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Robust Formation Control of Robot Manipulators with Inter-agent Constraints over Undirected Signed Networks

Pelin Şekercioğlu Bayu Jayawardhana Ioannis Sarras Antonio Loria Julien Marzat

Abstract—We address the problem of distributed control of a network of cooperative and competitive robot manipulators in end-effector coordinates. We propose a distributed bipartite formation controller that guarantees collision avoidance. In the considered setting two groups are formed and reach inter-group bipartite consensus or disagreement. On the other hand, the end-effectors achieve intra-group formation. To ensure that the end-effectors do not collide, we design gradient-based control laws using barrier-Lyapunov functions. In addition, the proposed controller ensures that the closed-loop system is robust to external disturbances. The latter are assumed to be generated by an exosystem, so they are effectively rejected by an internal-model-based compensator. More precisely, we establish asymptotic stability of the bipartite formation manifold. Finally, we illustrate our theoretical results via numerical simulations.

Index Terms— Formation consensus, signed networks, robotic manipulators, barrier-Lyapunov functions

I. INTRODUCTION

Formation control consists, roughly speaking, in making a group of physical systems adopt a formation and remain stable at an equilibrium, or move along a path, describing a common trajectory. This problem has been extensively studied, often relying on the bulk of literature on consensus control. However, the greater part of the literature considers only cooperative agents (in which case these form a network that can be modeled by a graph containing only links with positive weights) or considers linear models. Yet, robot manipulators are inherently nonlinear and there are many scenarios in which some agents may be competitive, so their interactions carry negative weights. Beyond applications involving robot manipulators, other scenarios that pertain to cooperative networks include herding control [1], [2]; social-networks theory [3], and aerospace applications [4].

The cooperative vs competitive nature of the links may be analyzed using the formalism of *signed networks* [3], in which the edges have both positive and negative weights. For some

of such networks, called structurally balanced¹, the achievable goal is *bipartite consensus* [3], in which all the agents converge to the same state in modulus but opposite in signs. See, e.g., [3], [5], [6], and [7].

In all of the previous references, however, generic first, second or higher-order linear models are used. These are less suitable for robot manipulators, which are most commonly modeled by the Euler-Lagrange equations. In that regard, the literature on control of multi-agent Euler-Lagrange systems is also rich, but most often only cooperative networks are considered. For instance, in [8], [9] the tracking-consensus problem for mobile robots with nonholonomic constraints is addressed, in [10] the formation control of flying spacecrafts, in [11] the synchronization of multi-Lagrangian systems, and in [12]–[15] the synchronization of multiple robot manipulators. Now in all of these references, the synchronization problem is studied in joint coordinates. Formation of manipulators in end-effector coordinates is considered in [16]–[18]. Nonetheless, in all of the previously cited references only networks of cooperative agents are considered. For signed networks, the bipartite consensus of networked robot manipulators is addressed, e.g., in [19]–[23], while the leader-follower bipartite consensus is studied in [24]–[27] (in the latter parametric uncertainty is also considered). In end-effectors coordinates the bipartite formation-control problem is considered in [28].

Now, besides the two aspects previously described, which relate to the network and systems' model (*i.e.*, the sign of the interconnections and the agents' dynamics), there are others that must be taken into account in the control of multi-agent robot systems. Two of these are the existence of constraints and the effect of external disturbances. Considering that a disturbance may be modeled by a multi-periodic signal [18] an effective method to compensate for its effect is the internal-model-based approach. See, e.g., [15], [18], [29]–[31] for works on consensus among cooperative robots, and [22]–[24] and [28] for works on cooperative networks of robot manipulators. Yet, none of the references cited above considers the presence of constraints.

In this paper, we are interested in the bipartite formation-control problem of end-effectors while ensuring inter-agent collision avoidance and maintenance of information exchange.

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¹A signed network is structurally balanced if all the nodes may be split into two disjoint subsets, where agents cooperative with each other are in the same subset and agents competitive with each other are in different ones [3].

These objectives are typically expressed as inter-agent constraints and are commonly addressed using artificial potential functions—see *e.g.*, [32]–[34]. Now, several articles address constrained consensus problems [35]–[37], but only a few works focus on constrained control problems for networks containing competitive interactions. For instance, in [1] the authors study connectivity-constrained multi-swarm herding, in [2] non-cooperative herding with connectivity maintenance is achieved, and the authors in [38] achieve bipartite flocking with collision avoidance and connectivity maintenance, using artificial potential functions. A barrier-Lyapunov-function-based controller is proposed in [39], which is a preliminary version of this paper, devoted to the the constrained leaderless bipartite formation problem over undirected signed networks of simple integrators. As a matter of fact, all the references mentioned above consider only first and second-order integrators.

