsumption [11], and instance checking [12].

DLs are intended for reasoning on static knowledge bases; however, in such cases, for example, ontologies in semantic web applications [15], are not static but evolve over time. Consider the case of a university’s global ranking. The Quacquarelli Symonds (QS) World University Rankings [16] provide an annually updated list based on some criteria of the top 1,400+ universities worldwide. Then, each year, an update is needed. The crucial and challenging problem for such an application is how to effectively and efficiently revise and update knowledge bases in a natural way.

The task of ontology evolution consists in incorporating new information into an ontology or deleting some axioms from it while taking into account possible changes. The first evolution task is called expansion [17], while the second is called contraction [18]. In general, in the two evolution tasks, the ontology to incorporate (or delete) is represented by a set of axioms that should satisfy some defined properties [19].

By incorporating the ontology, a negative interaction can occur, i.e., the new ontology can interact in an undesirable way with the axioms of the old one, causing the ontology or relevant parts of it to become unsatisfiable. Then, the new ontology cannot simply be added to the old one. Therefore, some changes must be made in the ontology to avoid negative interaction, e.g., handling the axioms that are in conflict with the ontology. This problem is similar to the belief revision problem in propositional logic [20], where the old belief is revised by adding new formulas to a knowledge base that can be sure or uncertain.

¹<https://www.w3.org/TR/owl2-overview/>

²<https://bioportal.bioontology.org/ontologies/SNOMEDCT>

³<http://geneontology.org/>

This problem is also defined as the knowledge change that is specified by the well-known Alchourrón, Gärdenfors, and Makinson postulates, (in short AGM postulates) [21]. Several works have been proposed to revise the DL ontology (e.g., [19, 22]). In particular, several model-based revision approaches have been proposed (e.g [23–25]), which consist of selecting a set of interpretations that are the closest to original belief bases. This revision operator for DLs is restricted to one part, either the ABox or TBox. However, in [26] it operates on the general KB. Revision with uncertain input has been studied at the syntactic level in [27]. It consists of adding a formula to the belief base at its prescribed level. The issue is complicated because the belief base needs to be changed; this is necessary to ensure that the added formula remains at the prescribed level and isn't implicitly inhibited by higher priority formulas that contradict it or pushed to higher priority levels by formulas that imply it [28].

In real-world applications, information is often provided by multiple sources, each with a different priority level reflecting their reliability. While each source is consistent, gathering them gives rise to a prioritized, inconsistent knowledge base [29]. Dealing with the inconsistency in the prioritized knowledge base has gained a lot of attention [30, 31]. In [32], a possibilistic extension of DL-Lite, a lightweight family of DLs that underlies *OWL2-QL*, has been proposed in a qualitative setting for handling inconsistent knowledge bases. Furthermore, in [33], they extended the possibilistic approach in [32] to the two fragments *DL-Lite_R* and *DL-Lite_F* and showed that the extension of the expressive power of DL-Lite is done without additional extra-computational costs. Revision in a prioritized DL KB has gained a lot of attention in recent years (e.g. [34]). In [35], they studied the revision of the DL-Lite knowledge base at the semantic level by conditioning possibility distributions. However, they showed that such conditioning provides in some scenarios some counter-intuitive results. Then, they studied revision at the syntactic level. Furthermore, in [17], Benferhat et. al studied the revision or evolution of the DL-lite knowledge base when the ABox is prioritized. They defined the “Prioritized Removed Sets Revision” approach that is based on inconsistency minimization in order to restore the consistency of such a knowledge base. In addition, an extension of the “Prioritized Removed Sets Revision” to *DL-Lite_R* prioritized knowledge bases is defined in [36].

In this paper, we study the revision of the prioritized \mathcal{EL} ontology process when a new piece of information that can be sure or uncertain is available. In order to encode and reason with such prioritized ontology, we define a possibilistic extension of \mathcal{EL} , denoted $\pi\text{-}\mathcal{EL}_\perp^+$. This extension guarantees the tractability of the reasoning process as the one of the standard \mathcal{EL}_\perp^+ ontology. To the best of our knowledge, there is no work that studies the syntactic and semantic evolution of prioritized \mathcal{EL} ontology. We first study revision of $\pi\text{-}\mathcal{EL}_\perp^+$ ontology semantically by conditioning the possibility distribution associated with \mathcal{EL}_\perp^+ interpretations by the new input information. Based on the type of input information and its interaction with the ontology, we define situations where conditioning in $\pi\text{-}\mathcal{EL}_\perp^+$ differs from the one of the standard

possibilistic settings, e.g, probability distributions, dempster belief functions and possibility distributions [37]. For that reason, we define algorithms for revision at the syntactic level of $\pi\text{-}\mathcal{EL}_{\perp}^{+}$ ontology. An interesting result we display in this paper is that revision operation is done efficiently in polynomial time. Revision of prioritized \mathcal{EL} ontologies with uncertain and inconsistent input information has several applications in different fields such as ontology reasoning and maintenance [38], semantic web and knowledge graphs [39], artificial intelligence and machine learning [40], Natural Language Processing (NLP) [41], medical informatics [42].

2 Related Work

This section provides an overview of the evolution (revision) of propositional logic knowledge bases and description logic ontologies, then move on to a discussion of the evolution (revision) of prioritized versions of these knowledge bases and ontologies.

2.1 Evolution in Propositional Logic Knowledge Base

In propositional logic, the notion of belief revision deals with the logical inconsistency that results from updating a knowledge base with the most recent information. In [21], Alchourrón, Gardenfors, and Markinson (AGM for short) defined a set of postulates to characterize a revision operator. In AGM’s study, the beliefs of an agent are expressed using a set of formulas that are closed under logical consequence, named belief set, i.e., A is a theory if $Cn(A) = A$, where Cn is a consequence operation. Therefore, a revision operator consists of converting a belief set and formulae to a belief set. The AGM theory defines three operators, namely expansion, revision, and contraction [22].

Expansion: is the act of inserting new input information into the ontology without any consideration for maintaining consistency.

Contraction: is the act of consistently removing formulas from the ontology instead of adding ones.

Revision: is the act of consistency adding new input information to the old ontology.

The AGM theory [21] is possibly the most influential work in the area of belief revision and knowledge evolution. It consists of introducing a set of rationality postulates that the operators of revision and contraction should satisfy. However, due to the difficulty of computing with belief sets since a belief set may contain a lot of sentences, many researchers have suggested using a belief base [43–46], which is a collection of sentences that are not closed under logical consequence, to express an agent’s beliefs. Dalal et al. in [47] defined the notion of the irrelevance of syntax, i.e., the revised knowledge base should not depend on the syntax of the original knowledge base or the new input information. Many evolution theories that adhere to Dalal’s principle and the AGM postulates have been studied in the literature [22, 48]. Winslett

in [49] classified the evolution of semantics into two methods: the model-based revision approaches (MBAs) and the formula-based approaches (FBAs).

- **The Model-Based Approaches (MBAs)** : The result of the revision of the knowledge base \mathcal{K} with the new input information ϕ under the MBAs is the set $(\mathcal{K} \circ \phi)$ of models, each satisfying ϕ and minimally changed from some models (or all models) of \mathcal{K} . There exist two types of MBAs, the first one is called local [50], and its idea is to go over a subset \mathcal{M} of models of \mathcal{K} and select the models \mathcal{I}' of ϕ that are minimally distant from \mathcal{M} [18]. The distance measure displayed in [51, 52] is based on a pre-order of models; it's used to remove models with unlikely results. The result models is not unique since we can have several models. For such reason, the possible model approach [53] with various variations is defined. This approach selects the models with minimal changes (the minimal is in terms of set inclusion). Another approach to reducing the result models number is to minimize the number of changes in the model of \mathcal{K} , this approach is called local cardinality-based semantics [25], and it is defined as follows: given two models \mathcal{I} and \mathcal{M} , the distance between the two models is given by the size of $Diff(\mathcal{I}, \mathcal{M})$. Note that $Diff(\mathcal{I}, \mathcal{M})$ contains the set of facts by which \mathcal{I} and \mathcal{M} disagree. More precisely, given two models \mathcal{I} and \mathcal{I}' , under a local cardinality-based semantics:

$$\mathcal{I} \text{ is closer to the model } \mathcal{M} \text{ than } \mathcal{I}' \text{ iff } |Diff(\mathcal{I}, \mathcal{M})| < |Diff(\mathcal{I}', \mathcal{M})|$$

The second type of MBAs, known as global [54], consists of selecting the models \mathcal{I}' of the new input ϕ that are minimally distant from all the set of models of the knowledge base \mathcal{K} . A global cardinality technique was employed by Dalal in his studies [47], which demonstrated its value for tasks like fault diagnostics in circuits where models of circuits with few faults are preferable. The result of the global MBA is also not unique, that is why Winslett in his study [55] proposed another method for reducing the number of the result models, called the global priority technique, which consists of dividing the set of predicates and functions into groups and categorized them, thus, predicates and functions with higher priorities are given greater weight than those with lower priorities when the distance between models is measured.

Similar to Winslett's operator, Borgida in [56] define its operator in the case when the input information ϕ is consistent with the knowledge base \mathcal{K} , then Borgida's revised theory is simply $\mathcal{K} \wedge \phi$.

- **Formula-Based Approaches (FBAs)**: The main idea of the revision process of the knowledge base \mathcal{K} with the new input ϕ under FBAs is to select the set of formulas \mathcal{K}_m in \mathcal{K} that have the greatest distance and are consistent with the new input ϕ . Ginsberg and their colleagues in [57, 58] showed that the revised knowledge base contains the maximal subset \mathcal{K}_m of \mathcal{K} that is consistent with ϕ plus ϕ . They also proved that all sets in \mathcal{K}_m are equally

plausible and the logical consequence \mathcal{Q} is inferred from each set. A general form of formula-based approach has been proposed by Nabel in [59]. He showed that due to their conceptual implicitness, syntax-based approaches have gained a lot of popularity. However, they have an impairment since the result of a revision operation relies on arbitrary syntactic distinctions, and to this end, such operations cannot be analyzed at the knowledge base [44]. It is important to note that the result knowledge base \mathcal{K}_m under FBAs is not unique. For such a reason, some works (e.g., [54, 60]) propose approaches to merge all the elements of \mathcal{K}_m into one set of formulas, which is then added to the input ϕ . These approaches are CrossProduct (CP), and When In Doubt Throw It Out (WIDTIO) [61]. In [62], Creignou et al. studied the two operators, namely *RSRG* (which stands for Removed Sets Revision Ginsberg) and *RSRW* (which stands for Removed Sets Revision Widtjo). These operators are similar to Ginsberg’s operators [58], however, instead of using set inclusion as a maximality criterion, they used set cardinality. The choice of subbases that are maximally consistent with regard to set cardinality does not necessarily guarantee that the most pertinent information will be chosen, and as a result, it is possible to overlook viable formulas based on the initial agent’s beliefs. For such a reason, Ktari et al. in [63] introduced the idea of credible belief base revision, which leads to two new formula-based revision operators, namely Credible Set Revision Ginsberg (*CSRG*) and Credible Set Revision Widtjo (*CSRW*). Such operators are presented in the same spirit as others in [62], which are based on selecting maximally consistent subsets with respect to credibility instead of set inclusion and cardinality.

2.2 Evolution of Description Logics Ontologies

The evolution process in the description logics ontology consists of removing or adding some input information to the old ontology while preserving its consistency [18].

Kang and Lau in [64] have studied the viability of adopting the belief revision as a foundation for DL ontology revision. In [19, 22], the authors generalized the AGM postulates by dropping all the assumptions that limit the direct application to DL ontologies and determining the necessary and sufficient conditions for logic to support AGM-compliant operators. However, none of them take into account the explicit creation of a revision operator. In [65], two revision operators, which are generalizations of the AGM postulates in DLs, are defined to deal with disjunctive \mathcal{ALC} knowledge bases.

In [66] a generic revision operator was proposed to address the incoherence problem. Nevertheless, this operator is not fine-grained in that it uses an incision function to remove an entire TBox axiom from a knowledge base as soon as it influences the coherence of the KB.

Furthermore, in [67], the authors introduced a formula-based approach for OWL-Lite ontologies, where the elimination of inconsistencies between the old knowledge and the new information is substantially syntax-dependent.

In the scenario where the new information consists of potentially negated ABox statements, Liu et al. in [68] explored the problem of ABox updates with empty TBoxes while taking into account a number of typical DLs from the \mathcal{ALC} family. They demonstrated that, while many description logics do not have updates, selecting the DL that includes a nominal concept guarantees the existence of updates. In addition, by selecting the right notion of update (projective semantic), it is even possible to compute updated ABoxes in polynomial time.

The authors in [69] investigated the evolution of ABox under integrity constraints in \mathcal{ALC} and DL-Lite. The updates in that paper are defined as the finite sequences of conditional insertions and deletions, where the pairs of nodes for which node or arc labels are inserted or deleted are chosen using complex DL formulas. They also studied the complexity of checking when a sequence of update operations preserves the integrity constraints.

In [70] they studied instance-level update and instance-level erasure in DL-Lite_F. They characterized the semantics of ABox update and erasure on the basis of the approaches proposed by Winslett and by Katsuno and Mendelzon in [49] and showed that ABox update is not closed under it. This result was extended in [71], when they defined new inexpressibility results for many other operators, including the operator from [70], and they studied both operations, namely expansion and contraction at knowledge base and ABox.

Regarding the revision of the terminological DL, Qi, and Du in [25] proposed three model-based revision operators for the DL KBs with empty ABoxes, by investigating Dalal's operator. They compared these operators and proved that one of them is more rational than the others. They also demonstrated that subsumption checking in DL-Lite under their revision operator is $P^{NP[O(\log n)]}$ -complete.

In [72] a modification is adopted to the MBAs to fit new semantics for DL KBs. The classical MBAs are based on classical interpretations. However, these new semantics is based on the so-called features, and they applied them to DL-Lite_{bool}^N. They showed that this method suffers from the same problems as classical MBA semantics.

In [73], the authors studied the WIDTIO in the DL-Lite_{A,id} and defined an algorithm that calculated the evolution of the knowledge base at the ABox-level that is done in polynomial time. In [74], they showed that maximal approximations of two well-known model-based revisions for DL-Lite_R can be computed using a syntactic algorithm. However, by applying the role feature axioms, such a coincidence of model-based and syntax-based approaches does not hold. That's why they identified conditions that guarantee a coincidence for DL-Lite_{FR}.

In [75], they studied formula-based approaches to ABox-level update in DL-Lite_A. That is, every update can be reformulated into a set of insertion and deletion instructions computable through a non-recursive data-log program to change the ABox while preserving its consistency with respect to the TBox. In [76], they proposed mechanisms to handle updates in OBDA systems, studied

update data at the ontology and source level, and showed that it is first-order rewritable. Also, they proved how evolution can be computed via non-recursive datalog.

2.3 Evolution of Prioritized Knowledge Base

In this section, we give an overview of the evolution of prioritized KB in propositional logic and then in DL ontologies.

- **Evolution of Prioritized Propositional Logic KB**

Gardenfors et al. in [77], extended the AGM framework to give a representation for changes that alter entrenchment orderings and belief sets as well. The idea of epistemic entrenchment revision was first proposed in [78] and then developed in [79], where they discovered how belief change strategies affect the corresponding preferences.

Benferhat and others in [80] studied the revision of prioritized belief bases with uncertain input information (ϕ, α) , that is semantically meaningful in the frameworks of possibility theory [27] and of Spohn's ordinal conditional functions [81]. They defined an efficient and simple syntactic implementation for both revision and contraction. They explained in their work the intuition behind the Darwiche and Pearl postulates [82]. The research demonstrated that when dealing with uncertain input, possibilistic revision aligns better with Darwiche and Pearl's postulates. The main difference is that with possibilistic revision, there are no restrictions on the uncertain input that can be used for the revision process. It is possible to enhance the suggested operators to address certain shortcomings commonly found in operators that follow the AGM and Darwiche and Pearl's postulates principles.

Qi et al. in [83] suggested a solution to address the issue of the "drowning effect". This issue arises because, when revising a belief base with a completely trustworthy formula, the outcome of the revision process omits formulas whose weights are below the inconsistency level of the updated base. This poses a problem as a high inconsistency level can result in a significant loss of information. Similar to the approach in reference [80], the operators introduced in the study [83] do not tackle this problem.

Jin and Thielscher in [84] proposed the independence postulate for iterated belief revision to address the limitations of the AGM and Darwiche and Pearl's postulates. These postulates require an agent to discard all of their previously acquired knowledge when they receive new information that contradicts their current beliefs, which is seen as a weakness. The independence postulate aims to overcome this weakness by allowing the agent to preserve some of their previous beliefs even in the face of contradicting information. Brewka in her study [85] studies the revision of the prioritized belief bases when the new input information is inconsistent with the old belief. She showed that if the new information is less reliable than conflicting old information, Even if she revises without specifying a degree of priority the postulate is violated. For such a reason, Flappa et al. in [86] proposed a

revision operator on a stratified belief base. This operator guarantees that under the revision process, the information is never lost but kept in a stratum or layer with data that is thought to be of lower importance. This idea is an extension of the one-level reuse of belief gave in [87].

Tamargo et al. in [88] studied how the belief base of an agent can be modified when new information that has a different degree of credibility is available. They proposed a framework for updating the credibility degree of the belief base within a MAS [89].

In [81], they studied belief revision with uncertain input and showed its close relationship with Jeffrey’s rule of conditioning [90]. Furthermore, in [91] the possibilistic counterparts to the revision by uncertain inputs have been defined.

- **Evolution of Prioritized DL Ontologies**

Qi and others in [92] introduced two revision operators in description logics, namely the weakening-based revision operator and its refinement. These operators are then used to develop an algorithm for addressing inconsistency in a stratified description logic knowledge base. The study demonstrates that if the weakening-based revision operator is used, the resulting knowledge base from the algorithm is semantically equivalent to the knowledge base produced by refined conjunctive maxi-adjustment (RCMA)[93]. Furthermore, in [94], they proposed a method for addressing inconsistencies in a prioritized DL knowledge base through a revision-based approach. Rather than relying on cardinality constraints for concepts, their approach involves weakening DL axioms, including both terminological and assertional axioms, by eliminating instances that contribute to the inconsistency.

Benferhat et al. in [35] introduced a new method called “Removed Sets Revision ”(RSR) for revising knowledge bases in DL-Lite, where the assertions are prioritized. This approach is influenced by belief base revision in propositional logic [95], and is based on minimizing inconsistencies. The process involves identifying the smallest possible subsets of assertions that need to be removed from the existing base to accept new information and ensure consistency. It is important to note that minimality is determined based on the cardinality of assertions rather than the inclusion of sets. This work was a continuation of the research presented in [96], which involved the modification of a group of prioritized propositional formulas. The extent to which a revision minimized the removal of a prioritized set was determined using lexicographic criteria, rather than set inclusion.

3 The \mathcal{EL} Family of Description Logics

The \mathcal{EL} description logic is a lightweight family of DLs that underlies the *OWL2-EL* profile. It aims to be both “expressive and lightweight ”. This reflects its goal of being able to represent a broad range of ontologies while

also being efficient in terms of reasoning. It is especially beneficial for ontologies that contain a large number of concepts and relationships for example the systematized nomenclature of medicine SNOMED CT and OpenGALEN.

In this section, we will present the syntax, semantics, and main reasoning tasks in \mathcal{EL} description logic.