In this paper, we consider the distributed bipartite formation-control problem of robot manipulators’ end-effectors under relative distance constraints and in the presence of disturbances. We consider a networked system of cooperative-competitive robot manipulators modeled by the Euler-Lagrange equations and interconnected over a structurally balanced undirected signed graph [3]. The desired formation goal is imposed on the manipulators’ end effectors. Such scenarios are motivated, for example, by applications in industrial robotics’ where robots share the same workspace but are assigned symmetric tasks by the team. Ideally, the robot manipulators should occupy the minimum space while evolving with guaranteed safety and increased reactivity.

Relative to [16]–[18], our results apply to networks having both cooperative and competitive interactions. Contrary to [19]–[28], in which the bipartite consensus problem of robot manipulators over signed networks is studied, we address the problem under inter-agent constraints. We consider inter-agent distance constraints on the end effectors, such as collision avoidance and connectivity maintenance. Relative to [1], [2], in which the control strategies rely on optimization techniques and to [38], in which artificial potential functions are used, we base our controller on the gradient of a barrier-Lyapunov function. In contrast to [1], [2], our controller applies to signed networks and in contrary to [38], a minimal safety distance between agents is ensured. Relative to [39], we consider Euler-Lagrange systems, not simple integrators, and we establish robustness with respect to external perturbations. To that end, we follow the frameworks of [15], [18], [29], to use an internal model to reject the disturbances, but contrary to these references, our work considers signed networks. Relative to [22]–[24] and [28], in which the presence of disturbances is considered, we also address collision avoidance and connectivity maintenance constraints. Our control design and analysis rely on the edge-based formulation for signed networks [40], which allows to recast the problem into one of stabilization of the origin in error coordinates. We establish asymptotic stability of the bipartite formation manifold using Lyapunov’s direct method.

Thus, relative to the existing literature, we contribute with a

robust bipartite formation control law that ensures that the manipulator’s end effectors achieve the desired formation while avoiding inter-agent collisions and staying in their sensors’ range. To the best of our knowledge, similar results are not available in the literature for robot manipulators containing competitive interactions.

II. MODEL AND PROBLEM FORMULATION

We describe in detail every aspect of the problem of bipartite formation of end-effectors with constraints and under perturbations, and present the models that we use.

A. Agents’ dynamics

Consider a network of N n -degrees-of-freedom robot manipulators modeled by the Euler-Lagrange equations.

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + \frac{\partial}{\partial q_i}U_i(q_i) = \tau_i + d_i, \quad i \leq N, \quad (1)$$

where $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n$ are the generalized joint position, velocity, and acceleration respectively, $M_i(q_i) \in \mathbb{R}^{n \times n}$ is the inertia matrix, $U : \mathbb{R}^n \rightarrow \mathbb{R}$ is the potential energy function, $\tau_i \in \mathbb{R}^p$ is the control input and $d_i \in \mathbb{R}^n$ is an external disturbance generated by an exosystem. As it is customary, we assume the following.

Assumption 1: The following properties hold.

1. There exist \underline{c}_i and $\bar{c}_i > 0$ such that, $\underline{c}_i I \leq M_i(q_i) \leq \bar{c}_i I$ for all $q_i \in \mathbb{R}^n$.
2. The matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric.
3. The Coriolis matrix $C_i(q_i, \dot{q}_i)$ is uniformly bounded in q_i . Moreover $|C_i(q_i, \dot{q}_i)\dot{q}_i| \leq k_{c_i}|\dot{q}_i|^2$ for $k_{c_i} > 0$.

As in [15] and [18], we consider that the external disturbances are modeled by

$$d_i = d_{M,i} + J_i(q_i)^\top d_{E,i}, \quad (2)$$

where $d_{M,i} \in \mathbb{R}^n$, $d_{E,i} \in \mathbb{R}^p$ and $J_i(q_i) \in \mathbb{R}^{n \times p}$ is the Jacobian matrix. The disturbance d_i is generated by an exosystem of the form

$$\dot{w}_{M,i} = S_{M,i}w_{M,i}, \quad d_{M,i} = C_{M,i}w_{M,i}, \quad (3a)$$

$$\dot{w}_{E,i} = S_{E,i}w_{E,i}, \quad d_{E,i} = C_{E,i}w_{E,i}, \quad i \leq N \quad (3b)$$

where $w_{M,i}, w_{E,i} \in \mathbb{R}^{l_i}$, $S_{M,i}, S_{E,i} \in \mathbb{R}^{l_i \times l_i}$ and $C_{M,i}, C_{E,i} \in \mathbb{R}^{n \times l_i}$. As in [29], we assume the following.

Assumption 2: The exosystems $S_{M,i}$ and $S_{E,i}$ are assumed to be neutrally stable, that is, all the eigenvalues of $S_{M,i}$ and $S_{E,i}$ are different and lie on the imaginary axis, and they are nonsingular. Moreover, they are assumed to be known.

Such an assumption is realistic for various human-robot-environment interactions because the disturbance is expressed as a sum of sinusoids—cf. [29], which is a truncated finite Fourier approximation of general external bounded disturbances.

B. Problem statement

We define now the problem of bipartite formation of manipulators' end-effectors. Let $x_i \in \mathbb{R}^p$ be the position of the i th manipulator's end-effector in the task space. The end-effector's position x_i can be mapped to its generalized joint coordinates via a nonlinear forward kinematics mapping [41]

$$x_i = x_{i_0} + h_i(q_i), \quad (4)$$

where x_{i_0} is the position of the manipulator's base and $h_i : \mathbb{R}^n \rightarrow \mathbb{R}^p$ is the mapping from joint-space to the task space. Differentiating (4) with respect to time, we obtain the relation between the task-space velocity and joint velocity [41]

$$\dot{x}_i = J_i(q_i)\dot{q}_i, \quad J_i(q_i) := \frac{\partial h_i(q_i)}{\partial q_i}\dot{q}_i, \quad (5)$$

with $J_i(q_i) \in \mathbb{R}^{n \times p}$ the Jacobian matrix of the forward kinematics.

The bipartite formation control problem consists in the end-effectors' positions of the cooperative agents reaching a desired geometric shape around a consensus value, while the end-effectors' positions of non-cooperative agents converge to another spatial configuration. The characteristics of the formation shape are defined through the relative biases b_i and b_j with respect to the consensus points. Formally, we can thus define the bipartite formation control objective as

$$\lim_{t \rightarrow \infty} \bar{x}_i(t) - \text{sgn}(a_{ij})\bar{x}_j(t) \rightarrow 0, \quad i, j \leq N, \quad (6)$$

where

$$\bar{x}_i := x_i - b_i, \quad (7)$$

and $a_{ij} \in \mathbb{R}$ is the adjacency weight between the two agents.

In an all-cooperative-agents setting, consensus means that all \bar{x}_i converge to the same value, but in this case, since some robot manipulators are cooperative and others are competitive, all end-effectors reach two symmetrical consensus values. For the purpose of control design and analysis, this boils down to making some synchronization errors to converge to zero. These errors correspond to the edges on the graph and are defined as

$$\bar{e}_k := \bar{x}_i - \text{sgn}(a_{ij})\bar{x}_j, \quad k \leq M, \quad (8)$$

where \bar{x}_i is defined in (7) and k denotes the index of the interconnection between the i th and j th end-effectors. Since a_{ij} is either positive or negative, the resulting network is modeled by a signed graph [3].

Assumption 3: The systems described in (1), which are interconnected via inputs τ_i , form a structurally balanced (see below), undirected, and connected signed graph.

Remark 1: Recall that a signed graph is *structurally balanced* if it may be split into two disjoint sets of vertices \mathcal{V}_1 and \mathcal{V}_2 , where $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ such that for every $i, j \in \mathcal{V}_p, p \in \{1, 2\}$, if $a_{ij} \geq 0$, while for every $i \in \mathcal{V}_p, j \in \mathcal{V}_q$, with $p, q \in \{1, 2\}, p \neq q$, if $a_{ij} \leq 0$. Otherwise, it is *structurally unbalanced* [3]. •

In addition, it is imposed that the controller τ_i must ensure that the end-effectors do not collide and remain within their

sensing ranges. This comes to ensuring that for any pair of communicating nodes ν_i and $\nu_j \in \mathcal{V}$, let $\delta_k := x_i - x_j$, let $R_k > 0$ and $\Delta_k > 0$ be defined with $k \leq M$, the following sets are invariant.