3.1 Syntax

Let (N_C, N_R, N_I) be three infinite and mutually disjoint sets, where N_C denotes a set of atomic concepts, N_R denotes a set of atomic roles and N_I denotes a set of individuals. The \mathcal{EL} concept expressions are built according to the following syntax:

$$C, D \rightarrow \top; A; C \sqcap D; \exists r.C$$

where $A \in N_C$, $r \in N_R$. For more details about the syntax and semantics of \mathcal{EL} see Table 1.

An \mathcal{EL} ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ contains a set of terminological axioms (also called TBox, denoted by \mathcal{T}). It refers to the part that deals with defining and specifying the concepts and relationships within the ontology. For example, to express the fact that all fathers are parents, we use the following axiom:

$$Father \sqsubseteq Parent$$

This is called the general concept inclusion (GCI) axiom, meaning that the concept of “father” is more specific than the concept of “parent”, or simply that “Father” is subsumed by “Parent”. Concept equivalence axioms of the form $Person \equiv Human$, which is equivalent to $Person \sqsubseteq Human$ and $Human \sqsubseteq Person$, assert that the two concepts have the same instance.

The assertional base (also called ABox and denoted by \mathcal{A}) is responsible for asserting specific facts or instances that are significant to the domain being modeled. The ABox contains a set of concept assertions of the form $C(a)$, for example, $Father(john)$ asserts that the instance $john$ belongs to the concept $Father$. The role assertions of the form $r(a, b)$ describe the relations between the given individuals. For example, $MotherOf(maria, Castro)$ states that $maria$ is a mother of $Castro$.

Several extensions of \mathcal{EL} have been considered [1, 12, 13]. For example, \mathcal{EL}^+ [97] extends \mathcal{EL} with role inclusion axioms of the form

$$s \sqsubseteq r$$

Meaning that s is a sub-role of r , in other words, every pair of individuals related by the role s is also related by the role r . In the role inclusion axiom, the role composition axioms can be defined to describe the role $AuntOf$. Intuitively, if Anna is a sister of Janna and Janna is a parent of Bob, then Anna is

an aunt of Bob. This type of interaction between the roles `sisterOf`, `parentOf`, and `auntOf` is captured by the complex role inclusion axiom:

$$\textit{sisterOf} \circ \textit{parentOf} \sqsubseteq \textit{auntOf}$$

It is important to note that the composition relation appears only on the left-hand side of complex role inclusions. The syntax and semantics of this fragment is described in Table 1

The logic \mathcal{EL}_{\perp}^+ [29] extends \mathcal{EL}^+ by allowing the use of the bottom concept \perp in concept expression. The bottom concept represents nothing and it is the dual of the top concept (\top). It is used to express the disjointness of concepts. For example:

$$\textit{Women} \sqcap \textit{Men} \sqsubseteq \perp$$

Means no individual can be at the same time women and men. This fragment can appear as logical conflict, i.e., $\textit{Women} \sqcap \textit{Men} \sqsubseteq \perp$ and $\textit{Women} \sqsubseteq \textit{Men}$. This sub-language is syntactically and semantically presented in Table 1

The DLs \mathcal{ELCO}_{\perp}^+ [13] extends \mathcal{EL}_{\perp}^+ with the use of nominal concepts of the form $\{a\}$. A nominal is a concept that has exactly one instance. For example, $\{\textit{Julien}\}$ is the concept whose only instance is the individual denoted by `Julien`. It is important to note that using the nominal concept, the concept assertion $\textit{Mother}(\textit{Anna})$ can be turned into a concept inclusion $\{\textit{Anna}\} \sqsubseteq \textit{Mother}$ and the role assertion $\textit{ParentOf}(\textit{Anna}, \textit{Bob})$ can be expressed using role inclusion of the form $\{\textit{Anna}\} \sqsubseteq \exists \textit{parentOf}.\{\textit{Bob}\}$. The syntax and semantics of this extension are represented in Table 1.

Finally, the extension of \mathcal{ELCO}_{\perp}^+ with concrete domains, range, reflexive role restrictions, and transitions is denoted by \mathcal{EL}^{++} [12] and it is indeed the core of the *OWL-EL* specification. In \mathcal{EL}^{++} DL, data types (also called concrete domains) can be used to define new concepts by referencing specific values, like strings or integers. For example:

$$\textit{Person} \sqcap \exists \textit{hasAge}.(>, 25) \sqcap \textit{hasName}.(=, \textit{Annas})$$

Describes the person whose name is `Annas` and whose age is greater than 25. Datatypes are described first by the domain from which their values can emerge, as well as by the relationships that can be formed to constrain their possible values. In our example, $(=, \textit{Annas})$ refers to the string `Annas` and $(>, 25)$ refers to the domain of real numbers, and the relation $>$ indicates the values that are strictly greater than 25. The syntax and semantics of a such family are shown in Table 1

3.2 Semantics

The semantics are expressed in terms of interpretations $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ which are made up of a non-empty interpretation domain $\Delta^{\mathcal{I}}$ and an interpretation function $\cdot^{\mathcal{I}}$ that maps:

- Each individual $a^{\mathcal{I}} \in N_I$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$

- Each concept $A \in N_C$ to a subset $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
- Each role $r \in N_R$ to a subset $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

Furthermore, the function $\cdot^{\mathcal{I}}$ is extended in a straightforward way for concept and role expressions as depicted in Table 1.

Table 1 Syntax and Semantics of \mathcal{EL} Description Logic

	Syntax	Semantics
Atomic concept	A	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
Atomic role	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
Individual	a	$a^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
Top	\top	$\Delta^{\mathcal{I}}$
Conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
Existential restriction	$\exists r.C$	$\{x \subseteq \Delta^{\mathcal{I}} \mid \exists y \subseteq \Delta^{\mathcal{I}} \text{ s.t. } (x, y) \subseteq r^{\mathcal{I}} \text{ and } y \subseteq C^{\mathcal{I}}\}$
Role chain	$r \circ s$	$\{\langle x, y \rangle \mid \exists z \subseteq \Delta^{\mathcal{I}} \text{ s.t. } \langle x, z \rangle \in r^{\mathcal{I}} \text{ and } \langle z, y \rangle \in s^{\mathcal{I}}\}$
Bottom	\perp	\emptyset
Nominal	$\{a\}$	$\{a^{\mathcal{I}}\}$
Concrete Domain	$p(f_1, \dots, f_k)$ for $p \in \mathcal{P}$	$x \in \Delta^{\mathcal{I}} \mid \exists y_1, \dots, y_k \in \Delta^{\mathcal{D}_j} : f_i^{\mathcal{I}}(x) = y_i \text{ for } 1 \leq i \leq k \wedge (y_1, \dots, y_k) \in p^{\mathcal{D}_j}$

An interpretation \mathcal{I} is said to be a model of (or satisfies) a General Concept Inclusion (GCI), role inclusion, or role composition axiom if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$, $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$, or $(r_1 \circ r_2)^{\mathcal{I}} \subseteq s^{\mathcal{I}}$, respectively. Similarly, \mathcal{I} satisfies a concept or role assertion if $a^{\mathcal{I}} \in C^{\mathcal{I}}$ or $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$, respectively. An interpretation \mathcal{I} is a model of an ontology \mathcal{O} if it satisfies all the axioms of \mathcal{O} . An ontology is said to be consistent if it has a model. Otherwise, it is inconsistent. An axiom ϕ is entailed by an ontology, denoted by $\mathcal{O} \models \phi$, if ϕ is satisfied by every model of \mathcal{O} . We say that C is subsumed by D w.r.t an ontology \mathcal{O} iff $\mathcal{O} \models C \sqsubseteq D$. Similarly, we say that a is an instance of C w.r.t \mathcal{O} iff $\mathcal{O} \models C(a)$. A concept C is said to be in unsatisfiable w.r.t. \mathcal{O} iff $\mathcal{O} \models C \sqsubseteq \perp$, otherwise C is said to be satisfiable.

In this paper, we will only consider assertion free \mathcal{EL}_{\perp}^+ , i.e. \mathcal{EL}_{\perp}^+ without concept assertions $C(a)$ and role assertions $R(a, b)$. Note that these assertions could be easily replaced via a mapping to new concept names or simply to nominal concepts if we consider $\mathcal{EL}\mathcal{O}_{\perp}^+$.

3.3 The Main Reasoning Tasks

Let C and D be concepts, \mathcal{O} be an \mathcal{EL} ontology, and a be an individual. Several reasoning tasks are defined in \mathcal{EL} description logic [11]:

- **Subsumption Checking:** Enable to find the subclass-superclass relationship. Meaning that C subsumes D ($C \sqsubseteq_{\mathcal{O}} D$) w.r.t \mathcal{O} , if $C^{\mathcal{I}} \sqsubseteq D^{\mathcal{I}}$ and $\mathcal{I} \models \mathcal{O}$

(NR ₀)	$\frac{C_1 \sqcap \top \sqcap C_2 \sqsubseteq D}{C_1 \sqcap C_2 \sqsubseteq D}$	
(NR ₁)	$\frac{C_1 \sqcap \perp \sqcap C_2 \sqsubseteq D}{\perp \sqsubseteq D}$	
(NR ₂)	$\frac{C \sqsubseteq D_1 \sqcap D_2}{C \sqsubseteq D_1 \quad C \sqsubseteq D_2}$	
(NR ₃)	$\frac{\exists r.C \sqsubseteq D}{C \sqsubseteq A \quad \exists r.A \sqsubseteq D} : C \notin N_c, A \text{ is a new concept}$	
(NR ₄)	$\frac{C \sqsubseteq D}{C \sqsubseteq A \quad A \sqsubseteq D} : C, D \notin N_c \cup \{\perp, \top\}, A \text{ is a new concept}$	
(NR ₅)	$\frac{B \sqsubseteq \exists r.C}{B \sqsubseteq \exists r.A \quad A \sqsubseteq C} : C \notin N_c, A \text{ is a new concept}$	
(NR ₆)	$\frac{C_1 \sqcap C \sqcap C_2 \sqsubseteq D}{C \sqsubseteq A \quad C_1 \sqcap A \sqcap C_2 \sqsubseteq D} : C \notin N_c \cup \{\perp, \top\}, A \text{ is a new concept}$	

Table 2 Normal Form Rules.

- **Instance Checking:** Verifies whether an individual is an instance of a given concept. Meaning that a is an instance of the concept C in \mathcal{A} w.r.t. \mathcal{O} if \mathcal{A} and \mathcal{O} have a common model.
- **Concept Satisfiability:** denotes that a DL concept has at least one individual that satisfies the condition or property that the concept represents. Meaning that C is satisfiable w.r.t the ontology \mathcal{O} if $\mathcal{I} \models \mathcal{O}$ and $C^{\mathcal{I}} \neq \emptyset$

In this paper, we consider the classification tasks, which consist in computing all the entailed subsumptions (and equivalences) that hold between atomic concepts of an ontology $\mathcal{O} \models A \sqsubseteq B$, with $(A \in N_c \cup \top)$ and $(B \in N_c \cup \perp)$. To handle provenance in DL languages with conjunctions, it's most natural to restrict the syntax to disallow conjunctions on the right side and define the semantics as usual. The \mathcal{EL} ontologies are typically expressed in normal form, so the main limitation in this language is the avoidance of qualified existential restrictions on the right side. In this paper, we follow the procedure given in [13]. First, we transform the ontology \mathcal{O} into the normal form using the normalization rules in Table 2. Note that \mathcal{O} is said to be in normal form if each of its axioms has one of the following forms:

$$\begin{aligned}
 A \sqsubseteq B, \quad A_1 \sqcap \dots \sqcap A_n \sqsubseteq B \\
 A \sqsubseteq \exists r.B, \quad \exists r.A \sqsubseteq B \\
 r \sqsubseteq s, \quad r_1 \circ r_2 \sqsubseteq s
 \end{aligned}$$

where $A, B \in N_C \cup \{\top, \perp\}$ and $A_i \in N_C$. Once the ontology is in normal form, the reasoning is performed using the set of inference rules [10, 12].

(IR ₀)	$\frac{}{A \sqsubseteq A} : A \in N_c \cup \{\perp, \top\}$
(IR ₁)	$\frac{}{C \sqsubseteq \top} : C \sqsubseteq \top$
(IR ₂)	$\frac{}{r} : r \in \mathcal{O}$
(IR ₃)	$\frac{C \sqsubseteq D' \quad D' \sqsubseteq D}{C \sqsubseteq D}$
(IR ₄)	$\frac{A \sqsubseteq B_1 \dots A \sqsubseteq B_n \quad B_1 \sqcap \dots \sqcap B_n \sqsubseteq B}{A \sqsubseteq B} : A, B, B_i \in N_c \cup \{\perp, \top\}$
(IR ₅)	$\frac{A \sqsubseteq \exists r.B \quad B \sqsubseteq C}{A \sqsubseteq \exists r.C} : A, B, C \in N_c \cup \{\perp, \top\}, C \neq \perp$
(IR ₆)	$\frac{A \sqsubseteq \exists r.B \quad B \sqsubseteq \perp}{A \sqsubseteq \perp} : A, B \in N_c \cup \{\perp, \top\}$
(IR ₇)	$\frac{A \sqsubseteq \exists r.B \quad r \sqsubseteq s}{A \sqsubseteq \exists s.B}$
(IR ₈)	$\frac{r_1 \sqsubseteq r_2 \quad r_2 \sqsubseteq r_3}{r_1 \sqsubseteq r_3}$
(IR ₉)	$\frac{A \sqsubseteq \exists r_1.B \quad B \sqsubseteq \exists r_2.A_2 \quad r_1 \circ r_2 \sqsubseteq s}{A_1 \sqsubseteq \exists s.A_2} : A_1, r_1, A_2 \in N_c \cup \{\perp, \top\}$

Table 3 Inference Rules.

Let $cl(\mathcal{O})$ be the closure of \mathcal{O} obtained by applying the normalization rule and inference rules given in Tables 2 and 3. The following proposition holds ([10, 12]).

Proposition 1 *Let \mathcal{O} be an assertion free \mathcal{EL}_{\perp}^+ ontology and let $A, B \in N_c$ be two concepts of \mathcal{O} . Then:*

1. $\mathcal{O} \models A \sqsubseteq B$ iff $A \sqsubseteq B \in cl(\mathcal{O})$ or $A \sqsubseteq \perp \in cl(\mathcal{O})$.
2. *The complexity of computing $cl(\mathcal{O})$ is polynomial.*

In the following section, we introduce the basics of possibility theory rephrased in the extension of \mathcal{EL} description logics setting.

3.4 Possibility theory

A possibility theory was introduced by L. A. Zadeh at the end of the seventies [98] and it was developed by many researchers such as Dubois et al. in [99]. The possibility theory differs from the probability theory in the use of the dual set functions called possibility and necessity measures.

Let \mathcal{L} be a description language and Ω be a universe of discourse on a set of DL interpretations, i.e. $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}) \in \Omega$. We introduce the semantics of possibility theory over DL interpretations.

Possibility Distribution A possibility distribution, denoted by π , is the main building block of the possibility theory. It is a mapping from the universe of discourse Ω to the unit interval $[0, 1]$ ⁴, that assigns to each interpretation $\mathcal{I} \in \Omega$ a possibility degree $\pi(\mathcal{I}) \in [0, 1]$ reflecting its compatibility or consistency w.r.t available knowledge. Their weights could be interpreted in two ways: a numerical interpretation when values have a real sense and an ordinal interpretation when values only reflect a total pre-order between the different states of the world. In this paper, we consider the latter interpretation, i.e., the qualitative setting. There are two cases:

- $\pi(\mathcal{I}) = 1$: means that \mathcal{I} is totally possible (i.e. fully consistent with available knowledge).
- $\pi(\mathcal{I}) = 0$: means that \mathcal{I} is impossible (i.e. fully inconsistent).

A possibility distribution π is normalized if it has at least one totally possible state, i.e., $\exists \mathcal{I} \in \Omega$ such that $\pi(\mathcal{I}) = 1$, otherwise, π is sub-normalized. The concept of normalization is important since it reflects the presence of conflicts in the set of available information. Finally, given two interpretations \mathcal{I} and \mathcal{I}' , we say that \mathcal{I} is more consistent or more compatible than \mathcal{I}' if $\pi(\mathcal{I}) > \pi(\mathcal{I}')$.

Possibility and necessity measures. Given a possibility distribution π , standard possibility theory offers two measures from 2^Ω to the interval $[0, 1]$ which discriminate between the plausibility and the certainty regarding an event $M \subseteq \Omega$. A possibility measure $\Pi(M) = \sup\{\pi(\mathcal{I}) : \mathcal{I} \in M\}$ evaluates to what extent M is compatible or plausible w.r.t available knowledge encoded by π . A necessity measure $N(M) = 1 - \Pi(\bar{M})$, which is a dual function to Π , evaluates to what extent M is certainty entailed from available knowledge encoded by π . When $N(M) = 1$, we say that M is certain. When $N(M) \in]0, 1[$, we say that M is somewhat certain. When $N(M) = 0$ and $N(\bar{M}) = 0$, we say that there is a total ignorance about M .

A necessity measure N satisfies the following properties for normalized possibility distributions:

$$\forall M \subseteq \Omega, \forall L \subseteq \Omega, N(M \cap L) = \min(N(M), N(L))$$

and

$$\forall M \subseteq \Omega, \forall L \subseteq \Omega, N(M \cup L) \geq \max(N(M), N(L))$$

Now we are able to introduce the possibility measure and its associated necessity measure for a DL axiom. Namely, Let ϕ be an \mathcal{EL}_\perp^+ axiom and $Mod(\phi)$ be the set of models of ϕ . The possibility measure and necessity measure associated with ϕ are defined respectively as follows:

$$\Pi(Mod(\phi)) = \sup_{\mathcal{I} \in \Omega} \{\pi(\mathcal{I}) : \mathcal{I} \models \phi\},$$

and

$$N(Mod(\phi)) = 1 - \sup_{\mathcal{I} \in \Omega} \{\pi(\mathcal{I}) : \mathcal{I} \not\models \phi\}.$$

where $\mathcal{I} \models \phi$ is the satisfaction relation defined in Table 1.

⁴In fact, it is a mapping from Ω to a totally ordered scale O . This scale may often be a finite set of integers or the unit interval $[0, 1]$ and encodes our knowledge on the real world. In general, one considers the interval $[0, 1]$.

3.5 Prioritized \mathcal{EL} Ontologies

In this section, we will define the syntax and semantics of the prioritized \mathcal{EL} ontology.

Syntax A prioritized \mathcal{EL} has been defined syntactically by equipping every axiom with a confidence degree to encode its certainty. This confidence degree is simply the necessity value of an axiom that reflects their certainty degree with the available knowledge. In other words, a prioritized \mathcal{EL} ontology, denoted by \mathcal{O}_π , is made by a set of prioritized \mathcal{EL} axioms of the form (ϕ, α) .

$$\mathcal{O}_\pi = \{(\phi_i, \alpha_i) : i = 1, n\}$$

Where α_i is known as a lower bound of necessity degree $N(\phi)$ (namely $N(\phi) \geq \alpha$). Note that the higher the degree α the more ϕ is certain. Note that the axioms with α_i 's equal to '0' are not explicitly represented in the ontology. Moreover, when all the degrees are equal to 1, \mathcal{O}_π coincides with a standard \mathcal{EL}_\perp^+ ontology \mathcal{O} .