$$\mathcal{I}_r := \{\delta_k \in \mathbb{R}^n : |\delta_k| < R_k, \quad \forall i, j \in \mathcal{V}_l, l \in \{1, 2\}\} \quad (9a)$$

$$\mathcal{I}_c := \{\delta_k \in \mathbb{R}^n : |\delta_k| > \Delta_k \quad \forall k \leq M\}, \quad (9b)$$

where \mathcal{I}_r is the set of proximity constraints and \mathcal{I}_c is the set of collision-avoidance constraints. Under these conditions, it is required to design a distributed bipartite formation control law of the form

$$\dot{\chi}_i = f_1(\bar{e}_k, q_i, \dot{q}_i, \chi_i)$$

$$\tau_i = f_2(\bar{e}_k, q_i, \dot{q}_i, \chi_i),$$

where χ_i is the disturbance compensator to be designed later, to achieve bipartite formation of end-effectors, such that,

$$\lim_{t \rightarrow \infty} \bar{e}_k(t) = 0, \quad \lim_{t \rightarrow \infty} \dot{q}_i(t) = 0, \quad k \leq M, i \leq N, \quad (10)$$

and the manipulators' end-effector's trajectories satisfy the proximity and collision-avoidance constraints, that is, $\delta_i(t) \in \mathcal{I}$ for all $t \geq 0$, with $\mathcal{I} := \mathcal{I}_r \cap \mathcal{I}_c$.

III. CONTROL DESIGN

The control approach is the following. We deal with the considered problem as the stabilization of the origin in edge coordinates [40], [42], which correspond exactly to the synchronization errors in (8). Then, in order to respect the inter-agent constraints, the control input is designed as the gradient of a so-called barrier-Lyapunov function—cf. [34], [42], [43]. Finally, in order to cope with disturbance, we use an internal model approach, similar to [15], [18], [29]. Next, we discuss in more detail each aspect of the control design.

A. Control in the absence of disturbance

A *barrier-Lyapunov function* (BLF) is defined in the following—cf. [34], [42], [43].

Definition 1: Consider the system $\dot{x} = f(x)$ and let \mathcal{I} be an open set containing the origin. A BLF is a positive definite \mathcal{C}^1 function $W : \mathcal{I} \rightarrow \mathbb{R}_{\geq 0}$, $x \mapsto W(x)$, satisfies $\nabla W(x)f(x) \leq 0$, where $\nabla W(x) := \partial W / \partial x$, and has the property that $W(x) \rightarrow \infty$, and $\nabla W(x) \rightarrow \infty$ as $x \rightarrow \partial \mathcal{I}$.

We proceed to introduce a barrier-Lyapunov function expressed in terms of the synchronization errors \bar{e}_k for the purposes of this paper. To that end, we remark that \bar{e}_k in (8) can also be expressed as

$$\bar{e}_k = \bar{x}_i - \bar{x}_j = \delta_k - \bar{b}_k, \quad i, j \in \mathcal{V}_p, \quad (11)$$

for a couple of cooperative agents and as

$$\bar{e}_k = \bar{x}_i + \bar{x}_j = \delta_k - \bar{b}_k + 2x_j, \quad i \in \mathcal{V}_p, j \in \mathcal{V}_q, \quad (12)$$

for a couple competitive agents, where $p, q \in \{1, 2\}, p \neq q$ and $\bar{b}_k = b_i - \text{sgn}(a_{ij})b_j$. Then, from (11) and (12), the constraint sets in (9) can be restated as follows

$$\mathcal{I}_r = \{\bar{e}_k \in \mathbb{R}^n : |\bar{e}_{x_k} + \alpha_k| < R_k, \quad \forall k \in \mathcal{E}_m\}, \quad (13a)$$

$$\mathcal{I}_c = \{\bar{e}_k \in \mathbb{R}^n : \Delta_k < |\bar{e}_{x_k} + \alpha_k|, \forall k \leq M\}, \quad (13b)$$

where α_k is defined as

$$\alpha_k := \delta_k - \bar{e}_k, \quad k \leq M. \quad (14)$$

Here, \mathcal{E}_m comprises the indices of m cooperative edges, where $m < M$, which are the edges with strictly positive weights. Now, we define the BLF $W_k : \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ for each $k \leq M$,