Example 1

$$\begin{aligned} ax_1 &= \langle \text{Fruits} \sqsubseteq \text{HealthyFoods}, 0.7 \rangle \\ ax_2 &= \langle \text{Avocado} \sqsubseteq \text{HealthyFats}, 1 \rangle \\ ax_3 &= \langle \text{Avocado} \sqsubseteq \text{Vegetable}, 0.2 \rangle \\ ax_4 &= \langle \text{Broccoli} \sqsubseteq \text{fruit}, 0.3 \rangle \\ ax_5 &= \langle \text{Quinoa} \sqsubseteq \text{fruit}, 0.4 \rangle \\ ax_6 &= \langle \text{Quinoa} \sqcap \text{Broccoli} \sqsubseteq \perp, 0.5 \rangle \\ ax_7 &= \langle \exists \text{hasZeroSugar.Soda} \sqsubseteq \text{HealthyFoods}, 0.6 \rangle \\ ax_8 &= \langle \text{Cookies} \sqsubseteq \text{Sweets}, 0.8 \rangle \\ ax_9 &= \langle \text{Sweets} \sqsubseteq \text{UnhealthyFoods}, 0.9 \rangle \end{aligned}$$

In this ontology, each axiom is attached with a necessity degree that evaluates its degree of certainty. Typically in this example, the axiom $\langle \text{Broccoli} \sqsubseteq \text{fruit}, 0.3 \rangle$ indicates that the broccoli is considered as a fruit with a degree equal or greater than 0.3, and the axiom $\langle \text{Avocado} \sqsubseteq \text{HealthyFats}, 1 \rangle$ indicates that the avocado is a healthy food with a degree equals to 1. Note that we use 1 to indicate the fully certain information.

Definition 1 Let \mathcal{O}_π be the prioritized \mathcal{EL}_\perp^+ ontology, and $\alpha \in [0.1]$. The notion of α -cut of \mathcal{O}_π (resp. strict α -cut), denoted by $\mathcal{O}_{\pi \geq \alpha}$ (resp. $\mathcal{O}_{\pi > \alpha}$), is defined as the set of \mathcal{EL}_\perp^+ formulas in \mathcal{O}_π having degrees at least equal (resp. strictly greater than) α .

A prioritized \mathcal{EL}_\perp^+ ontology, denoted \mathcal{O}_π , is consistent if the standard \mathcal{EL}_\perp^+ ontology \mathcal{O} , obtained by omitting the weights of \mathcal{O}_π , is consistent. If the ontology is inconsistent, then the degree of inconsistency is syntactically defined as follows:

Definition 2 The inconsistency degree of \mathcal{O}_π is syntactically defined as follows:

$$Inc(\mathcal{O}_\pi) = \max\{\alpha : \mathcal{O}_{\pi \geq \alpha} \text{ is inconsistent}\}$$

Semantics. At the semantics level, the prioritized \mathcal{EL}_\perp^+ ontology \mathcal{O}_π is represented by a possibility distribution, denoted by $\pi_{\mathcal{O}}$, defined over the set of DL interpretations, namely $\Omega = \{\mathcal{I}_1, \dots, \mathcal{I}_n\}$. The possibility distribution $\pi_{\mathcal{O}}$ assigns to each interpretation $\mathcal{I} \in \Omega$ a possibility degree $\pi(\mathcal{I}) \in]0, 1]$ reflecting what extent this latter satisfies the axioms of the ontology. More formally,

Definition 3 The possibility distribution $\pi_{\mathcal{O}}$ associated with \mathcal{EL}_\perp^+ ontology \mathcal{O}_π is defined as follows:

$$\forall \mathcal{I} \in \Omega, \pi(\mathcal{I}) = \begin{cases} 1 & \text{if } \forall (\phi_i, \alpha_i) \in \mathcal{O}_\pi, \mathcal{I} \models \phi_i \\ 1 - \max\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{O}_\pi, \mathcal{I} \not\models \phi\} & \text{otherwise.} \end{cases}$$

An interpretation \mathcal{I} that satisfies all the axioms in \mathcal{O}_π will have highest possibility degree, (namely $\pi(\mathcal{I}) = 1$). When the interpretation falsifies some axioms in \mathcal{O}_π , then $\pi(\mathcal{I})$ depends on the axiom having the maximum necessity α .

Example 2 Let $\Omega = \{\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_4, \mathcal{I}_5, \mathcal{I}_6\}$ be a set of interpretations. The following table gives the possibility distribution calculated by Definition 3.

Table 4 Possibility Distribution Given by Definition 3

\mathcal{I}	\mathcal{I}	$\pi(\mathcal{I})$
\mathcal{I}_1	Satisfies all the axioms of \mathcal{O}_π	$\pi(\mathcal{I}_1) = 1$
\mathcal{I}_2	falsifies ax_4, ax_5	$\pi(\mathcal{I}_2) = 0.6$
\mathcal{I}_3	falsifies ax_6	$\pi(\mathcal{I}_3) = 0.5$
\mathcal{I}_4	falsifies ax_8	$\pi(\mathcal{I}_4) = 0.2$
\mathcal{I}_5	falsifies ax_7, ax_6	$\pi(\mathcal{I}_5) = 0.4$
\mathcal{I}_6	falsifies ax_2, ax_1, ax_9	$\pi(\mathcal{I}_6) = 0.1$

The Table 4 provides the possibility distribution for different interpretations of a prioritized \mathcal{EL}_\perp^+ ontology \mathcal{O}_π . The possibility distribution is calculated based on the priority weights of the axioms in the ontology. Each row in the table represents an interpretation \mathcal{I} along with its associated possibility degree $\pi(\mathcal{I})$. The first row (\mathcal{I}_1) represents an interpretation that satisfies all the axioms of the ontology. Since it adheres to all the rules of the ontology, its possibility degree is 1, which means it is considered certain. The second row (\mathcal{I}_2) represents an interpretation that falsifies axioms ax_4 and ax_5 . As it violates some constraints of the ontology, its possibility degree is 0.6. The other rows follow similar schema, where each interpretation's possibility degree reflects its level of adherence to the axioms of the ontology.

A prioritized ontology \mathcal{O}_π is consistent if there is an interpretation \mathcal{I} that satisfies all the axioms of \mathcal{O}_π . Otherwise, the ontology is inconsistent and their inconsistency is:

$$Inc(\mathcal{O}_\pi) = 1 - \max_{\mathcal{I} \in \Omega} \{\pi(\mathcal{I}) : \mathcal{I} \not\models \phi\}$$

Example 3 Let \mathcal{I}'_1 and \mathcal{I}'_2 be two interpretations. Assume that \mathcal{I}'_1 satisfies the axiom (Avocado \sqsubseteq HealthyFats, 1), But does not satisfy all the other axioms. Then $\pi(\mathcal{I}'_1) = 0.1$. Assume that \mathcal{I}'_2 does not satisfy the axioms (Cookies \sqsubseteq UnhealthyFoods, 0.5), but it satisfies the others then $\pi(\mathcal{I}'_2) = 1 - 0.5 = 0.5$. The inconsistency degree is $Inc(\mathcal{O}_\pi) = 1 - 0.5 = 0.5$

Reasoning in Prioritized \mathcal{EL}_\perp^+ Ontology The main question considered in this paper is how to compute the possibilistic entailment of the prioritized \mathcal{EL}_\perp^+ ontology [29]. In the first step, we transform the ontology into its normal form using the normalization rules in Table 5. In this table, we provide a set of normalization rules to transform complex concept inclusion axioms attached to a prioritized possibility degrees (which called promise and they are above the horizontal line) into simpler and normalized axioms (which called conclusion and they are under the horizontal line), which makes the reasoning process more tractable in prioritized \mathcal{EL}_\perp^+ ontologies.

To compute the possibilistic entailment, we define the inference rules in Table 6. Inference rules, which are commonly referred to as logical rules of inference, serve as foundational principles in deductive reasoning, allowing us to draw conclusions based on established premises or axioms. These rules play a crucial role in formal logic, ensuring the soundness and validity of logical arguments. Typically expressed in the form of "if-then" statements, inference rules present the conditions or premises in the "if" part and the logically deduced conclusion in the "then" part, based on the information provided by the given premises.

The rules in Tables 6, 5 are obtained using the following Lemma.

Lemma 1 Considering that \mathcal{O}_π is the prioritized \mathcal{EL}_\perp^+ ontology that includes the two axioms (ϕ, α_1) and (ϕ, α_2) then \mathcal{O}_π and $\mathcal{O}'_\pi = \{\mathcal{O}_\pi \setminus \{(\phi, \alpha_1), (\phi, \alpha_2)\}\} \cup \{(\phi, \max(\alpha_1, \alpha_2))\}$ are equivalent in the sense that they induce the same possibility distribution, i.e., $\forall \mathcal{I} \in \Omega, \pi_{\mathcal{O}}(\mathcal{I}) = \pi_{\mathcal{O}'}(\mathcal{I})$.

In the following proposition, we show that the prioritized \mathcal{EL}_\perp^+ ontology \mathcal{O}_π and the new ontology \mathcal{O}'_π , obtained by applying the normalization rules in Table 5 into \mathcal{O}_π , induce the same possibility distribution.

Proposition 2 Let \mathcal{O}_π be the prioritized \mathcal{EL}_\perp^+ ontology and \mathcal{O}'_π be the ontology obtained by applying the normalization rules in Table 5 into \mathcal{O}_π . Then \mathcal{O}_π and \mathcal{O}'_π induce the same possibility distribution.

(PNR ₀)	$\frac{(C_1 \sqcap \top \sqcap C_2 \sqsubseteq D, \alpha)}{(C_1 \sqcap C_2 \sqsubseteq D, \alpha)}$	
(PNR ₁)	$\frac{(C_1 \sqcap \perp \sqcap C_2 \sqsubseteq D, \alpha)}{(\perp \sqsubseteq D, \alpha)}$:
(PNR ₂)	$\frac{(C \sqsubseteq D_1 \sqcap D_2, \alpha)}{(C \sqsubseteq D_1, \alpha) (C \sqsubseteq D_2, \alpha)}$	
(PNR ₃)	$\frac{(\exists r.C \sqsubseteq D, \alpha)}{(C \sqsubseteq A, 1) (\exists r.A \sqsubseteq D, \alpha)}$: $C \notin N_c$, A is a new concept
(PNR ₄)	$\frac{(C \sqsubseteq D, \alpha)}{(C \sqsubseteq A, 1) (A \sqsubseteq D, \alpha)}$: $C, D \notin N_c \cup \{\perp, \top\}$, A is a new concept
(PNR ₅)	$\frac{(B \sqsubseteq \exists r.C, \alpha)}{(B \sqsubseteq \exists r.A, 1) (A \sqsubseteq C, \alpha)}$: $C \notin N_c$, A is a new concept
(PNR ₆)	$\frac{(C_1 \sqcap C \sqcap C_2 \sqsubseteq D, \alpha)}{(C \sqsubseteq A, 1) (C_1 \sqcap A \sqcap C_2 \sqsubseteq D, \alpha)}$: $C \notin N_c \cup \{\perp, \top\}$, A is a new concept

Table 5 Prioritized normalization rules.

In the following proposition, we study the inference rules of the prioritized \mathcal{EL}_\perp^+ ontology given in Table 6.

Proposition 3 *Let \mathcal{O}_π be the prioritized \mathcal{EL}_\perp^+ ontology and $cl(\mathcal{O}_\pi)$ be its closed ontology, obtained by applying the rules given in Tables 5 and 6 into \mathcal{O}_π . Then \mathcal{O}_π and $cl(\mathcal{O}_\pi)$ induce the same possibility distribution.*

Example 4 Considering the prioritized \mathcal{EL}_\perp^+ ontology that is given in Example 1. Applying the rule **PIR**₃ into the axioms $\langle \text{Cookies} \sqsubseteq \text{Sweets}, 0.8 \rangle$ and $\langle \text{Sweets} \sqsubseteq \text{UnhealthyFoods}, 0.9 \rangle$ leads to obtain the new axiom $\langle \text{Cookies} \sqsubseteq \text{UnhealthyFoods}, \min(0.8, 0.9) = 0.8 \rangle$

The following proposition studies the prioritized entailment, which is given for subsumption relation as is the main reasoning task in \mathcal{EL} .

Proposition 4 *Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology and $cl(\mathcal{O}_\pi)$ be its closure by applying the normalization rules in Table 5 and inference rules in Table 6. Let A and B be two concepts of \mathcal{O}_π . Then $\mathcal{O}_\pi \models (A \sqsubseteq B, \alpha)$ if*

- $(A \sqsubseteq B, \beta) \in cl(\mathcal{O}_\pi)$ with $\beta \geq \alpha$
- $(A \sqsubseteq \perp, \beta) \in cl(\mathcal{O}_\pi)$ with $\beta \geq \alpha$

Ontologies are usually represented by various distinct aspects, circumstances, and viewpoints. As a result, unifying, updating, and modifying multiple pieces of knowledge may lead to inconsistencies and conflicts in the logical representation. In the next section, we will present solutions for

(PIR ₀)	$\overline{(A \sqsubseteq A, 1)}$	$: A \in N_c \cup \{\perp, \top\}$
(PIR ₁)	$\overline{(C \sqsubseteq \top, 1)}$	$: C \sqsubseteq \top$
(PIR ₂)	$\overline{(r, 1)}$	$: r \in \mathcal{O}$
(PIR ₃)	$\frac{(C \sqsubseteq D', \alpha_1) \quad (D' \sqsubseteq D, \alpha_2)}{(C \sqsubseteq D, \min(\alpha_1, \alpha_2))}$	
(PIR ₄)	$\frac{(A \sqsubseteq B_1 \dots A \sqsubseteq B_n, \alpha_1) \quad (B_1 \sqcap \dots \sqcap B_n \sqsubseteq B, \alpha_2)}{(A \sqsubseteq B, \min(\alpha_1, \alpha_2))}$	$: A, B, B_i \in N_c \cup \{\perp, \top\}$
(PIR ₅)	$\frac{(A \sqsubseteq \exists r.B, \alpha_1) \quad (B \sqsubseteq C, \alpha_2)}{(A \sqsubseteq \exists r.C, \min(\alpha_1, \alpha_2))}$	$: A, B, C \in N_c \cup \{\perp, \top\}, C \neq \perp$
(PIR ₆)	$\frac{(A \sqsubseteq \exists r.B, \alpha) \quad (B \sqsubseteq \perp, 1)}{(A \sqsubseteq \perp, \alpha)}$	$: A, B \in N_c \cup \{\perp, \top\}$
(PIR ₇)	$\frac{(A \sqsubseteq \exists r.B, \alpha_1) \quad (r \sqsubseteq s, \alpha_2)}{(A \sqsubseteq \exists s.B, \min(\alpha_1, \alpha_2))}$	
(PIR ₈)	$\frac{(r_1 \sqsubseteq r_2, \alpha_1) \quad (r_2 \sqsubseteq r_3, \alpha_2)}{(r_1 \sqsubseteq r_3, \min(\alpha_1, \alpha_2))}$	
(PIR ₉)	$\frac{(A \sqsubseteq \exists r_1.B, \alpha_1) \quad (B \sqsubseteq \exists r_2.A_2, \alpha_2) \quad (r_1 \circ r_2 \sqsubseteq s, \alpha_3)}{(A_1 \sqsubseteq \exists s.A_2, \min(\alpha_1, \alpha_2, \alpha_3))}$	$: A_1, r_1, A_2 \in N_c \cup \{\perp, \top\}$

Table 6 Prioritized inference rules.

addressing these issues and provide the appropriate context for discovering and developing our solutions.

4 Evolution of Prioritized \mathcal{EL}_{\perp}^+ Ontology

The revision process consists of incorporating a new ontology into the existing one while preserving its consistency. Similar to the AGM postulate [21], the revision process is based on three main ideas:

- The principle of prioritization, according to which a new ontology is given priority over existing ones.
- The principle of consistency states that after using the revision operator, the resulting ontology must be consistent.
- The principle of minimal change states that only a few formulas from the old ontology should be changed during the revision process.

In the following sections, we will study the syntactical and semantic evolution of prioritized \mathcal{EL}_{\perp}^+ ontology.

4.1 Semantics Evolution of Prioritized \mathcal{EL}_{\perp}^+ Ontology

Let \mathcal{O}_{π} be the prioritized \mathcal{EL}_{\perp}^+ ontology and $\pi_{\mathcal{O}_{\pi}}$ be its attached possibility distribution obtained by Definition 3. Considering that \mathcal{O}_{π} is consistent, which means that there is a model \mathcal{I} that satisfies all the axioms of \mathcal{O}_{π} and the

possibility distribution is normalized. Let (ϕ, u) be the new input information. Then, two cases hold: the input is totally reliable (namely $u = 1$) or uncertain (namely $0 < u < 1$). The revision process in the prioritized \mathcal{EL}_{\perp}^+ ontology consists of adding new input information to \mathcal{O}_{π} while preserving its consistency.

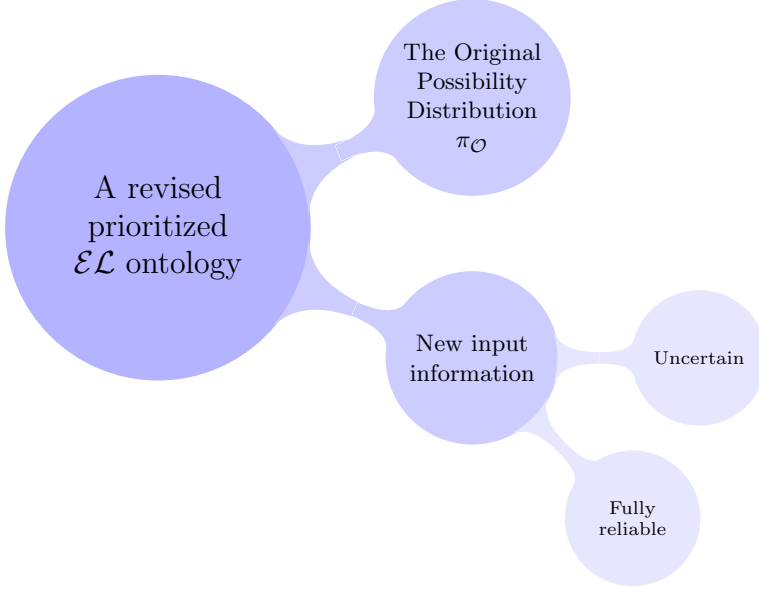


Fig. 1 Semantic Revision of Prioritized \mathcal{EL} Ontology

Figure 1 shows that the revision at the semantic level takes as input the original possibility distribution $\pi_{\mathcal{O}_{\pi}}$ and the new input information and transforms $\pi_{\mathcal{O}_{\pi}}$ into a revised possibility distribution. Section 4.1.1 will study the semantic revision with sure (fully reliable) input information, namely ϕ . Then, the revision with uncertain input, namely (ϕ, u) is defined. In both cases the revised possibility distribution is defined by the following expression:

$$\pi'_{\mathcal{O}_{\pi}} = \pi_{\mathcal{O}_{\pi}}(\cdot \mid (\phi)) \text{ (resp. } \pi'_{\mathcal{O}_{\pi}} = \pi_{\mathcal{O}_{\pi}}(\cdot \mid (\phi, u)) \text{)}$$

The input is considered as an assumption that must be satisfied by the revised possibility distribution $\pi'_{\mathcal{O}_{\pi}}$. Note that $\pi'_{\mathcal{O}_{\pi}}$ should satisfy the following logical properties:

- **P1:** $\pi'(\mathcal{I}) = 1$
- **P2:** if $\pi(\mathcal{I}) = 0$ then $\pi'(\mathcal{I}) = 0$
- **P3** $\Pi'(\phi) = 1$ and $N'(\phi) \geq u$

The first logical property, i.e., **P1** ensures that each revised possibility distribution is normalized, implying that the resulting ontology is consistent. The

second property **P2** means that an impossible interpretation remains impossible after conditioning. According to **P3**, the reliable input should be derived from the ontology with at least the prescribed necessity level.