$$W_k(\bar{e}_k) = \frac{1}{2} [|\bar{e}_k|^2 + B_k(\bar{e}_k)], \quad (15)$$

where $B_k(\bar{e}_k)^2$ is the sum of two functions satisfying Definition 1, each of them encoding the constraints in (13), respectively, $B_k(\bar{e}_k) = \frac{1}{2}(1 + \sigma_k)B_{r_k}(\bar{e}_k) + B_{c_k}(\bar{e}_k)$, where $B_{c_k}(\bar{e}_k) \rightarrow \infty$ as $|\bar{e}_k| \rightarrow \Delta_k$ and $B_{r_k}(\bar{e}_k) \rightarrow \infty$ as $|\bar{e}_k| \rightarrow R_k$ for all k . In the latter, $\sigma_k = 1$ if $k \in \mathcal{E}_m$, i.e., if the interaction is cooperative, and $\sigma_k = -1$ otherwise. Furthermore, $B_k(\bar{e}_k)$ is non-negative and satisfies $B_k(0) = 0$. Additionally, it tends to infinity as $|\bar{e}_k| \rightarrow \Delta_k$ for all edges and as $|\bar{e}_k| \rightarrow R_k$ for $k \in \mathcal{E}_m$. However, considering the constraints defined in (13), the barrier function has to be adjusted to ensure that the solution lies within the interior of the constraint sets in (13) and to guarantee the system's convergence to the desired point. To achieve this, we employ the concept of the *gradient recentered barrier function* [44]. Let $\widehat{W}_k : \mathcal{I}_c \times \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ for competitive interactions and $\widehat{W}_k : \mathcal{I}_c \cap \mathcal{I}_r \times \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ for cooperative ones be defined as

$$\widehat{W}_k(\alpha_k, \bar{e}_k) := W_k(\bar{e}_k + \alpha_k) - W_k(\alpha_k) - \frac{\partial W_k}{\partial s}(\alpha_k) \bar{e}_k, \quad (16)$$

which satisfies $\widehat{W}_k(\alpha_k, 0) = 0$, $\nabla_{\bar{e}_k} \widehat{W}_k(\alpha_k, 0) = 0$, where $\nabla_{\bar{e}_k} \widehat{W}_k = \frac{\partial \widehat{W}_k}{\partial \bar{e}_k}$, and $\widehat{W}_k(\alpha_k, \bar{e}_k) \rightarrow \infty$ as $|\delta_k| \rightarrow \Delta_k$ for $k \leq M$, and as $|\delta_k| \rightarrow R_k$ for all $k \in \mathcal{E}_m$. Moreover, $\widehat{W}_k(\alpha_k, \bar{e}_k)$ satisfies $\frac{\kappa_1}{2} \bar{e}_k^2 \leq \widehat{W}_k(\alpha_k, \bar{e}_k) \leq \kappa_2 [\nabla_{\bar{e}_k} \widehat{W}_k]^2$.

Then, we introduce the BLF-gradient-based bipartite formation control law given by

$$\begin{aligned} \tau_i^* = & -k_{1_i} J_i(q_i)^\top \left[\sum_{k=1}^M [E_s]_{ik} \nabla_{\bar{e}_k} \widehat{W}_k + \sum_{k=1}^M [\mathbb{E}]_{ik} \nabla_{\alpha_k} \widehat{W}_k \right] \\ & - k_{2_i} \dot{q}_i + \frac{\partial}{\partial q_i} U_i(q_i), \end{aligned} \quad (17)$$

where $k_{1_i} > 0$, $k_{2_i} > 0$ for all $i \leq N$,

$$\mathbb{E} = E - E_s, \quad (18)$$

E is the incidence matrix of the cooperative version of the considered network³, and E_s the incidence matrix of the considered signed network. We recall that E_s describes the interaction topology of the network and is defined as follows

²A particular choice for $B_k(\bar{e}_k)$ is given in Section V.

³A structurally balanced graph may be transformed into a traditional cooperative one using the gauge transformation—see [3], [40].

for a structurally balanced signed network.

$$[E_s]_{ik} := \begin{cases} +1, & \text{if } v_i \text{ is the initial node of the edge } \varepsilon_k; \\ -1, & \text{if } v_i, v_j \text{ are cooperative such that} \\ & v_i, v_j \in \mathcal{V}_l, l \in \{1, 2\} \text{ and } v_i \text{ is the} \\ & \text{terminal node of the edge } \varepsilon_k; \\ +1, & \text{if } v_i, v_j \text{ are competitive such that} \\ & v_i \in \mathcal{V}_p, v_j \in \mathcal{V}_q, p, q \in \{1, 2\}, p \neq q \text{ and} \\ & v_i \text{ is the terminal node of the edge } \varepsilon_k; \\ 0, & \text{otherwise,} \end{cases}$$

where $\varepsilon_k = \{v_i, v_j\}$, $k \leq M$, $i, j \leq N$ are arbitrarily oriented edges and \mathcal{V}_1 and \mathcal{V}_2 are the two disjoint sets of vertices. The first two terms in the control law in (17) are needed to ensure the bipartite formation of end-effectors while respecting the inter-agent constraints imposed on the task space. The second term is needed specifically because of the use of the gradient recentered barrier function and the presence of competitive interactions between agents. The third term is needed to control the joint velocity. It consists of a damping term to stabilize the joint velocity at zero. The last term is to compensate for the gravitational force.