Based on the logical properties **P1** – **P3**, two definitions of conditioning have been proposed in qualitative and quantitative settings [27]. In this paper, we study the case of qualitative ordering, which is based on conditioning using the min operator [29].

4.1.1 Min-based Prioritized \mathcal{EL}_\perp^+ Possibility Distribution Conditioning

In this section, we study first the case where the input information is fully reliable, i.e., $u = 1$. This means that any interpretation \mathcal{I} that does not satisfy ϕ , i.e. falsifies ϕ , is declared impossible ($\pi(\mathcal{I}) = 0$). The semantic evolution consists in conditioning the possibility distribution of the ontology (Definition 3) with the new input $(\phi, 1)$. Based on this type of input, two situations hold: either the input is consistent with the ontology or it is inconsistent. In both situations the revised possibility distribution $\pi'_{\mathcal{O}_\pi}$ is defined as follows:

Definition 4 Let \mathcal{O}_π be the prioritized \mathcal{EL}_\perp^+ ontology and ϕ be the new input information. The revised possibility distribution $\pi'_{\mathcal{O}_\pi}$ is defined as follows:

$$\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot \mid (\phi)) = \begin{cases} 1 & \text{if } \pi(\mathcal{I}) = \Pi(\phi) \text{ and } \mathcal{I} \models \phi \\ \pi(\mathcal{I}) & \text{if } \pi(\mathcal{I}) < \Pi(\phi) \\ 0 & \text{if } \mathcal{I} \not\models \phi \text{ otherwise} \end{cases}$$

The definition 4 defines the process of revising the possibility distribution ($\pi'_{\mathcal{O}_\pi}$) of a prioritized \mathcal{EL}_\perp^+ ontology (\mathcal{O}_π) when new input information ϕ is fully reliable ($u = 1$). The process starts by conditioning the original possibility distribution of the ontology ($\pi_{\mathcal{O}_\pi}$) with the new input $(\phi, 1)$. There are three possible cases that define how the possibility values of interpretations are updated:

1. If $\pi(\mathcal{I}) = \Pi(\phi)$ and \mathcal{I} satisfies ϕ , meaning that ϕ is true in \mathcal{I} and its possibility value is the greatest among all interpretations satisfying ϕ , then the possibility value of \mathcal{I} remains 1.
2. If $\pi(\mathcal{I}) < \Pi(\phi)$, meaning that the possibility value of \mathcal{I} for ϕ is smaller than the greatest possibility value among interpretations satisfying ϕ , then the possibility value of \mathcal{I} remains unchanged.
3. If \mathcal{I} does not satisfy ϕ , indicating that ϕ is false in \mathcal{I} , then the possibility value of \mathcal{I} is set to 0.

Example 5 We continue with the prioritized \mathcal{EL}_\perp^+ ontology that is given in Example 1. Let $\langle \text{Quinoa} \sqsubseteq \text{Briccoli}, 1 \rangle$ be the new input information. The revised possibility distribution $\pi_1(\mathcal{I})$ is given in Table 8.

Table 7 Possibility Distribution Given by Definition 4

\mathcal{I}	\mathcal{I}	$\pi(\mathcal{I})$	$\pi_1(\mathcal{I})$
\mathcal{I}_1	Satisfies all the axioms of \mathcal{O}_π	1	0
\mathcal{I}_2	falsifies ax_4, ax_5	0.6	0.6
\mathcal{I}_3	falsifies ax_6	0.5	0
\mathcal{I}_4	falsifies ax_8	0.2	0.2
\mathcal{I}_5	falsifies ax_7, ax_6	0.4	0.4
\mathcal{I}_6	falsifies ax_2, ax_1, ax_9	0.1	0.1

Based on this example, we have a priori $\Pi(\text{Quinoa} \sqsubseteq \text{Briccoli}) = 0.5$ since $\mathcal{I}_3, \mathcal{I}_5 \models (\text{Quinoa} \sqsubseteq \text{Briccoli})$. Then, the best models of the new input get the best possibility degree (i.e., degree equals to 1). However, the countermodels will get zero degree.

The following proposition shows that the revised possibility distribution satisfies the postulates **P1- P2**.

Proposition 5 *Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology and $\pi_{\mathcal{O}_\pi}$ be its associated possibility distribution. Let (ϕ, u) be the new input information. Then $\pi_1(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\cdot \mid \phi)$ obtained by Definition 4 satisfies the postulates **P1- P2**.*

Conditioning of $\pi_{\mathcal{O}_\pi}$ with Uncertain Input Information (ϕ, u) The revision of the prioritized \mathcal{O}_π ontology with uncertain input information of the form (ϕ, u) consists of updating the possibility distribution associated with \mathcal{O}_π interpretations based on the new evidence. Depending on whether the input ϕ is consistent or not with $\pi_{\mathcal{O}_\pi}$, the revised possibility distribution $\pi'_{\mathcal{O}_\pi}(\cdot \mid_m(\phi, u))$ is defined in the following definition.

Definition 5 Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology, and $\pi_{\mathcal{O}_\pi}$ be the associated possibility distribution. Let (ϕ, u) be the uncertain input. The min-based conditioning in a prioritized \mathcal{EL}_\perp^+ is defined as follows:

$$\forall \mathcal{I} \models \phi, \pi_{\mathcal{O}_\pi}(\cdot \mid_m(\phi, u)) = \begin{cases} 1 & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) = \Pi(\phi) \\ 1 - u & \text{if } \Pi(\phi) \leq \pi_{\mathcal{O}_\pi} \leq 1 - u \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$$

$$\forall \mathcal{I} \not\models \phi, \pi_{\mathcal{O}_\pi}(\cdot \mid_m(\phi, u)) = \begin{cases} 1 - u & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) = N_\pi(\phi) \\ 1 - u & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) > 1 - u \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$$

The definition 5 explains the process of min-based conditioning in a prioritized \mathcal{EL}_\perp^+ ontology (\mathcal{O}_π) when uncertain input (ϕ, u) is provided. The process involves updating the possibility values of different interpretations in the ontology depending on whether they satisfy ϕ or not. When we update the possibility values, the following cases are defined:

1. If an interpretation \mathcal{I} satisfies ϕ (i.e., $\mathcal{I} \models \phi$), then the possibility value of \mathcal{I} after conditioning is expressed based on the following conditions:
 - If the original possibility distribution of ϕ is equal to the possibility value of ϕ among all interpretations ($\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \Pi(\phi)$), then the possibility value of \mathcal{I} remains 1.
 - If the original possibility value of \mathcal{I} for ϕ lies between the minimum and maximum possibility values of ϕ (i.e., $\Pi(\phi) \leq \pi_{\mathcal{O}_\pi}(\mathcal{I}) \leq 1 - u$), then the possibility value of \mathcal{I} is updated to $(1 - u)$, where u is the uncertainty level of the input (ϕ, u) .
 - Otherwise, the possibility value of \mathcal{I} remains unchanged.
2. If an interpretation \mathcal{I} does not satisfy ϕ (i.e., $\mathcal{I} \not\models \phi$), then the possibility value of \mathcal{I} after conditioning is defined based on the following conditions:
 - If the original possibility distribution of ϕ is equal to the minimum possibility value of ϕ among all interpretations ($\pi_{\mathcal{O}_\pi}(\mathcal{I}) = N\pi(\phi)$), then the possibility value of \mathcal{I} is updated to $(1 - u)$.
 - If the original possibility distribution is greater than $(1 - u)$, then the possibility value of \mathcal{I} is updated to $(1 - u)$.
 - Otherwise, the possibility value of \mathcal{I} remains unchanged.

Example 6 We continue with the prioritized \mathcal{EL}_\perp^+ ontology that is given in Example 1 and Example 2. Considering the two separated inputs that must be accepted, the first one $\langle \text{Quinoa} \sqsubseteq \text{Broccoli}, 0.9 \rangle$ and the second one $\langle \text{Quinoa} \sqsubseteq \text{Broccoli}, 0.2 \rangle$. The revised possibility distribution $\pi'_{\mathcal{O}_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I} \mid_m (\text{Quinoa} \sqsubseteq \text{Broccoli}, 0.9))$ and $\pi''_{\mathcal{O}_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I} \mid_m (\text{Quinoa} \sqsubseteq \text{Broccoli}, 0.2))$ is given in Table 8. The red color represents the possibility values that is updated to $(1 - u)$. The green color represents the unchanged possibility values and the blue color represents the high possibility values namely $\pi_{\mathcal{O}_\pi} = 1$. In the initial possibility distribution $\pi_{\mathcal{O}_\pi}(\mathcal{I})$, we have

Table 8 Possibility Distribution Given by Definition 5

\mathcal{I}	\mathcal{I}	$\pi(\mathcal{I})$	$\pi'(\mathcal{I})$	$\pi''(\mathcal{I})$
\mathcal{I}_1	Satisfies all the axioms of \mathcal{O}_π	1	0.1	0.8
\mathcal{I}_2	falsifies ax_4, ax_5	0.6	0.1	0.6
\mathcal{I}_3	falsifies ax_6	0.5	1	1
\mathcal{I}_4	falsifies ax_8	0.2	0.1	0.2
\mathcal{I}_5	falsifies ax_7, ax_6	0.4	0.4	0.8
\mathcal{I}_6	falsifies ax_2, ax_1, ax_9	0.1	0.1	0.1

$\Pi(\text{Quinoa} \sqsubseteq \text{Broccoli}) = 0.6$. Therefore, its necessity degree is equal to 0. Then the new information should be satisfied to increase its necessity degree to 0.9. In the second scenario, the necessity degree of the input should be lowered to 0.2. As shown in Table 8, the interpretations $\mathcal{I}_3, \mathcal{I}_5$ are models of $(\text{Quinoa} \sqsubseteq \text{Broccoli})$, which means that $\Pi(\text{Quinoa} \sqsubseteq \text{Broccoli}) = 0.5$. However $\langle \mathcal{I}_1, \mathcal{I}_3, \mathcal{I}_4, \mathcal{I}_6 \rangle \not\models (\text{Quinoa} \sqsubseteq \text{Broccoli})$, which means that $N_\pi(\text{Quinoa} \sqsubseteq \text{Broccoli}) = 1$.

The following proposition shows that the revised possibility distribution using uncertain input of the form (ϕ, u) satisfies the postulates **P1- P3**.

Proposition 6 *Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology and $\pi_{\mathcal{O}_\pi}$ be its associated possibility distribution. Let (ϕ, u) be the new input information. Then $\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot \mid (\phi, u))$ obtained by Definition 5 satisfies the postulates **P1- P3**.*

The conditioning suggested in Definition 5 is not entirely satisfactory, as it can lead to counterintuitive outcomes, as demonstrated in the upcoming example. Conditioning in Definition 5 is effective only when the input contradicts the ontology or is initially deduced with a weight lower than the prescribed weight. In such cases, the revision involves merely adding the input to the ontology (a form of expansion). Nonetheless, in Definition 5 conditioning is ineffective when the input is initially deduced with a weight higher than the prescribed weight. This scenario is exemplified in the following case.

Example 7 We continue with the ontology in Example 1. Applying the inference rules in Table 6, we have the following ontology $\mathcal{O}' = \mathcal{O}_\pi \cup \langle \text{cookies} \sqsubseteq \text{UnhealthyFoods}, 0.8 \rangle$ since it is produced from the two axioms $\langle \text{Cookies} \sqsubseteq \text{Sweets}, 0.8 \rangle$ and $\langle \text{Sweets} \sqsubseteq \text{UnhealthyFoods}, 0.9 \rangle$. Considering the two situations: The first, when the input is $\langle \text{cookies} \sqsubseteq \text{UnhealthyFoods}, 0.9 \rangle$ and the second when the input is $\langle \text{cookies} \sqsubseteq \text{UnhealthyFoods}, 0.2 \rangle$. The possibility distribution is given in Table 9. The inter-

Table 9 Possibility Distribution Given by Definition 5

\mathcal{I}	\mathcal{I}	$\pi(\mathcal{I})$	$\pi'(\mathcal{I})$	$\pi''(\mathcal{I})$
\mathcal{I}_1	Satisfies all the axioms of \mathcal{O}_π	1	1	1
\mathcal{I}_2	falsifies ax_4, ax_5	0.6	0.6	0.8
\mathcal{I}_3	falsifies ax_6	0.5	0.5	0.5
\mathcal{I}_4	falsifies ax_8	0.2	0.1	0.8
\mathcal{I}_5	falsifies ax_7, ax_6	0.4	0.4	0.8
\mathcal{I}_6	falsifies ax_2, ax_1, ax_9	0.1	0.1	0.1

pretations $\mathcal{I}_4, \mathcal{I}_6$ are not models of $\langle \text{cookies} \sqsubseteq \text{UnhealthyFoods} \rangle$, then $N_\pi(\text{cookies} \sqsubseteq \text{UnhealthyFoods}) = 0.2$. However, the interpretations $\mathcal{I}_1, \mathcal{I}_2, \mathcal{I}_3, \mathcal{I}_5$ satisfy the axiom $\langle \text{cookies} \sqsubseteq \text{UnhealthyFoods} \rangle$, then $\Pi(\text{cookies} \sqsubseteq \text{UnhealthyFoods}) = 1$. To make sure that $N'(\text{cookies} \sqsubseteq \text{UnhealthyFoods})$ equals 0.9, the degrees of possibility for the interpretations \mathcal{I}_4 and \mathcal{I}_6 are adjusted to 0.1 by subtracting 0.9 from 1. When the input is $\text{cookies} \sqsubseteq \text{UnhealthyFoods} = 0.2$, then there is a problem in the possibility distribution $\pi''(\mathcal{I})$ associated to the interpretation \mathcal{I}_4 . Therefore, in order to have a necessity degree of $\text{cookies} \sqsubseteq \text{UnhealthyFoods}$ equal to 0.2, we must lower at least the necessity degree of $\text{Sweets} \sqsubseteq \text{cookies}$ to 0.2 since it is lower than that of $\text{cookies} \sqsubseteq \text{UnhealthyFoods}$. However, if the necessity degree of $\text{Sweets} \sqsubseteq \text{cookies}$ is lowered to 0.2, then the corresponding possibility distribution $\pi(\mathcal{I})$ after modification will not be equivalent to that given in Table 9. Thus, conditioning with Definition 5 does not capture the syntactic revision that will be detailed in the following section.

4.2 The Evolution of Prioritized \mathcal{EL}_{\perp}^+ Ontology at Syntactic Level

Revision at the syntactic level is the act of adding new information (ϕ, u) to the ontology \mathcal{O}_{π} . Regarding the type of input information, we first study revision when the input information is inconsistent with the ontology, and then, when the new input is consistent, as shown in Figure 2.

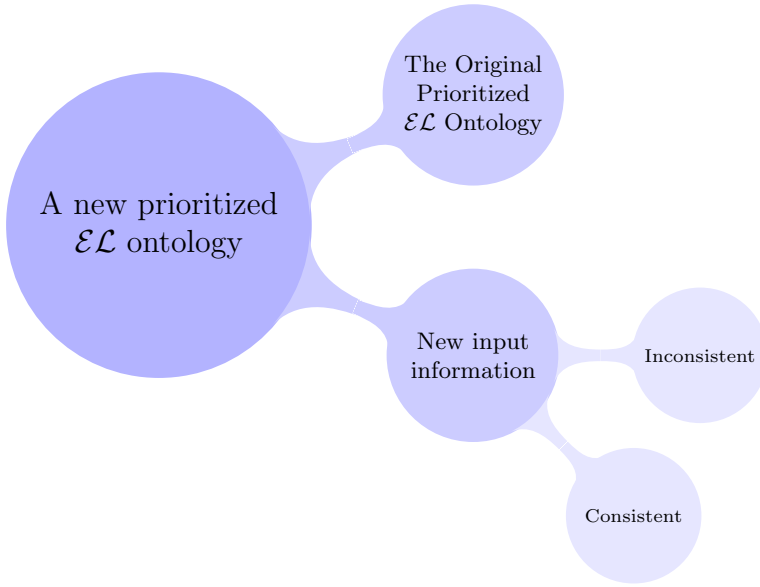


Fig. 2 Syntactic Revision of Prioritized \mathcal{EL} Ontology

4.2.1 Revision with Inconsistent Input Information

During the revision process of the prioritized \mathcal{EL}_{\perp}^+ ontology with inconsistent input information (ϕ, u) , two situations occur:

1. The input (ϕ, u) is inhibited by higher-priority axioms that contradict it.
2. The input (ϕ, u) is not inhibited.

The resulting ontology \mathcal{O}'_{π} is obtained in both cases by adding the new input (ϕ, u) to the old ontology \mathcal{O}_{π} .

The syntactical revision presented in Figure 3 is performed using the following steps:

- *Prioritized Ontology*: This is the starting point of the algorithm. It represents the initial prioritized ontology.
- *Add New Input $(\phi, 1)$* : In this step, we add the new fully reliable input information $(\phi, 1)$ to the ontology.

Algorithm 1: Revision with Inconsistent Input

input: The original ontology \mathcal{O}_π and the uncertain input (ϕ, u)

output: A consistent ontology \mathcal{O}''_π

1 **begin**

 // \mathcal{O}_π contains a set of axioms

2 $\mathcal{O}'_\pi \leftarrow \emptyset;$

 // Add the new information with a higher level of certainty to \mathcal{O}_π .

3 $\mathcal{O}'_\pi \leftarrow \mathcal{O}_\pi \cup (\phi, 1);$

 // Calculate the inconsistency degree λ_{Inc} of the new ontology.

4 **for** $i = 1$ **to** n **do**

5 $\lambda_{Inc} \leftarrow \max\{\alpha_i : \mathcal{O}'_{\pi \geq \alpha} \text{ is inconsistent}\};$

 // Delete the axioms that have a certain degree less than or equal to the
 inconsistency λ_{Inc} of the augmented ontology \mathcal{O}'_π

 // \mathcal{O}_{Inc} is a subset of the ontology contains the axioms less than the
 inconsistency degree

6 $\mathcal{O}_1 \leftarrow \mathcal{O}'_\pi \setminus \mathcal{O}_{Inc};$

 // Add the new information at its prescribed level to the obtained ontology

7 $\mathcal{O}''_\pi \leftarrow \mathcal{O}_1 \cup (\phi, u)$

8 **return** $\mathcal{O}''_\pi;$

- *Compute Inconsistency Degree λ_{Inc} :* calculate the inconsistency degree λ_{Inc} of the original prioritized ontology with the new input information added in the previous step.
- *Delete Axioms with Priority $\leq \lambda_{Inc}$:* delete the axioms from the augmented ontology that have a priority level less than or equal to the inconsistency degree λ_{Inc} .
- *Ontology Consistent:* This is a decision point. It checks whether the ontology is now consistent after deleting the conflicting axioms. If the ontology is consistent, the algorithm proceeds to the next step. Otherwise, it goes back to recompute the inconsistency degree and repeat the process.
- *Add New Input at Prescribed Level:* If the ontology is consistent, the algorithm adds the new input (ϕ, u) to the ontology at its prescribed certainty level u . This is the final step where the revised ontology is created.
- *Revised Ontology:* This is the end of the flowchart, representing the revised prioritized ontology after adding and removing the necessary axioms based on the new input and inconsistency degree.

The previous steps guaranteed the consistency of the obtained ontology \mathcal{O}''_π from the Algorithm 1.

The following proposition gives the possibility distribution $\pi_{\mathcal{O}'_\pi}$ of the obtained ontology \mathcal{O}'_π using Definition 5.

Proposition 7 *Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology, and $\pi_{\mathcal{O}_\pi}$ be the associated possibility distribution. Let (ϕ, u) be the new input information and $\lambda = Inc(\mathcal{O}'_\pi)$,*

where $\mathcal{O}'_\pi = \mathcal{O}_\pi \cup \{(\phi, 1)\}$. Then, the new prioritized ontology \mathcal{O}'_π is defined as follows:

$$\mathcal{O}'_\pi = \{(\phi, u)\} \cup \{(\Phi, \alpha) : (\Phi, \alpha) \in \mathcal{O}_\pi \text{ and } \alpha > \lambda\}$$

The associated possibility distribution obtained by conditioning defined in Definition 5 is as follows:

$$\forall \mathcal{I} \in \Omega, \pi'_{\mathcal{O}'_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I} \mid_m (\phi, u))$$

With $\pi_{\mathcal{O}_\pi}(\mathcal{I} \mid_m (\phi, u))$ represents the possibility distribution obtained by $\pi_{\mathcal{O}_\pi}$ with min-based conditioning in Definition 5

Example 8 We continue with Example 2 and Example 6. Considering the input $\langle \text{Quinoa} \sqsubseteq \text{Broccoli}, 0.9 \rangle$. Then the inconsistency of the augmented ontology, i.e., $\lambda = \text{Inc}(\mathcal{O}_\pi \cup \langle \text{Quinoa} \sqsubseteq \text{Broccoli}, 1 \rangle) = 0.5$. Therefore, the axioms $\langle \text{Quinoa} \sqsubseteq \text{Broccoli}, 0.9 \rangle$ is not inhibited with the axioms that have priority levels that contradict it. Therefore, the obtained ontology $\mathcal{O}'_\pi = \{ \langle \text{Quinoa} \sqsubseteq \text{Broccoli}, 0.9 \rangle, \langle \text{Fruit} \sqsubseteq \text{HealthyFoods}, 0.7 \rangle, \langle \exists \text{hasZeroSugar.Soda} \sqsubseteq \text{HealthyFoods}, 0.6 \rangle, \langle \text{Cookies} \sqsubseteq \text{Sweets}, 0.8 \rangle, \langle \text{Avocado} \sqsubseteq \text{HealthyFoods} \rangle$ and their associated possibility distributions is represented in Table 8. Now when the input information is $\langle \text{Quinoa} \sqsubseteq \text{Broccoli}, 0.2 \rangle$, then the axioms $\langle \text{Quinoa} \sqsubseteq \text{Broccoli}, 0.2 \rangle$ is inhibited with the axioms that have priority levels that contradict it and their possibility distribution is displayed in Table 8.

The following proposition studies the computational complexity of the Algorithm 1.

Proposition 8 *The computational of the Algorithm 1 is efficient and done in polynomial time.*

4.2.2 Revision with Consistent Input Information

Two cases should be taken into consideration during the revision process with consistent input:

1. The input information (ϕ, u) is inferred from the ontology, i.e., $\mathcal{O}_\pi \models (\phi, u)$.
2. The input (ϕ, u) not inferred from \mathcal{O}_π , i.e., $\mathcal{O}_\pi \not\models (\phi, u)$.

In the first case, when the input (ϕ, u) is inferred from the ontology, two scenarios exist depending on the necessity measure of the input, namely the a priori necessity measure $N(\phi, u) = p$ and the prescribe necessity measure $N'(\phi, u) = u$.

- If $p > u$, then the information is inferred to a degree greater than its prescribed level.
- If $p < u$, then the information is inferred to a degree less than its prescribed level.

In \mathcal{EL}_\perp^+ ontology, to determine the extent to which the input (ϕ, u) is deduced from the prioritized ontology, i.e. $\mathcal{O} \models (\phi, u)$ where $p > u$ or $u > p$, we first add to the ontology \mathcal{O}_π the assumption that the input ϕ false. The

Algorithm 2: Revision with Consistent Input

input: The original ontology \mathcal{O}_π and the assumption ϕ false

output: A consistent ontology \mathcal{O}'_π

```
1 begin
   //  $\mathcal{O}_\pi$  contains a set of axioms
2    $\mathcal{O}'_\pi \leftarrow \emptyset$ 
    $\mathcal{S} \leftarrow \emptyset$ 
   // Add the assumption that  $\phi$  is false with the highest priority level to
    $\mathcal{O}_\pi$ .
3    $\mathcal{O}'_\pi \leftarrow \mathcal{O}_\pi \cup (\phi, 1)$ 
   // Calculate the inconsistency degree  $p$  of the new ontology.
4   for  $i = 1$  to  $n$  do
5      $p \leftarrow \max\{\alpha_i : \mathcal{O}'_{\pi \geq \alpha} \text{ is inconsistent}\}$ 
     while  $u > p$  do
6        $\mathcal{O}'_\pi \leftarrow \mathcal{O}_\pi \cup (\phi, u)$ 
7       if  $(\mathcal{S} \geq \mathcal{O}_{\pi \geq u}), (\mathcal{S} \leq \mathcal{O}_{\pi \leq p}), (\mathcal{S} \models \phi)$  then
8          $\mathcal{S} \leftarrow (\phi, u)$ 
9 return  $\mathcal{O}'_\pi$ 
```

construction of the new ontology in Algorithm 2 is done using steps in Figure 4:

1. Add the assumption that ϕ false to the prioritized ontology \mathcal{O}_π with a higher priority level, namely $u = 1$.
2. Compute the inconsistency degree of the new ontology, $\mathcal{O}'_\pi = \mathcal{O}_\pi \cup (\phi, 1)$, which is equal to p , i.e., $\text{Inc}(\mathcal{O}'_\pi) = p$
3. Two situations hold:
 - 3.1** If $u > p$, then the new ontology is $\mathcal{O}'_\pi = \mathcal{O}_\pi \cup \{(\phi, u)\}$
 - 3.2** When $u < p$, two cases hold:
 - Reduce the weights of axioms that are between u and p to u .
 - Identify the set $\mathcal{S} \in \mathcal{O}_\pi$ of axioms responsible of the implication of ϕ and having the certainty degree between u and p and shift down their degree to u .

The two cases lead to inferring ϕ with its prescribed level. It is obvious that the revision process has no effect on the initial weights associated with the axioms of the ontology \mathcal{O}_π if $\mathcal{O}_\pi \models \phi$ with $p < u$. However, the second case guarantees a minimal change of the ontology since it only changes the weights of axioms responsible for inferring ϕ .

In the proposition that follows, we look at the first case of syntactical revision where the input is derived from ontology and $u < p$.

Proposition 9 *Let \mathcal{O}_π be the prioritized \mathcal{EL}_\perp^+ ontology and (ϕ, u) be the uncertain input. Considering that \mathcal{O}'_π is the augmented ontology by the assumption that*

ϕ is false. Then the degree of inconsistency of \mathcal{O}'_π is $\lambda_{inc} = Inc(\mathcal{O}'_\pi)$. The revised prioritized \mathcal{EL}_\perp^+ ontology is defined as follows:

$$\begin{aligned} \mathcal{O}'_\pi = & \{(\phi, u)\} \cup \{(\phi, \alpha) : (\phi, \alpha) \in \mathcal{O}_\pi \text{ and } u > \lambda_{inc}\} \\ & \cup \{(\phi, \alpha) : (\phi, \alpha) \in \mathcal{O}_\pi \text{ and } \alpha < u\} \\ & \cup \{(\phi, \alpha) : (\phi, \alpha) \in \mathcal{O}_\pi \text{ and } u \leq \alpha \leq \lambda_{inc}\} \end{aligned}$$

Their associated possibility distributions using the min operator defined in Definition 5 are defined as follows:

$$\forall \mathcal{I} \in \Omega, \pi_{\mathcal{O}'_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I} \mid_m (\phi, u))$$

The value of u represents the degree of uncertainty or possibility associated with the new input information, and the proposition defines the process of updating the prioritized ontology and associated possibility distribution based on this new information. The new prioritized ontology \mathcal{O}'_π is formed by adding the new input information (ϕ, u) to the old ontology \mathcal{O}_π , and also including any other formulas with their associated possibility α from \mathcal{O}_π , where α is greater than a threshold value λ . The threshold value λ is defined as $Inc(\mathcal{O}'_\pi)$. In other words, formulas in \mathcal{O}_π with a possibility value greater than λ are preserved in the new ontology. In proposition 9 the axioms having priority degrees between u and λ_{Inc} should be reduced to u . However, using the second case, we can improve the results of Proposition 9 by first selecting the set of axioms $\mathcal{S} \in \mathcal{O}_\pi$ that implies ϕ . There exist, semantically, four sets of interpretations when the input is satisfied:

- $\mathcal{I} \models \mathcal{S}$ and $\mathcal{I} \models \mathcal{O}_\pi \setminus \mathcal{S}$
- $\mathcal{I} \models \mathcal{S}$ and $\mathcal{I} \not\models \mathcal{O}_\pi \setminus \mathcal{S}$
- $\mathcal{I} \not\models \mathcal{S}$ and $\mathcal{I} \models \mathcal{O}_\pi \setminus \mathcal{S}$
- $\mathcal{I} \not\models \mathcal{S}$ and $\mathcal{I} \not\models \mathcal{O}_\pi \setminus \mathcal{S}$

In the following definition, we improve the Definition 5 by providing a new definition of the min-based conditioning of prioritized \mathcal{EL}_\perp^+ possibility distribution.

Definition 6 Let \mathcal{O}_π be the prioritized \mathcal{EL}_\perp^+ ontology and $\pi_{\mathcal{O}}$ be its associated possibility distribution. considering the uncertain input information (ϕ, u) . Let $\mathcal{S} \subseteq \mathcal{O}_\pi$ be the set of axioms that inferred ϕ . Assume that $\alpha' = \max\{\alpha : (\phi, \alpha) \in \mathcal{O}_\pi \setminus \mathcal{S} \text{ and } \mathcal{I} \not\models \phi\}$. The min-based conditioning is defined as follows:

- $\forall \mathcal{I} \models (\phi \cup \mathcal{S}), \pi(\cdot \mid_m (\phi, u)) = \begin{cases} 1 & \text{if } \pi(\mathcal{I}) = \Pi(\phi) \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$
- $\forall \mathcal{I} \models \phi \cup (\mathcal{O}_\pi \setminus \mathcal{S}), \mathcal{I} \not\models \mathcal{S}, \pi(\cdot \mid_m (\phi, u)) = \begin{cases} 1 - u & \text{if } \pi(\mathcal{I}) = N(\phi) \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$
- $\forall \mathcal{I} \not\models \phi, \mathcal{I} \not\models \mathcal{S}, \mathcal{I} \not\models \mathcal{O}_\pi \setminus \mathcal{S}, \pi(\cdot \mid_m (\phi, u)) = \begin{cases} 1 - u & \text{if } \pi(\mathcal{I}) = N(\phi) \text{ and } 1 - \alpha' \geq 1 - u \\ 1 - \alpha' & \text{if } \pi(\mathcal{I}) = N(\phi) \text{ and } 1 - \alpha' \leq 1 - u \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$

$$\bullet \forall \mathcal{I} \neq \phi, \pi(\cdot |_{\mathcal{I}}(\phi, u)) = \begin{cases} 1 - u & \text{if } \pi(\mathcal{I}) = N(\phi) \\ 1 - \alpha' & \text{if } \pi(\mathcal{I}) > 1 - u \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$$

It is worth noting that Definition 6 improves Definition 5 while maintaining the same logical properties as **P1**, **P2** and **P3**.

Proposition 10 *Let \mathcal{O}_π be a prioritized \mathcal{EL} ontology and $\pi_{\mathcal{O}_\pi}$ its associated possibility distribution. Let (ϕ, u) be the new input information. Therefore: $\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot |_{\mathcal{I}}(\phi, u))$ obtained by the Definition 6 satisfies the logical properties **P1**, **P2** and **P3***

The following proposition gives the formal syntax of \mathcal{O}'_π and its associated possibility distribution $\pi'_{\mathcal{O}'_\pi}$ obtained with the Definition 6.

Proposition 11 *Let \mathcal{O}_π be the prioritized \mathcal{EL}^+_\perp ontology and $\pi_{\mathcal{O}_\pi}$ be its associated possibility distribution. Let (ϕ, u) be the uncertain input. Considering that \mathcal{O}'_π is new ontology obtained by adding the assumption ϕ false to \mathcal{O}_π . We have $\lambda_{inc} = Inc(\mathcal{O}'_\pi)$. The revised ontology is defined as follows:*

$$\mathcal{O}'_\pi = \{(\phi, u)\} \cup \{\mathcal{O}_\pi \setminus \mathcal{S}\} \cup \{(\phi, \alpha) : (\phi, \alpha) \in \mathcal{S} \text{ and } \alpha > \lambda_{inc}\} \cup \{(\phi, u) : (\phi, \lambda_{inc}) \in \mathcal{S} \text{ and } \lambda_{inc} = \alpha\}.$$

Its associated possibility distribution, denoted by $(\pi'_{\mathcal{O}'_\pi})$, obtained by the min-based conditioning defined in Definition 6 is as follows:

$$\forall \mathcal{I} \in \Omega, \pi'_{\mathcal{O}'_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I} |_{\mathcal{I}}(\phi, u))$$

5 Conclusion

In this paper, we investigated the evolution of prioritized \mathcal{EL} ontologies in the presence of new information that can be certain or uncertain. We propose an extension of \mathcal{EL} description logic, named \mathcal{EL}^+_\perp , within possibility theory to encode such knowledge, which provides a natural way to deal with the ordinal scale. We defined the evolution process at the semantic and syntactic levels. Finally, we propose a polynomial syntactic counterpart of the evolution process while preserving the consistency of the ontology.

6 Declarations

- **Competing Interests:** The authors declare no competing interests.
- **Ethical and informed consent for data used:** Not applicable.
- **Rim mohamed** is responsible for thinking about the idea and writing paper.
Zied Bouraoui is responsible for discussing the idea of the paper.
Zied Loukil is responsible for discussing the idea of the paper.
Faiez Gargouri is responsible for discussing the idea of the paper.

- Data availability and access: “No datasets were generated or analyzed during the current study”.

Appendix A

Proposition 1. Let \mathcal{O} be an assertion free \mathcal{EL}_\perp^+ ontology and let $A, B \in N_c$ be two concepts of \mathcal{O} . Then:

1. $\mathcal{O} \models A \sqsubseteq B$ iff $A \sqsubseteq B \in cl(\mathcal{O})$ or $A \sqsubseteq \perp \in cl(\mathcal{O})$.
2. The complexity of computing $cl(\mathcal{O})$ is polynomial.

Proof Suppose that $\mathcal{O} \models A \sqsubseteq B$. This means that in every model of \mathcal{O} , every instance of the concept A is also an instance of the concept B . By the definition of the closure operator $cl(\mathcal{O})$, we have $A \sqsubseteq B \in cl(\mathcal{O})$. Now, Considering that $A \sqsubseteq B \in cl(\mathcal{O})$ or $A \sqsubseteq \perp \in cl(\mathcal{O})$. If $A \sqsubseteq B \in cl(\mathcal{O})$, it means that $A \sqsubseteq B$ is a logical consequence of \mathcal{O} . Thus, in every model of \mathcal{O} , every instance of concept A is also an instance of concept B , which implies $\mathcal{O} \models A \sqsubseteq B$. Similarly, if $A \sqsubseteq \perp \in cl(\mathcal{O})$, it means that $A \sqsubseteq \perp$ is a logical consequence of \mathcal{O} . Since \perp represents the bottom concept (the unsatisfiable concept), this implies that there are no instances of concept A in any model of \mathcal{O} . Hence, $\mathcal{O} \models A \sqsubseteq B$ holds trivially.

To prove that the complexity of computing $cl(\mathcal{O})$ is polynomial, we need to show that there exists a polynomial-time algorithm that can compute the closure $cl(\mathcal{O})$. Since \mathcal{O} is an assertion-free \mathcal{EL}_\perp^+ ontology, it consists of concept inclusions of the form $A \sqsubseteq B$ and role inclusions of the form $r \sqsubseteq s$. We can construct an initial closure set $cl(\mathcal{O})$ that contains all the concept inclusions of \mathcal{O} . Then, for each concept inclusion $A \sqsubseteq B$ in $cl(\mathcal{O})$, we check if $A \sqsubseteq \perp$ is a logical consequence of \mathcal{O} . If it is, we add $A \sqsubseteq \perp$ to $cl(\mathcal{O})$.

This process of checking and adding concept inclusions can be done in polynomial time since the size of $cl(\mathcal{O})$ is bounded by the size of \mathcal{O} . Hence, the complexity of computing $cl(\mathcal{O})$ is polynomial. \square

Lemma 1. Considering that \mathcal{O}_π is the prioritized \mathcal{EL}_\perp^+ ontology that includes the two axioms (ϕ, α_1) and (ϕ, α_2) then \mathcal{O}_π and $\mathcal{O}'_\pi = \{\mathcal{O}_\pi \setminus \{(\phi, \alpha_1), (\phi, \alpha_2)\}\} \cup \{(\phi, \max(\alpha_1, \alpha_2))\}$ are equivalent in the sense that they induce the same possibility distribution, i.e., $\forall \mathcal{I} \in \Omega, \pi_{\mathcal{O}}(\mathcal{I}) = \pi_{\mathcal{O}'_\pi}(\mathcal{I})$.

Proof The proof is given immediately from the definition of the possibility distribution in Definition 3 \square

Proposition 2. Let \mathcal{O}_π be the prioritized \mathcal{EL}_\perp^+ ontology and \mathcal{O}'_π be the ontology obtained by applying the normalization rules in Table 5 into \mathcal{O}_π . Then \mathcal{O}_π and \mathcal{O}'_π induce the same possibility distribution.

Sketch of proof. We give the proof for some rules. The proof of the other follows similarly.

(PNR₀). Let \mathcal{I} be an interpretation. Considering that $\mathcal{I} \models (C_1 \sqcap \top \sqcap C_2 \sqsubseteq D, \alpha)$ by definition of satisfaction relation (for instance see Table 1), we have:

$$(C_1 \sqcap \top \sqcap C_2)^{\mathcal{I}} \subseteq (D)^{\mathcal{I}}$$

Hence

$$((C_1)^{\mathcal{I}} \cap (\Delta)^{\mathcal{I}} \cap (C_2)^{\mathcal{I}}) \subseteq (D)^{\mathcal{I}}$$

Then

$$((C_1)^{\mathcal{I}} \cap (C_2)^{\mathcal{I}}) \subseteq D^{\mathcal{I}}$$

which means that $\mathcal{I} \models C_1 \sqcap C_2 \sqsubseteq D$, and $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \pi_{\mathcal{O}'_\pi}(\mathcal{I})$.
If $\mathcal{I} \not\models (C_1 \sqcap \top \sqcap C_2 \sqsubseteq D, \alpha)$, the proof follows similarly.