B. Robust control redesign

In order to deal with the disturbances modeled by (2) and (3), we design an estimator of d_i . For that, we use an internal model-based approach [15], [18], [29]. Let

$$\dot{\chi}_{1_i} = A_{M,i} \chi_{1_i} - B_{M,i} u_i, \quad (19a)$$

$$\dot{\chi}_{2_i} = A_{E,i} \chi_{2_i} - B_{E,i} J_i(q_i) u_i, \quad (19b)$$

where $\chi_{1_i} \in \mathbb{R}^{l_i}$, $\chi_{2_i} \in \mathbb{R}^{l_i}$, $A_{M,i} \in \mathbb{R}^{l_i \times l_i}$, $A_{E,i} \in \mathbb{R}^{l_i \times l_i}$, $B_{M,i} \in \mathbb{R}^{l_i \times n}$, $B_{E,i} \in \mathbb{R}^{l_i \times n}$, $u_i \in \mathbb{R}^n$ is the input to the internal model dynamics, which is defined later, $A_{M,i} + A_{M,i}^\top = 0$, $A_{E,i} + A_{E,i}^\top = 0$ and the pairs $(A_{M,i}, B_{M,i})$ and $(A_{E,i}, B_{E,i})$ are observable. We also assume, as in [15] and [18] that the eigenvalues of the matrix $S_{M,i}$ in (3) and $A_{M,i}$ and the eigenvalues of $S_{E,i}$ in (3) and $A_{E,i}$ are identical. In this case, there exist transformation matrices $T_{M,i} \in \mathbb{R}^{l_i \times p_i}$ and $T_{E,i} \in \mathbb{R}^{l_i \times p_i}$, such that

$$T_{M,i} S_{M,i} = A_{M,i} T_{M,i}, \quad B_{M,i}^\top T_{M,i} + C_{M,i} = 0 \quad (20a)$$

$$T_{E,i} S_{E,i} = A_{E,i} T_{E,i}, \quad B_{E,i}^\top T_{E,i} + C_{E,i} = 0. \quad (20b)$$

Then, we can rewrite (19) in the compact form as

$$\dot{\chi}_i = A_i \chi_i - B_i(q_i) u_i, \quad (21)$$

where $\chi_i = [\chi_{1_i} \quad \chi_{2_i}]^\top$, $A_i = \begin{bmatrix} A_{M,i} & 0 \\ 0 & A_{E,i} \end{bmatrix}$ and $B_i(q_i) = [B_{M,i} \quad J_i(q_i)^\top B_{E,i}]$.

Next, the control law is redesigned using χ_i , and the input u_i will be defined later, using the internal model. Then, we define the following estimation error coordinates: $\tilde{\chi}_i$, for the estimate of the disturbance, and \tilde{d}_i , for the disturbance.

$$\tilde{\chi}_i = \chi_i - T_i w_i \quad (22a)$$

$$\tilde{d}_i = B_i^\top(q_i) \chi_i + d_i, \quad (22b)$$

where $T_i = [T_{M,i} \ T_{E,i}]$ and $w_i = [w_{M,i} \ w_{E,i}]^\top$. Taking the derivative of (22a) and using (3) for (22b), we obtain

$$\begin{aligned}\dot{\tilde{\chi}}_i &= \dot{\chi}_i - T_i \dot{w}_i \\ \tilde{d}_i &= B_i(q_i)^\top \chi_i + C_i w_i.\end{aligned}$$

Replacing (21) and (3) then using (22a) in the first equation and using (20) and (22a) in the second equation, we obtain the following error dynamics.

$$\begin{aligned}\dot{\tilde{\chi}}_i &= A_i \chi_i - B_i(q_i) u_i - T_i S_i w_i \\ &= A_i \tilde{\chi}_i - B_i(q_i) u_i\end{aligned}\quad (23a)$$

$$\begin{aligned}\tilde{d}_i &= B_i(q_i)^\top \chi_i - B_i(q_i)^\top T_i w_i \\ &= B_i(q_i)^\top (\chi_i - T_i w_i) = B_i(q_i)^\top \tilde{\chi}_i.\end{aligned}\quad (23b)$$

The equations in (23) are important because they define a passive map from u_i to \tilde{d}_i . To see that, consider the storage function $H_i(\tilde{\chi}_i) = \frac{1}{2} |\tilde{\chi}_i|^2$. Its derivative gives

$$\begin{aligned}\dot{H}_i(\tilde{\chi}_i) &= \tilde{\chi}_i^\top \dot{\tilde{\chi}}_i = \frac{1}{2} \tilde{\chi}_i^\top (A_i + A_i^\top) \tilde{\chi}_i - \tilde{\chi}_i^\top B_i(q_i) u_i \\ &= -\tilde{\chi}_i^\top B_i(q_i) u_i,\end{aligned}$$

since $A_i + A_i^\top = 0$. Thus, the system in (23) is lossless from the input u_i to the output $\tilde{d}_i = B_i^\top \tilde{\chi}_i$. We use this observation in the control analysis.