(PNR₁). Let \mathcal{I} be an interpretation. Considering that $\mathcal{I} \models (C_1 \sqcap \perp \sqcap C_2 \sqsubseteq D, \alpha)$ by definition of satisfaction relation in Table 1, we have:

$$(C_1 \sqcap \perp \sqcap C_2)^{\mathcal{I}} \subseteq (D)^{\mathcal{I}}$$

Hence

$$((C_1)^{\mathcal{I}} \cap (\perp)^{\mathcal{I}} \cap (C_2)^{\mathcal{I}}) \subseteq (D)^{\mathcal{I}}$$

Then

$$((C_1^{\mathcal{I}} \cap \emptyset \cap (C_2)^{\mathcal{I}}) \subseteq D^{\mathcal{I}}$$

As a result

$$(\emptyset) \subseteq D^{\mathcal{I}}$$

which means that $\mathcal{I} \models \perp \subseteq D$, and $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \pi_{\mathcal{O}'_\pi}(\mathcal{I})$.
If $\mathcal{I} \not\models C_1 \sqcap \perp \sqcap C_2 \sqsubseteq D$, the proof follows similarly.

(PNR₂): let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology where $\mathcal{O}_\pi = \{(C \sqsubseteq D_1 \sqcap D_2, \alpha)\}$. Let \mathcal{O}'_π be the prioritized ontology obtained after applying PNR_2 to the ontology, i.e., $\mathcal{O}'_\pi = \{(C \sqsubseteq D_1, \alpha) \text{ and } (C \sqsubseteq D_2, \alpha)\}$. Then \mathcal{O}_π and \mathcal{O}'_π induce the same possibility distribution.

Let \mathcal{I} be the interpretation that satisfies $(C \sqsubseteq D_1 \sqcap D_2, \alpha)$, namely $\mathcal{I} \models (C \sqsubseteq D_1 \sqcap D_2, \alpha)$ by definition of satisfaction relation in Table 1, we have:

$$(C)^{\mathcal{I}} \subseteq (D_1 \sqcap D_2)^{\mathcal{I}}$$

Hence

$$(C)^{\mathcal{I}} \subseteq (D_1)^{\mathcal{I}} \text{ and } (C)^{\mathcal{I}} \subseteq (D_2)^{\mathcal{I}}$$

This means that

$$\mathcal{I} \models C \sqsubseteq D_1 \text{ and } \mathcal{I} \models C \sqsubseteq D_2$$

Therefore $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \pi_{\mathcal{O}'_\pi}(\mathcal{I})$.

Conversely, assume that $\mathcal{I} \models (C \sqsubseteq D_1, \alpha)$ and $\mathcal{I} \models (C \sqsubseteq D_2, \alpha)$ by definition of satisfaction relation, we have $C^{\mathcal{I}} \subseteq D_1^{\mathcal{I}}$ and $C^{\mathcal{I}} \subseteq D_2^{\mathcal{I}}$ then $C^{\mathcal{I}} \subseteq (D_1^{\mathcal{I}} \cap D_2^{\mathcal{I}})$, therefore $\mathcal{I} \models C \sqsubseteq D_1 \sqcap D_2$ and $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \pi_{\mathcal{O}'_\pi}(\mathcal{I})$. The other case when $\mathcal{I} \not\models C \sqsubseteq D_1 \sqsubseteq D_2$ follows similarly. \square

Proposition 3. Let \mathcal{O}_π be the prioritized \mathcal{EL}_\perp^+ ontology and $cl(\mathcal{O}_\pi)$ be its closed ontology, obtained by applying the rules given in Tables 5 and 6 into \mathcal{O}_π . Then \mathcal{O}_π and $cl(\mathcal{O}_\pi)$ induce the same possibility distribution.

Proof The proof comes down to deduce that the application of normalization rules and inference rules does not change the possibility distribution, it is enough to keep repeating the application of rules and see the possibility distribution. For the normalization rules (Table 5) the proof is given above. Now for the inference rules, considering that $(C_1 \sqsubseteq C_2, \alpha_1), (C_2 \sqsubseteq C_3, \alpha_2) \in \mathcal{O}_\pi$. Let us show that by applying the rule (PIR₃) into \mathcal{O}_π the closed ontology $cl(\mathcal{O}_\pi)$ contains the axiom $(C_1 \sqsubseteq C_3, \min(\alpha_1, \alpha_2)) \in cl(\mathcal{O}_\pi)$ and the possibility distribution does not change, namely $cl(\mathcal{O}_\pi) = \mathcal{O}_\pi \sqcup \{C_1 \sqsubseteq C_3, \min(\alpha_1, \alpha_2)\}$ are equivalent. Let $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ a DL interpretation. We have four cases:

- $\mathcal{I} \models C_1 \sqsubseteq C_2$ and $\mathcal{I} \models C_2 \sqsubseteq C_3$ by definition of satisfaction relation in Table 1, we have $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$ and $C_2^{\mathcal{I}} \subseteq C_3^{\mathcal{I}}$ hence $C_1^{\mathcal{I}} \subseteq C_3^{\mathcal{I}}$ which means that $\mathcal{I} \models C_1 \sqsubseteq C_3$. Therefore $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \pi_{cl(\mathcal{O}_\pi)}(\mathcal{I})$
- $\mathcal{I} \models C_1 \sqsubseteq C_2$ and $\mathcal{I} \not\models C_2 \sqsubseteq C_3$ let $\mathcal{O}_\pi'' = \mathcal{O}_\pi \setminus \{(C_1 \sqsubseteq C_2, \alpha_1) \text{ and } (C_2 \sqsubseteq C_3, \alpha_2)\}$ then

$$\begin{aligned} \pi_{\mathcal{O}_\pi}(\mathcal{I}) &= \min(\pi_{\mathcal{O}_\pi''}, 1 - \alpha_2) \\ &= \min(\pi_{\mathcal{O}_\pi''}, 1 - \alpha_2, 1 - \min(\alpha_1, \alpha_2)) \\ &= \pi_{cl(\mathcal{O}_\pi)}(\mathcal{I}) \end{aligned}$$
- $\mathcal{I} \not\models C_1 \sqsubseteq C_2$ and $\mathcal{I} \models C_2 \sqsubseteq C_3$: follow similarly.
- $\mathcal{I} \not\models C_1 \sqsubseteq C_2$ and $\mathcal{I} \not\models C_2 \sqsubseteq C_3$ let $\mathcal{O}_\pi'' = \mathcal{O}_\pi \setminus \{(C_1 \sqsubseteq C_2, \alpha_1), (C_2 \sqsubseteq C_3, \alpha_2)\}$ we have $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \min(\pi_{\mathcal{O}_\pi''}(\mathcal{I}), 1 - \alpha_1, 1 - \alpha_2)$

$$\begin{aligned} &= \min(\pi_{\mathcal{O}_\pi''}(\mathcal{I}), 1 - \alpha_1, 1 - \alpha_2, 1 - \min(\alpha_1, \alpha_2)) \\ &= \pi_{cl(\mathcal{O}_\pi)}(\mathcal{I}) \end{aligned}$$

□

Proposition 4. Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology and $cl(\mathcal{O}_\pi)$ be its closure by applying the normalization rules in Table 5 and inference rules in Table 6. Let A and B be two concepts of \mathcal{O}_π . Then $\mathcal{O}_\pi \models (A \sqsubseteq B, \alpha)$ if

- $(A \sqsubseteq B, \beta) \in cl(\mathcal{O}_\pi)$ with $\beta \geq \alpha$
- $(A \sqsubseteq \perp, \beta) \in cl(\mathcal{O}_\pi)$ with $\beta \geq \alpha$

Proof The proof is immediate. It is given from the definition of the inference rules given in Table 6. □

Proposition 5. Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology and $\pi_{\mathcal{O}_\pi}$ be its associated possibility distribution. Let (ϕ, u) be the new input information. Then $\pi_1(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\cdot \mid (\phi))$ obtained by Definition 4 satisfies the postulates **P1- P2**.

Proof The proof is as follows:

- **P1:** $\pi'(\mathcal{I}) = 1$: To prove this, we need to show that $\pi'(\mathcal{I}) = 1$ for any interpretation \mathcal{I} . If $\pi(\mathcal{I}) = \Pi(\phi)$ and $\mathcal{I} \models \phi$, then by the definition of $\pi'_{\mathcal{O}_\pi}$, we have $\pi'(\mathcal{I}) = 1$. If $\pi(\mathcal{I}) < \Pi(\phi)$, then by the definition of $\pi'_{\mathcal{O}_\pi}$, we have $\pi'(\mathcal{I}) = \pi(\mathcal{I}) > 0$. Since π is a possibility distribution, we have $\pi(\mathcal{I}) \leq 1$. Therefore, $\pi'(\mathcal{I}) > 0$ and $\pi'(\mathcal{I}) \leq 1$, which implies $\pi'(\mathcal{I}) = 1$. If $\mathcal{I} \not\models \phi$, then by the definition of $\pi'_{\mathcal{O}_\pi}$, we have $\pi'(\mathcal{I}) = 0 \leq 1$. Therefore, $\pi'(\mathcal{I}) = 1$ in all cases, which satisfies **P1**.
- **P2:** if $\pi(\mathcal{I}) = 0$ then $\pi'(\mathcal{I}) = 0$: To prove this, we need to show that if $\pi(\mathcal{I}) = 0$ for an interpretation \mathcal{I} , then $\pi'(\mathcal{I}) = 0$. If $\pi(\mathcal{I}) = 0$, then by the definition of possibility distribution, \mathcal{I} is completely impossible, i.e., $\pi(\mathcal{I}) = 0$ implies $\Pi(\neg\mathcal{I}) = 1$. Therefore, $\Pi(\phi \wedge \neg\mathcal{I}) = \Pi(\phi) \wedge \Pi(\neg\mathcal{I}) = \Pi(\phi) \wedge 0 = 0$. This means that $\phi \wedge \neg\mathcal{I}$ is completely impossible, i.e., $\mathcal{I} \not\models \phi$ implies $\phi \wedge \neg\mathcal{I}$ is completely impossible. By the definition of $\pi'_{\mathcal{O}_\pi}$, if $\mathcal{I} \not\models \phi$, then $\pi'(\mathcal{I}) = 0$. Therefore, $\pi'(\mathcal{I}) = 0$ for any interpretation \mathcal{I} with $\pi(\mathcal{I}) = 0$, which satisfies **P2**.

□

Proposition 6. Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology and $\pi_{\mathcal{O}_\pi}$ be its associated possibility distribution. Let (ϕ, u) be the new input information. Then $\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot \mid (\phi, u))$ obtained by Definition 5 satisfies the postulates **P1- P3**.

Proof The proof is as follows:

- To prove that the above definition satisfies P1, we need to show that for any interpretation \mathcal{I} , the possibility distribution $\pi'(\mathcal{I})$ obtained after min-based conditioning satisfies $\pi'(\mathcal{I}) = 1$.

Let (ϕ, u) be the uncertain input, and let \mathcal{I} be an interpretation such that $\mathcal{I} \models \phi$. Then, according to the definition of min-based conditioning, we have:

$$\pi'(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I} \mid_m (\phi, u)) = \begin{cases} 1 & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) = \Pi(\phi) \\ 1 - u & \text{if } \Pi(\phi) \leq \pi_{\mathcal{O}_\pi} \leq 1 - u \\ \pi_{\mathcal{O}_\pi}(\mathcal{I}) & \text{Otherwise} \end{cases}$$

Since $\mathcal{I} \models \phi$, we have $\pi_{\mathcal{O}_\pi}(\mathcal{I}) \geq \Pi(\phi)$, so the first two cases do not apply. Therefore, we have:

$$\pi'(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I}) = 1$$

On the other hand, if \mathcal{I} is such that $\mathcal{I} \not\models \phi$, then we have:

$$\forall \mathcal{I} \not\models \phi, \pi_{\mathcal{O}_\pi}(\cdot \mid_m (\phi, u)) = \begin{cases} 1 - u & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) = N_\pi(\phi) \\ 1 - u & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) > 1 - u \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$$

Since $\pi_{\mathcal{O}_\pi}(\mathcal{I}) \leq N_\pi(\phi)$ if $\mathcal{I} \not\models \phi$, the first case does not apply. Since $\pi_{\mathcal{O}_\pi}(\mathcal{I}) \leq 1 - u$ by definition of $N_\pi(\phi)$, the second case does not apply either. Therefore, we have:

$$\pi'(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I}) = 1$$

In both cases, we have $\pi'(\mathcal{I}) = 1$, which completes the proof that the above definition satisfies P1.

- **P2:** if $\pi(\mathcal{I}) = 0$ then $\pi'(\mathcal{I}) = 0$. Let $\mathcal{I} \not\models \phi$ and $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = 0$. Then we have:

$$\forall \mathcal{I} \not\models \phi, \pi_{\mathcal{O}_\pi}(\cdot \mid_m (\phi, u)) = \begin{cases} 1 - u & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) = N_\pi(\phi) \\ 1 - u & \text{if } \pi_{\mathcal{O}_\pi}(\mathcal{I}) > 1 - u \\ \pi(\mathcal{I}) & \text{Otherwise} \end{cases}$$

Therefore, the property **P2** is satisfied.

- To prove that Definition 5 satisfies P3, we need to show that: $\Pi'(\phi) = 1$ and $N'(\phi) \geq u$. To show that $\Pi'(\phi) = 1$, we need to show that for all interpretations \mathcal{I} that satisfy ϕ , $\pi'(\mathcal{I}) = 1$. There are two cases to consider:

1. $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \Pi(\phi)$ In this case, we have:
 $\pi_{\mathcal{O}_\pi}(\cdot \mid_m (\phi, u)) = 1$ then, $\pi'(\mathcal{I}) = 1$

2. $\Pi(\phi) \leq \pi_{\mathcal{O}_\pi}(\mathcal{I}) \leq 1 - u$ In this case, we have: $\pi_{\mathcal{O}_\pi}(\cdot \mid_m (\phi, u)) = 1 - u$ then, $\pi'(\mathcal{I}) = 1 - u < 1$

However, this case cannot happen because of the assumption that $\Pi(\phi) > 1 - u$. Therefore, we have shown that for all interpretations \mathcal{I} that satisfy ϕ , $\pi'(\mathcal{I}) = 1$, and hence $\Pi'(\phi) = 1$. Now, to show that $N'(\phi) \geq u$, we need to show that for all interpretations \mathcal{I} that do not satisfy ϕ , $\pi'(\mathcal{I}) \leq 1 - u$. There are two cases to consider:

1. $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = N\pi(\phi)$ In this case, we have: $\pi_{\mathcal{O}_\pi}(\cdot \mid_m (\phi, u)) = 1 - u$, then $\pi'(\mathcal{I}) = 1 - u \leq 1 - u$
2. $\pi_{\mathcal{O}_\pi}(\mathcal{I}) > 1 - u$ In this case, we have: $\pi_{\mathcal{O}_\pi}(\cdot \mid_m (\phi, u)) = 1 - u$ then, $\pi'(\mathcal{I}) = 1 - u \leq 1 - u$

Therefore, we have shown that for all interpretations \mathcal{I} that do not satisfy ϕ , $\pi'(\mathcal{I}) \leq 1 - u$, and hence $N'(\phi) \geq u$. □

Proposition 7. Let \mathcal{O}_π be a prioritized \mathcal{EL}_\perp^+ ontology, and $\pi_{\mathcal{O}_\pi}$ be the associated possibility distribution. Let (ϕ, u) be the new input information and $\lambda = \text{Inc}(\mathcal{O}'_\pi)$, where $\mathcal{O}'_\pi = \mathcal{O}_\pi \cup \{(\phi, 1)\}$. Then, the new prioritized ontology \mathcal{O}'_π is defined as follows:

$$\mathcal{O}'_\pi = \{(\phi, u)\} \cup \{(\Phi, \alpha) : (\Phi, \alpha) \in \mathcal{O}_\pi \text{ and } \alpha > \lambda\}$$

The associated possibility distribution obtained by conditioning defined in Definition 5 is as follows:

$$\forall \mathcal{I} \in \Omega, \pi'_{\mathcal{O}'_\pi}(\mathcal{I}) = \pi_{\mathcal{O}_\pi}(\mathcal{I} \mid_m (\phi, u))$$

With $\pi_{\mathcal{O}_\pi}(\mathcal{I} \mid_m (\phi, u))$ represents the possibility distribution obtained by $\pi_{\mathcal{O}_\pi}$ with min-based conditioning in Definition 5

Proof First, point out that $\Pi(\phi) = 1 - \lambda$. Let \mathcal{I} be an interpretation that falsifies the input information ϕ , then for each interpretation that falsified ϕ , $\pi_{\mathcal{O}_\pi} = \pi_{\mathcal{O}'_\pi}(\mathcal{I}) = 0$. Now, considering that \mathcal{I} satisfies ϕ , then, if $\pi_{\mathcal{O}_\pi}(\mathcal{I}) = \Pi(\phi) = 1 - \lambda$, this means that the interpretation \mathcal{I} satisfies all the axioms with a weight greater than $1 - \lambda$, hence \mathcal{I} satisfies all the axioms of \mathcal{O}'_π . As a result $\pi_{\mathcal{O}'_\pi}(\mathcal{I}) = 1$. Considering now that $\pi_{\mathcal{O}_\pi}(\mathcal{I}) < 1 - \lambda$, then: □

$$\begin{aligned} \pi_{\mathcal{O}_\pi}(\mathcal{I}) &= \min\{1 - \alpha : (\Phi, \alpha) \in \mathcal{O}_\pi \text{ and } \mathcal{I} \not\models \Phi\} \\ &= \min\{\min\{1 - \alpha : (\Phi, \alpha) \in \mathcal{O}_\pi \text{ and } \mathcal{I} \not\models \Phi \text{ and } \alpha > \lambda\}, \\ &\quad \min\{1 - \alpha : (\Phi, \alpha) \in \mathcal{O}_\pi \text{ and } \mathcal{I} \not\models \Phi \text{ and } \alpha \leq \lambda\}\} \\ &= \min\{1 - \alpha : (\Phi, \alpha) \in \mathcal{O}_\pi \text{ and } \mathcal{I} \not\models \Phi \text{ and } \alpha > \lambda\} \text{ (since } \pi_{\mathcal{O}_\pi} < 1 - \lambda) \\ &= \pi_{\mathcal{O}'_\pi} \end{aligned}$$

Proposition 8. The computational of the Algorithm 1 is efficient and done in polynomial time.

Proof The computation of the obtained ontology by the Algorithm 1 is efficient. Its complexity is the same as the one of computing the inconsistency of $\pi\mathcal{EL}_{\perp}^{+}$ ontology that is given in [29], which is done in polynomial time. \square

Proposition 9. Let \mathcal{O}_{π} be the prioritized \mathcal{EL}_{\perp}^{+} ontology and (ϕ, u) be the uncertain input. Considering that \mathcal{O}'_{π} is the augmented ontology by the assumption that ϕ is false. Then the degree of inconsistency of \mathcal{O}'_{π} is $\lambda_{inc} = Inc(\mathcal{O}'_{\pi})$. The revised prioritized \mathcal{EL}_{\perp}^{+} ontology is defined as follows:

$$\begin{aligned}\mathcal{O}'_{\pi} = & \{(\phi, u)\} \cup \{(\phi, \alpha) : (\phi, \alpha) \in \mathcal{O}_{\pi} \text{ and } u > \lambda_{inc}\} \\ & \cup \{(\phi, \alpha) : (\phi, u) \in \mathcal{O}_{\pi} \text{ and } \alpha < u\} \\ & \cup \{(\phi, \alpha) : (\phi, \alpha) \in \mathcal{O}_{\pi} \text{ and } u \leq \alpha \leq \lambda_{inc}\}\end{aligned}$$

Their associated possibility distributions using the min operator defined in Definition 5 are defined as follows:

$$\forall \mathcal{I} \in \Omega, \pi_{\mathcal{O}'_{\pi}}(\mathcal{I}) = \pi_{\mathcal{O}_{\pi}}(\mathcal{I} \mid_m (\phi, u))$$

Proof Considering the case when the ontology \mathcal{O}_{π} is inconsistent with $\neg\phi$, this means that $\Pi(\phi) = 1$ and $\Pi(\neg\phi) = 1 - \lambda_{inc}$. Let \mathcal{I} be an interpretation that satisfies ϕ . Then: When $\mathcal{I} \not\models \phi$, then:

$$\begin{aligned}\pi_{\mathcal{O}_{\pi}}(\mathcal{I}) &= Min\{\min\{1 - \alpha_i : (\phi, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i > \lambda_{inc}\} \\ &\quad \min\{1 - \alpha_i : (\phi, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i \leq \lambda_{inc}\}\} \\ &= Min\{\min\{1 - \alpha_i : (\phi_i, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i > \lambda_{inc}\}, \\ &\quad \min\{1 - \alpha_i : (\phi_i, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i \leq \lambda_{inc}\}\} \\ &= \min\{1 - \alpha_i : (\phi_i, \alpha_i) \in \mathcal{O}_{\pi} \text{ text } \mathcal{I}\phi_i\} \\ &= \pi_{\mathcal{O}_{\pi}}(\mathcal{I})\end{aligned}$$

$$\pi_{\mathcal{O}_{\pi}}(\mathcal{I}) = Min\{1 - a, \min\{1 - \alpha_i : (\phi_i, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i > \lambda_{inc}\}\}$$

We have two scenarios:

- When $\pi_{\mathcal{O}_{\pi}}(\mathcal{I}) = \Pi(\neg\phi) = 1 - \lambda_{inc}$ which means that the interpretation \mathcal{I} satisfies all the axioms of the ontology \mathcal{O}_{π} having weight strictly greater than λ_{inc} , then:

$$\min\{1 - \alpha_i : (\phi_i, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i > \lambda_{inc}\} = 1$$

Therefore $\pi_{\mathcal{O}_{\pi}}(\mathcal{I}) = 1 - u$

- When $\pi_{\mathcal{O}_{\pi}}(\mathcal{I}) < 1 - \lambda_{inc}$, then:

$$\begin{aligned}\pi_{\mathcal{O}'_{\pi}}(\mathcal{I}) &= Min\{1 - u, \min\{1 - \alpha_i : (\phi, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i > \lambda_{inc}\}\} \\ &= Min\{1 - u, \{\min\{1 - \alpha_i : (\phi_i, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i > \lambda_{inc}\}, \\ &\quad \min\{1 - \alpha_i : (\phi_i, \alpha_i) \in \mathcal{O}_{\pi} \text{ and } \mathcal{I} \not\models \phi_i \text{ and } \alpha_i \leq \lambda_{inc}\}\} \\ &\quad (\text{since } \pi_{\mathcal{O}_{\pi}}(\mathcal{I}) < 1 - \lambda_{inc}) \\ &= Min\{1 - u, \pi_{\mathcal{O}_{\pi}}(\mathcal{I})\}\end{aligned}$$

□

Proposition 10. Let \mathcal{O}_π be a prioritized \mathcal{EL} ontology and $\pi_{\mathcal{O}_\pi}$ its associated possibility distribution. Let (ϕ, u) be the new input information. Therefore: $\pi'_{\mathcal{O}_\pi} = \pi_{\mathcal{O}_\pi}(\cdot \mid_m (\phi, u))$ obtained by the Definition 6 satisfies the logical properties **P1**, **P2** and **P3**

Proof To prove that the given definition of min-based conditioning satisfies properties **P1**, **P2**, and **P3**, we need to show that the updated possibility distribution π' satisfies these properties.

P1: $\pi'(\mathcal{I}) = 1$

Let \mathcal{I} be any interpretation. We need to show that $\pi'(\mathcal{I}) = 1$. There are four cases to consider:

- Case 1: $\mathcal{I} \models \phi \cup \mathcal{S}$ In this case, $\pi'(\mathcal{I}) = \pi(\mathcal{I})$ by the first condition in the definition of min-based conditioning. Since π is a possibility distribution and $\mathcal{I} \models \phi \cup \mathcal{S}$, we have $\pi(\mathcal{I}) \geq \Pi(\phi) > 0$, which implies that $\pi'(\mathcal{I}) = \pi(\mathcal{I}) > 0$.
- Case 2: $\mathcal{I} \models \phi \cup (\mathcal{O}_\pi \setminus \mathcal{S})$ and $\mathcal{I} \not\models \mathcal{S}$
In this case, $\pi'(\mathcal{I}) = \pi(\mathcal{I})$ by the second condition in the definition of min-based conditioning. Since π is a possibility distribution and $\mathcal{I} \models \phi \cup (\mathcal{O}_\pi \setminus \mathcal{S})$, we have $\pi(\mathcal{I}) \geq N(\phi) > 0$, which implies that $\pi'(\mathcal{I}) = \pi(\mathcal{I}) > 0$.
- Case 3: $\mathcal{I} \models \phi$ and $\mathcal{I} \not\models \mathcal{S}$ and $\mathcal{I} \not\models \mathcal{O}_\pi \setminus \mathcal{S}$ and $\pi(\mathcal{I}) = N(\phi)$ and $1 - \alpha' \geq 1 - u$
In this case, $\pi'(\mathcal{I}) = 1 - u$ by the third condition in the definition of min-based conditioning. Since $1 - \alpha' \geq 1 - u$, we have $u \geq \alpha'$. Also, since $\pi(\mathcal{I}) = N(\phi) > 0$ and $\mathcal{I} \models \phi$, we have $\Pi(\phi) > 0$. Therefore, $\pi(\mathcal{I}) \geq \Pi(\phi)$, which implies that $\pi'(\mathcal{I}) = 1 - u < 1$.
- Case 4: $\mathcal{I} \not\models \phi$ and $\pi(\mathcal{I}) > 1 - u$
In this case, $\pi'(\mathcal{I}) = 1 - \alpha'$ by the fourth condition in the definition of min-based conditioning. Since $\pi(\mathcal{I}) > 1 - u$, we have $\alpha' < u$, which implies that $1 - \alpha' > 1 - u$. Also, since $\pi(\mathcal{I}) > 0$, we have $\pi'(\mathcal{I}) = 1 - \alpha' < 1$.

P2: if $\pi(\mathcal{I}) = 0$ then $\pi'(\mathcal{I}) = 0$. Let's consider the different cases for min-based conditioning:

- Case 1: $\mathcal{I} \models \phi \cup \mathcal{S}$: In this case, by definition of min-based conditioning, we have $\pi'(\mathcal{I}) = \pi(\mathcal{I})$, since $\pi(\mathcal{I})$ is either 1 or greater than 0 (if it were 0, then \mathcal{I} would not be a model of $\phi \cup \mathcal{S}$). Therefore, if $\pi(\mathcal{I}) = 0$, then we also have $\pi'(\mathcal{I}) = 0$, which satisfies P2.
- Case 2: $\mathcal{I} \models \phi \cup (\mathcal{O}_\pi \setminus \mathcal{S})$, $\mathcal{I} \not\models \mathcal{S}$, and $\pi(\mathcal{I}) = N(\phi)$: In this case, by definition of min-based conditioning, we have $\pi'(\mathcal{I}) = 1 - u$. Therefore, if $\pi(\mathcal{I}) = 0$, then we also have $\pi'(\mathcal{I}) = 0$, which satisfies P2.
- Case 3: $\mathcal{I} \models \phi \cup (\mathcal{O}_\pi \setminus \mathcal{S})$, $\mathcal{I} \not\models \mathcal{S}$, and $\pi(\mathcal{I}) > N(\phi)$: In this case, by definition of min-based conditioning, we have $\pi'(\mathcal{I}) = \pi(\mathcal{I})$, since $\pi(\mathcal{I})$ is either 1 or greater than $N(\phi)$. Therefore, if $\pi(\mathcal{I}) = 0$, then we also have $\pi'(\mathcal{I}) = 0$, which satisfies P2.
- Case 4: $\mathcal{I} \models \phi$, $\mathcal{I} \not\models \mathcal{S}$, $\mathcal{I} \not\models \mathcal{O}_\pi \setminus \mathcal{S}$, and $\pi(\mathcal{I}) = N(\phi)$: In this case, by definition of min-based conditioning, we have $\pi'(\mathcal{I}) = 1 - u$ if $1 - \alpha' \geq 1 - u$,

and $\pi'(\mathcal{I}) = 1 - \alpha'$ if $1 - \alpha' < 1 - u$. Therefore, if $\pi(\mathcal{I}) = 0$, then we also have $\pi'(\mathcal{I}) = 0$, which satisfies P2.

- Case 5: $\mathcal{I} \not\models \phi$ and $\pi(\mathcal{I}) > 1 - u$: In this case, by definition of min-based conditioning, we have $\pi'(\mathcal{I}) = \pi(\mathcal{I})$, since $\pi(\mathcal{I})$ is either 1 or greater than $1 - u$. Therefore, if $\pi(\mathcal{I}) = 0$, then we also have $\pi'(\mathcal{I}) = 0$, which satisfies P2.

P3: $\Pi'(\phi) = 1$ and $N'(\phi) \geq u$ Considering the case where $\mathcal{S} = \mathcal{O}_\pi$. Then $\mathcal{O}_\pi \models \phi$, so we have $\Pi(\phi) = 1$ and $N(\phi) = 0$. Therefore, by definition of min-based conditioning, we have $\Pi'(\phi) = 1$ and $N'(\phi) = 0 \geq u$, which satisfies P3. Now, let's consider the case where $\mathcal{S} \subsetneq \mathcal{O}_\pi$. We know that ϕ is not entailed by \mathcal{O}_π , so we have $N(\phi) > 0$. We need to show that $\Pi'(\phi) = 1$ and $N'(\phi) \geq u$. First, consider the case where $\alpha' \geq 1 - u$. Then, we have $\Pi(\phi) \geq 1 - \alpha' > u$, since $\Pi(\phi) \leq 1 - \alpha'$ and $\alpha' \geq 1 - u$. Therefore, by definition of min-based conditioning, we have $\Pi'(\phi) = 1$ and $N'(\phi) \geq u$. Now, let's consider the case where $\alpha' < 1 - u$. In this case, we have $\Pi(\phi) \leq 1 - \alpha' < 1 - u$, since $\Pi(\phi) \geq 1 - \alpha'$ and $\alpha' < 1 - u$. Therefore, by definition of min-based conditioning, we have $\Pi'(\phi) = 1$ and $N'(\phi) \geq u$. Therefore, in both cases, we have $\Pi'(\phi) = 1$ and $N'(\phi) \geq u$, which satisfies P3. □

References

- [1] Baader, F., Horrocks, I., Lutz, C., Sattler, U.: An Introduction to Description Logic. Cambridge University Press, (2017)
- [2] Xiao, G., Calvanese, D., Kontchakov, R., Lembo, D., Poggi, A., Rosati, R., Zakharyashev, M.: Ontology-based data access: A survey. (2018). International Joint Conferences on Artificial Intelligence
- [3] Pletscher-Frankild, S., Pallejà, A., Tsafou, K., Binder, J.X., Jensen, L.J.: Diseases: Text mining and data integration of disease-gene associations. *Methods* **74**, 83–89 (2015)
- [4] Khan, S.U.: Transformation of semantic networks into frames. *International Journal of Innovation, Management and Technology* **4**(1), 21 (2013)
- [5] Lehmann, F.: Semantic networks. *Computers & Mathematics with Applications* **23**(2-5), 1–50 (1992)
- [6] Alfonseca, M.: Frames, semantic networks, and object-oriented programming in apl2. *IBM journal of research and development* **33**(5), 502–510 (1989)
- [7] Barwise, J.: An introduction to first-order logic, (1977). In: *Studies in Logic and the Foundations of Mathematics* vol. 90, pp. 5–46. Elsevier
- [8] Batini, C., Lenzerini, M., Navathe, S.B.: A comparative analysis of methodologies for database schema integration. *ACM computing surveys*

(CSUR) **18**(4), 323–364 (1986)

- [9] Krisnadhi, A., Lutz, C.: Data complexity in the el family of description logics. In: Logic for Programming, Artificial Intelligence, and Reasoning: 14th International Conference, LPAR 2007, Yerevan, Armenia, October 15-19, 2007. Proceedings 14, pp. 333–347 (2007). Springer
- [10] Baader, F., Brandt, S., Lutz, C.: Pushing the el envelope. In: IJCAI, vol. 5, pp. 364–369 (2005)
- [11] Baader, F., Lutz, C., Suntisrivaraporn, B.: Is tractable reasoning in extensions of the description logic el useful in practice. In: M4M-05, vol. 450 (2005). Citeseer
- [12] Baader, F., Brandt, S., Lutz, C.: Pushing the el envelope further (2008)
- [13] Kazakov, Y., Krötzsch, M., Simancik, F.: Practical reasoning with nominals in the el family of description logics. In: KR (2012)
- [14] Borgwardt, S., Peñaloza, R.: Consistency reasoning in lattice-based fuzzy description logics. *International Journal of Approximate Reasoning* **55**(9), 1917–1938 (2014)
- [15] Hepp, M., De Leenheer, P., De Moor, A., Sure, Y.: *Ontology Management: Semantic Web, Semantic Web Services, and Business Applications*, (2007). Springer
- [16] Universities, Q.T.: Qs world university rankings. University Rankings. Business & Management Studies. Disponivel : <https://www.topuniversities.com/universityrankings/university-subject-rankings/2020/business-management-studies> **4** (2011)
- [17] Benferhat, S., Bouraoui, Z., Tabia, K.: On the revision of prioritized dl-lite knowledge bases. In: Straccia, U., Cali, A. (eds.) *Scalable Uncertainty Management - 8th International Conference, SUM 2014*, Oxford, UK, September 15-17, 2014. Proceedings. *Lecture Notes in Computer Science*, vol. 8720, pp. 22–36. Springer. https://doi.org/10.1007/978-3-319-11508-5_3. https://doi.org/10.1007/978-3-319-11508-5_3
- [18] Calvanese, D., Kharlamov, E., Nutt, W., Zheleznyakov, D.: Evolution of dl-lite knowledge bases. In: Proc. of the 9th Int. Semantic Web Conf.(ISWC), vol. 6496, pp. 112–128 (2010)
- [19] Flouris, G., Plexousakis, D., Antoniou, G.: Generalizing the agm postulates: preliminary results and applications. In: NMR, pp. 171–179 (2004)

- [20] Benferhat, S., Dubois, D., Prade, H., Williams, M.-A.: A practical approach to revising prioritized knowledge bases. *Studia Logica* **70**(1), 105–130 (2002)
- [21] Alchourrón, C.E., Gärdenfors, P., Makinson, D.: On the logic of theory change: Partial meet contraction and revision functions. *The journal of symbolic logic* **50**(2), 510–530 (1985)
- [22] Flouris, G., Plexousakis, D., Antoniou, G.: On applying the agm theory to dls and owl. In: *The Semantic Web–ISWC 2005: 4th International Semantic Web Conference, ISWC 2005, Galway, Ireland, November 6–10, 2005. Proceedings 4*, pp. 216–231 (2005). Springer
- [23] Liu, H., Lutz, C., Milicic, M., Wolter, F.: Updating description logic aboxes. *KR* **6**, 46–56 (2006)
- [24] De Giacomo, G., Lenzerini, M., Poggi, A., Rosati, R.: On the approximation of instance level update and erasure in description logics. In: *AAAI*, pp. 403–408 (2007)
- [25] Qi, G., Du, J.: Model-based revision operators for terminologies in description logics. In: *Twenty-First International Joint Conference on Artificial Intelligence* (2009)
- [26] Wang, Z., Wang, K., Topor, R.: A new approach to knowledge base revision in dl-lite. In: *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 24 (2010)
- [27] Dubois, D., Prade, H.: Possibility theory: qualitative and quantitative aspects. *Quantified representation of uncertainty and imprecision*, 169–226 (1998)
- [28] Halashek-Wiener, C., Parsia, B., Sirin, E.: Description logic reasoning with syntactic updates. In: *On the Move to Meaningful Internet Systems 2006: CoopIS, DOA, GADA, and ODBASE: OTM Confederated International Conferences, CoopIS, DOA, GADA, and ODBASE 2006, Montpellier, France, October 29–November 3, 2006. Proceedings, Part I*, pp. 722–737 (2006). Springer
- [29] Mohamed, R., Loukil, Z., Bouraoui, Z.: Qualitative-based possibilistic el ontology. In: *PRIMA 2018* (2018)
- [30] Delgrande, J.P., Dubois, D., Lang, J.: Iterated revision as prioritized merging. *KR* **6**, 210–220 (2006)
- [31] Özçep, Ö.L.: Iterated ontology revision by reinterpretation. arXiv preprint arXiv:1603.09194 (2016)

- [32] Benferhat, S., Bouraoui, Z.: Possibilistic dl-lite. In: Liu, W., Subrahmanian, V.S., Wijsen, J. (eds.) Scalable Uncertainty Management - 7th International Conference, SUM 2013, Washington, DC, USA, September 16-18, 2013. Proceedings. Lecture Notes in Computer Science, vol. 8078, pp. 346–359. Springer. https://doi.org/10.1007/978-3-642-40381-1_27. https://doi.org/10.1007/978-3-642-40381-1_27
- [33] Benferhat, S., Bouraoui, Z.: Min-based possibilistic dl-lite. *J. Log. Comput.* **27**(1), 261–297 (2017). <https://doi.org/10.1093/logcom/exv014>
- [34] Benferhat, S., Bouraoui, Z., Papini, O., Würbel, E.: Prioritized assertional-based removed sets revision of dl-lite belief bases. *Annals of Mathematics and Artificial Intelligence* **79**(1-3), 45–75 (2017)
- [35] Benferhat, S., Bouraoui, Z., Papini, O., Würbel, E.: A prioritized assertional-based revision for dl-lite knowledge bases. In: *Logics in Artificial Intelligence: 14th European Conference, JELIA 2014, Funchal, Madeira, Portugal, September 24-26, 2014. Proceedings 14*, pp. 442–456 (2014). Springer
- [36] Benferhat, S., Bouraoui, Z., Papini, O., Würbel, E.: Assertional-based removed set revision of dl-liter belief bases. In: *International Symposium on Artificial Intelligence and Mathematics, ISAIM 2014, Fort Lauderdale, FL, USA, January 6-8, 2014* (2014). http://www.cs.uic.edu/pub/Isaim2014/WebPreferences/ISAIM2014_Benferhat.etal.pdf
- [37] Charitopoulos, V.M., Dua, V.: A unified framework for model-based multi-objective linear process and energy optimisation under uncertainty. *Applied Energy* **186**, 539–548 (2017)
- [38] Canito, A., Corchado, J., Marreiros, G.: A systematic review on time-constrained ontology evolution in predictive maintenance. *Artificial Intelligence Review* **55**(4), 3183–3211 (2022)
- [39] Yahya, M., Breslin, J.G., Ali, M.I.: Semantic web and knowledge graphs for industry 4.0. *Applied Sciences* **11**(11), 5110 (2021)
- [40] Khanzode, K.C.A., Sarode, R.D.: Advantages and disadvantages of artificial intelligence and machine learning: A literature review. *International Journal of Library & Information Science (IJLIS)* **9**(1), 3 (2020)
- [41] Kang, Y., Cai, Z., Tan, C.-W., Huang, Q., Liu, H.: Natural language processing (nlp) in management research: A literature review. *Journal of Management Analytics* **7**(2), 139–172 (2020)
- [42] Haux, R.: Medical informatics: past, present, future. *International journal of medical informatics* **79**(9), 599–610 (2010)

- [43] Nebel, B.: *How Hard Is It to Revise a Belief Base?*, (1998). Springer
- [44] Nebel, B., *et al.*: Syntax-based approaches to belief revision. *Belief revision* **29**, 52–88 (1992)
- [45] Hansson, S.: New operators for theory change. *Theoria* **55**(2), 114–132 (1989)
- [46] Fuhrmann, A.: Theory contraction through base contraction. *Journal of philosophical logic*, 175–203 (1991)
- [47] Dalal, M.: Investigations into a theory of knowledge base revision: preliminary report. In: *Proceedings of the Seventh National Conference on Artificial Intelligence*, vol. 2, pp. 475–479 (1988). Citeseer
- [48] Wassermann, R.: On agm for non-classical logics. *Journal of Philosophical Logic* **40**, 271–294 (2011)
- [49] Winslett, M.: A framework for comparison of update semantics. In: *Proceedings of the Seventh ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems*, pp. 315–324 (1988)
- [50] Forbus, K.D.: Introducing actions into qualitative simulation. Technical report, ILLINOIS UNIV AT URBANA DEPT OF COMPUTER SCIENCE (1988)
- [51] Hegner, S.: Specification and implementation of programs for updating incomplete information databases. In: *Proceedings of the Sixth ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems*, pp. 146–158 (1987)
- [52] Reiter, R.: A theory of diagnosis from first principles. *Artificial intelligence* **32**(1), 57–95 (1987)
- [53] Winslett, M.S.: *Reasoning About Action Using a Possible Models Approach* vol. 1427-1432. Department of Computer Science, University of Illinois at Urbana-Champaign (1988)
- [54] Eiter, T., Gottlob, G.: On the complexity of propositional knowledge base revision, updates, and counterfactuals. In: *Proceedings of the Eleventh ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems*, pp. 261–273 (1992)
- [55] Chou, T.S., Winslett, M.: A model-based belief revision system. *Journal of Automated Reasoning* **12**, 157–208 (1994)
- [56] Borgida, A.: Language features for flexible handling of exceptions in information systems. *ACM Transactions on Database Systems (TODS)* **10**(4),

565–603 (1985)

- [57] Fagin, R., Ullman, J.D., Vardi, M.Y.: On the semantics of updates in databases. In: Proceedings of the 2nd ACM SIGACT-SIGMOD Symposium on Principles of Database Systems, pp. 352–365 (1983)
- [58] Ginsberg, M.L.: Counterfactuals. *Artificial intelligence* **30**(1), 35–79 (1986)
- [59] Nebel, B.: Belief revision and default reasoning: Syntax-based approaches. *KR* **91**, 417–428 (1991)
- [60] Fagin, R., Kuper, G.M., Ullman, J., Vardi, M.Y.: Updating logical databases. Technical report, STANFORD UNIV CA DEPT OF COMPUTER SCIENCE (1984)
- [61] Liberatore, P., Schaerf, M.: The complexity of model checking for belief revision and update. In: *AAAI/IAAI*, Vol. 1, pp. 556–561 (1996)
- [62] Creignou, N., Ktari, R., Papini, O.: Complexity of model checking for cardinality-based belief revision operators. In: *Symbolic and Quantitative Approaches to Reasoning with Uncertainty: 14th European Conference, ECSQARU 2017, Lugano, Switzerland, July 10–14, 2017, Proceedings*, pp. 387–397 (2017). Springer
- [63] Ktari, R., Boujelben, M.A.: On the use of evidence theory in belief base revision. *arXiv preprint arXiv:2009.11640* (2020)
- [64] Kang, S.H., Lau, S.K.: Ontology revision using the concept of belief revision. In: *Knowledge-Based Intelligent Information and Engineering Systems: 8th International Conference, KES 2004, Wellington, New Zealand, September 20-25, 2004, Proceedings, Part III 8*, pp. 8–15 (2004). Springer
- [65] Qi, G., Liu, W., Bell, D.A.: Knowledge base revision in description logics. In: *Logics in Artificial Intelligence: 10th European Conference, JELIA 2006 Liverpool, UK, September 13-15, 2006 Proceedings 10*, pp. 386–398 (2006). Springer
- [66] Qi, G., Haase, P., Huang, Z., Ji, Q., Pan, J.Z., Völker, J.: A kernel revision operator for terminologies-algorithms and evaluation. In: *ISWC*, vol. 8, pp. 419–434 (2008)
- [67] Haase, P., Stojanovic, L.: Consistent evolution of owl ontologies. In: *The Semantic Web: Research and Applications: Second European Semantic Web Conference, ESWC 2005, Heraklion, Crete, Greece, May 29–June 1, 2005. Proceedings 2*, pp. 182–197 (2005). Springer

- [68] Liu, H., Lutz, C., Miličić, M., Wolter, F.: Foundations of instance level updates in expressive description logics. *Artificial Intelligence* **175**(18), 2170–2197 (2011)
- [69] Ahmetaj, S., Calvanese, D., Ortiz, M., Šimkus, M.: Managing change in graph-structured data using description logics. *ACM Transactions on Computational Logic (TOCL)* **18**(4), 1–35 (2017)
- [70] Giacomo, G.D., Lenzerini, M., Poggi, A., Rosati, R.: On instance-level update and erasure in description logic ontologies. *J. Log. Comput.* **19**(5), 745–770 (2009). <https://doi.org/10.1093/logcom/exn051>
- [71] Zheleznyakov, D., Kharlamov, E., Nutt, W., Calvanese, D.: On expansion and contraction of dl-lite knowledge bases. *J. Web Semant.* **57** (2019). <https://doi.org/10.1016/j.websem.2018.12.002>
- [72] Wang, Z., Wang, K., Topor, R.W.: A new approach to knowledge base revision in dl-lite. In: Fox, M., Poole, D. (eds.) *Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence, AAAI 2010, Atlanta, Georgia, USA, July 11-15, 2010*. AAAI Press. <http://www.aaai.org/ocs/index.php/AAAI/AAAI10/paper/view/1786>
- [73] Lenzerini, M., Savo, D.F.: On the evolution of the instance level of dl-lite knowledge bases. In: Rosati, R., Rudolph, S., Zakharyashev, M. (eds.) *Proceedings of the 24th International Workshop on Description Logics (DL 2011), Barcelona, Spain, July 13-16, 2011*. CEUR Workshop Proceedings, vol. 745. CEUR-WS.org. http://ceur-ws.org/Vol-745/paper_11.pdf
- [74] Qi, G., Wang, Z., Wang, K., Fu, X., Zhuang, Z.: Approximating model-based abox revision in dl-lite: Theory and practice. In: Bonet, B., Koenig, S. (eds.) *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence, January 25-30, 2015, Austin, Texas, USA*, pp. 254–260. AAAI Press. <http://www.aaai.org/ocs/index.php/AAAI/AAAI15/paper/view/9830>
- [75] Giacomo, G.D., Oriol, X., Rosati, R., Savo, D.F.: Updating dl-lite ontologies through first-order queries. In: Groth, P., Simperl, E., Gray, A.J.G., Sabou, M., Krötzsch, M., Lécué, F., Flöck, F., Gil, Y. (eds.) *The Semantic Web - ISWC 2016 - 15th International Semantic Web Conference, Kobe, Japan, October 17-21, 2016, Proceedings, Part I*. Lecture Notes in Computer Science, vol. 9981, pp. 167–183 (2016). https://doi.org/10.1007/978-3-319-46523-4_11. https://doi.org/10.1007/978-3-319-46523-4_11
- [76] Giacomo, G.D., Lembo, D., Oriol, X., Savo, D.F., Teniente, E.: Practical update management in ontology-based data access. In: d’Amato, C., Fernández, M., Tamma, V.A.M., Lécué, F., Cudré-Mauroux, P.,

- Sequeda, J.F., Lange, C., Heflin, J. (eds.) *The Semantic Web - ISWC 2017 - 16th International Semantic Web Conference, Vienna, Austria, October 21-25, 2017, Proceedings, Part I. Lecture Notes in Computer Science*, vol. 10587, pp. 225–242. Springer. <https://doi.org/10.1007/978-3-319-68288-4.14>. <https://doi.org/10.1007/978-3-319-68288-4.14>
- [77] Gärdenfors, P., Rott, H., Gabbay, D., Hogger, C., Robinson, J.: Belief revision. *Computational Complexity* **63**(6), 35–132 (1995)
- [78] Rott, H.: Two methods of constructing contractions and revisions of knowledge systems. *Journal of philosophical logic*, 149–173 (1991)
- [79] Bochman, A.: *Evolutionary belief change: Priorities, conditionals and argumentation*. (2013)
- [80] Benferhat, S., Dubois, D., Prade, H., Williams, M.: A practical approach to revising prioritized knowledge bases. *Stud Logica* **70**(1), 105–130 (2002). <https://doi.org/10.1023/A:1014658309853>
- [81] Spohn, W.: Ordinal conditional functions: A dynamic theory of epistemic states. In: *Causation in Decision, Belief Change, and Statistics: Proceedings of the Irvine Conference on Probability and Causation*, pp. 105–134 (1988). Springer
- [82] Darwiche, A., Pearl, J.: On the logic of iterated belief revision. *Artificial intelligence* **89**(1-2), 1–29 (1997)
- [83] Qi, G.: A semantic approach for iterated revision in possibilistic logic. In: *AAAI*, pp. 523–528 (2008)
- [84] Jin, Y., Thielscher, M.: Iterated belief revision, revised. *Artificial Intelligence* **171**(1), 1–18 (2007)
- [85] Brewka, G.: Belief revision in a framework for default reasoning. In: *The Logic of Theory Change: Workshop, Konstanz, FRG, October 13–15, 1989 Proceedings*, pp. 206–222 (1991). Springer
- [86] Falappa, M.A., Garcia, A.J., Kern-Isberner, G., Simari, G.R.: Stratified belief bases revision with argumentative inference. *Journal of Philosophical Logic* **42**, 161–193 (2013)
- [87] Falappa, M.A., Kern-Isberner, G., Simari, G.R.: Belief revision, explanations and defeasible reasoning. *Artificial Intelligence Journal* **141**(1-28), 10–1016 (2002)
- [88] Tamargo, L.H.: Knowledge dynamics in multi-agent systems: Plausibility, belief revision and forwarding information. *AI Communications* **25**(4),

391–393 (2012)

- [89] Tamargo, L.H., Falappa, M.A., Garcia, A.J., Simari, G.R.: A change model for credibility partial order. In: Scalable Uncertainty Management: 5th International Conference, SUM 2011, Dayton, OH, USA, October 10-13, 2011. Proceedings 5, pp. 317–330 (2011). Springer
- [90] Ichihashi, H., Tanaka, H.: Jeffrey-like rules of conditioning for the dempster-shafer theory of evidence. *International Journal of Approximate Reasoning* **3**(2), 143–156 (1989)
- [91] Dubois, D., Prade, H.: A synthetic view of belief revision with uncertain inputs in the framework of possibility theory. *International Journal of Approximate Reasoning* **17**(2-3), 295–324 (1997)
- [92] Qi, G., Liu, W., Bell, D.A.: A revision-based approach to handling inconsistency in description logics. *Artif. Intell. Rev.* **26**(1-2), 115–128 (2006). <https://doi.org/10.1007/s10462-007-9044-3>
- [93] Meyer, T.A., Lee, K., Booth, R.: Knowledge integration for description logics. In: Veloso, M.M., Kambhampati, S. (eds.) Proceedings, The Twentieth National Conference on Artificial Intelligence and the Seventeenth Innovative Applications of Artificial Intelligence Conference, July 9-13, 2005, Pittsburgh, Pennsylvania, USA, pp. 645–650. AAAI Press / The MIT Press. <http://www.aaai.org/Library/AAAI/2005/aaai05-101.php>
- [94] Qi, G., Liu, W., Bell, D.: A revision-based approach to handling inconsistency in description logics. *Artificial Intelligence Review* **26**, 115–128 (2006)
- [95] Papini, O.: A complete revision function in propositional calculus. In: Proceedings of the 10th European Conference on Artificial Intelligence, pp. 339–343 (1992)
- [96] Benferhat, S., da Costa Pereira, C., Tettamanzi, A.: Hybrid possibilistic conditioning for revision under weighted inputs. In: Raedt, L.D., Bessiere, C., Dubois, D., Doherty, P., Frasconi, P., Heintz, F., Lucas, P.J.F. (eds.) ECAI 2012 - 20th European Conference on Artificial Intelligence. Including Prestigious Applications of Artificial Intelligence (PAIS-2012) System Demonstrations Track, Montpellier, France, August 27-31, 2012. *Frontiers in Artificial Intelligence and Applications*, vol. 242, pp. 151–156. IOS Press. <https://doi.org/10.3233/978-1-61499-098-7-151>. <https://doi.org/10.3233/978-1-61499-098-7-151>
- [97] Baader, F., Horrocks, I., Sattler, U.: Description logics. In: Staab, S., Studer, R. (eds.) *Handbook on Ontologies*. *International Handbooks on Information Systems*, pp. 3–28. Springer

- [98] Zadeh, L.A.: Fuzzy sets as a basis for a theory of possibility. *Fuzzy sets and systems* **1**(1), 3–28 (1978)
- [99] Dubois, D., Prade, H.: Possibility theory. *Scholarpedia* **2**(10), 2074 (2007)

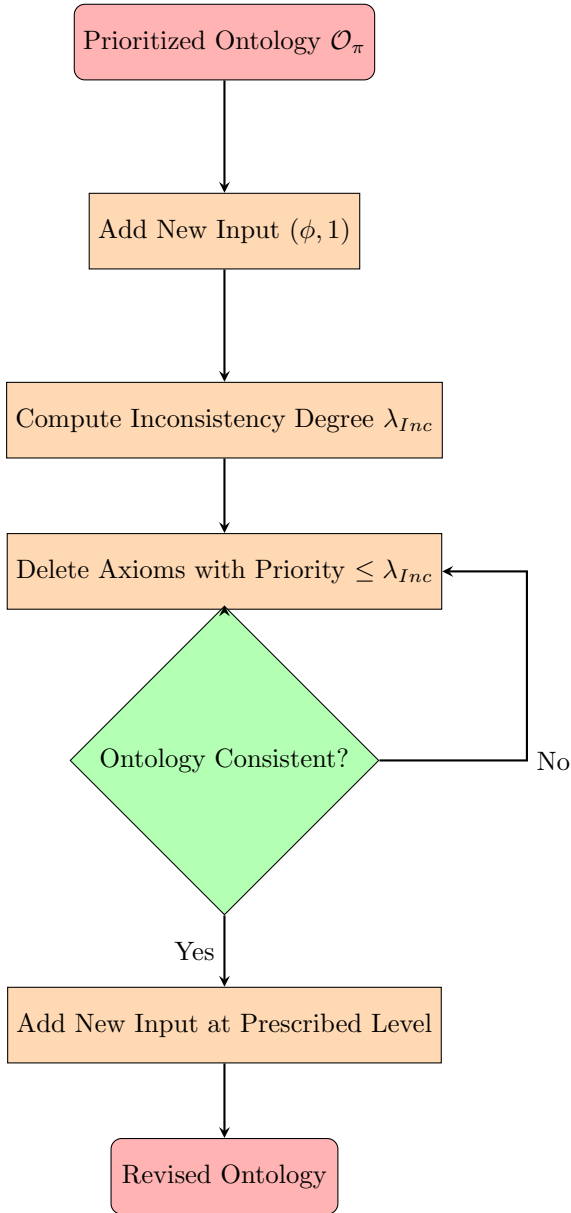


Fig. 3 Syntactical Revision with Inconsistent Input

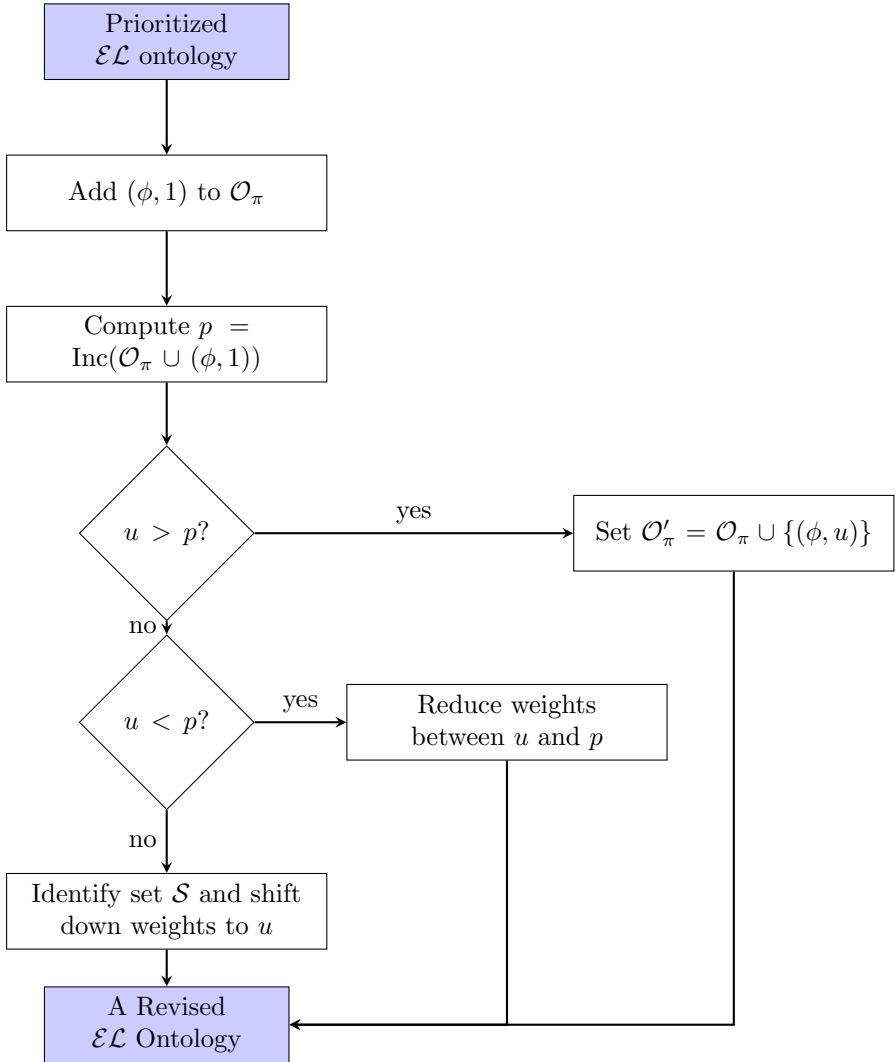


Fig. 4 Syntax Revision with Consistent Input