Now, to robustify the controller, we add a term to compensate for the disturbances. Then, the control law in (17) is redesigned into

$$\tau_i = \tau_i^* + B_i^\top(q_i) \chi_i, \quad (24)$$

where the last term counteracts the effect of external disturbances.

IV. STABILITY ANALYSIS

A. Asymptotic stability in the absence of disturbance

We analyze the stability of the bipartite formation manifold for the closed-loop system (1) interconnected by the control law (17). To that end, using the definition of the incidence matrix, we represent the synchronization errors in (8) and α_k defined in (14), in vector form

$$\bar{e} = E_s^\top \bar{x}, \quad (25a)$$

$$\alpha = E^\top x - E_s^\top \bar{x}. \quad (25b)$$

Then, after (16), we define

$$\bar{W}(\alpha, \bar{e}) = \sum_{k=1}^M \widehat{W}_k(\alpha_k, \bar{e}_k), \quad (26)$$

to write the closed-loop system (1)–(17) in the compact form

$$\begin{aligned}\ddot{q} &= -M(q)^{-1} \left[C(q, \dot{q}) \dot{q} + K_1 J(q)^\top [E_s \otimes I_n] \nabla_{\bar{e}} \bar{W}(\alpha, \bar{e}) \right. \\ &\quad \left. + K_1 J(q)^\top [E \otimes I_n] \nabla_{\alpha} \bar{W}(\alpha, \bar{e}) + [K_2 \otimes I_n] \dot{q} \right],\end{aligned}\quad (27)$$

where $q = [q_i]$, $M(q) = \text{blkdiag}[M_i(q_i)]$, $C(q, \dot{q}) = \text{blkdiag}[C_i(q_i, \dot{q}_i)]$, $K_1 = \text{diag}(k_{1i})$, $K_2 = \text{diag}(k_{2i})$ and $J(q)^\top = \text{blkdiag}[J_i(q_i)^\top]$, $\forall i \leq N$.

Proposition 1: Consider N robot manipulators modeled by (1), with $d_i = 0$ and satisfying the Assumptions 1 and 3, in closed-loop with the distributed control law (17), with k_{1i} , $k_{2i} > 0$, for all $i \leq N$. Then, for any given formation shape reachable by the end-effectors, the set $\{(\bar{e}, \dot{q}) = (0, 0)\}$ is asymptotically stable for any initial conditions such that, for any $k \leq M$, $\delta_k \in \mathcal{I} = \mathcal{I}_r \cap \mathcal{I}_c$ —see (9)—and $|\alpha_k(0)| > \Delta_k$. In addition, the set \mathcal{I} is forward invariant, *i.e.*, the constraints hold for all $t \geq 0$. \square

Proof: After Assumption 3, the considered graph is undirected and connected, so it contains a spanning tree. Then, as for the more ordinary scenario of consensus, the result may be assessed by analyzing the dynamics of the agents that belong to the spanning-tree—see [40], [42], [43]. To obtain the closed-loop equations in spanning-tree coordinates, following the latter, we first recall that

$$E_s = [E_{t_s} \ E_{c_s}], \quad (28)$$

where $E_{t_s} \in \mathbb{R}^{N \times N-1}$ is the incidence matrix representing the edges of the spanning tree, corresponding to the spanning-tree graph \mathcal{G}_t , and $E_{c_s} \in \mathbb{R}^{N \times M-(N-1)}$ is the incidence matrix representing the remaining edges, corresponding to $\mathcal{G}_c := \mathcal{G} \setminus \mathcal{G}_t$. Consequently, after (25a) and (28), the errors can be expressed as $\bar{e} = [(E_{t_s}^\top \bar{x})^\top (E_{c_s}^\top \bar{x})^\top]^\top$, which gives $\bar{e} := [\bar{e}_t^\top \ \bar{e}_c^\top]^\top$. Furthermore, for a structurally balanced signed graph, there exists a matrix R_s such that

$$E_s = E_{t_s} R_s, \quad (29)$$

where $R_s := [I_{N-1} \ T_s]$ and $T_s := (E_{t_s}^\top E_{t_s})^{-1} E_{t_s}^\top E_{c_s}$ —see Proposition 1 in [39]. Notably, the following relationship between the synchronization errors \bar{e} and the spanning-tree errors \bar{e}_t holds:

$$\bar{e} = (E_{t_s} R_s)^\top \bar{x} = R_s^\top \bar{e}_t, \quad (30)$$

so the bipartite formation objective (10) is achieved if $\bar{e}_t \rightarrow 0$ and $\dot{q} \rightarrow 0$. On the other hand, a similar relation holds for α defined in (25b):

$$\alpha = \mathbb{E}^\top x + E_s^\top b, \quad (31)$$

where \mathbb{E} is defined in (18). The matrix \mathbb{E} corresponds only to competitive edges. Thus, akin to (28), we can write $\mathbb{E} = [\mathbb{E}_t \ \mathbb{E}_c]$ and $\alpha = [\alpha_t^\top \ \alpha_c^\top]^\top$. Thus,

$$\mathbb{E} = \mathbb{E}_t R_s \quad (32)$$

and

$$\alpha = R_s^\top [\mathbb{E}_t^\top x + E_{t_s}^\top b] = R_s^\top \alpha_t. \quad (33)$$

Next, to express the control law in spanning-tree coordinates, we introduce

$$\tilde{W}(\alpha_t, \bar{e}_t) := \bar{W}(R_s^\top \alpha_t, R_s^\top \bar{e}_t).$$

That is, in view of (30) and (33), $\tilde{W}(\alpha_t, \bar{e}_t)$ denotes the same quantity as the right-hand-side of (26), but in spanning-tree coordinates, so \tilde{W} maps $\mathcal{I}_{c_t} \cap \mathcal{I}_{r_t} \times \mathbb{R}^N \rightarrow \mathbb{R}_{\geq 0}$, where

$$\mathcal{I}_{r_t} := \{\bar{e}_{t_k} \in \mathbb{R}^n : |r_{s_k}^\top [\bar{e}_{t_k} + \alpha_{t_k}]| < R_k, \forall k \in \mathcal{E}_m\}, \quad (34)$$

$$\mathcal{I}_{c_t} := \{\bar{e}_{t_k} \in \mathbb{R}^n : \Delta_k < |r_{s_k}^\top [\bar{e}_{t_k} + \alpha_{t_k}]|, \forall k \leq M\}, \quad (35)$$

and r_{s_k} is the k th column of R_s . These sets define the constraints in spanning-tree coordinates.

Thus, the gradient-based control terms now read

$$\begin{aligned}\nabla_{\bar{e}_t} \tilde{W} &\equiv \frac{\partial \tilde{W}(\alpha, \bar{e})^\top}{\partial \bar{e}} \frac{\partial \bar{e}}{\partial \bar{e}_t} = \nabla_{\bar{e}} \tilde{W}^\top R_s^\top, \\ \nabla_{\alpha_t} \tilde{W} &\equiv \frac{\partial \tilde{W}(\alpha, \bar{e})^\top}{\partial \alpha} \frac{\partial \alpha}{\partial \alpha_t} = \nabla_{\alpha} \tilde{W}^\top R_s^\top\end{aligned}\quad (36)$$

and, in spanning-tree edge coordinates, Eq. (27) becomes

$$\begin{aligned}\ddot{q} = & -M(q)^{-1} \left[C(q, \dot{q})\dot{q} + K_1 J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W}(\alpha_t, \bar{e}_t) \right. \\ & \left. + K_1 J(q)^\top [\mathbb{E}_t \otimes I_n] \nabla_{\alpha_t} \tilde{W}(\alpha_t, \bar{e}_t) + [K_2 \otimes I_n] \dot{q} \right].\end{aligned}\quad (37)$$

The rest of the proof consists in establishing asymptotic stability of the origin $\{(\bar{e}_t, \dot{q}) = (0, 0)\}$ and forward invariance of the set $\mathcal{I}_t := \mathcal{I}_{c_t} \cap \mathcal{I}_{r_t}$, for the trajectories of (37). First, consider the Lyapunov function candidate

$$V(\alpha_t, \bar{e}_t, \dot{q}) = \tilde{W}(\alpha_t, \bar{e}_t) + \frac{1}{2} \dot{q}^\top M(q) \dot{q}, \quad (38)$$

where $M(q) = M(q)^\top$. The derivative of (38) satisfies

$$\begin{aligned}\dot{V} = & \nabla_{\bar{e}_t} \tilde{W}^\top [E_{t_s} \otimes I_n]^\top J \dot{q} + \nabla_{\alpha_t} \tilde{W}^\top [\mathbb{E}_t \otimes I_n]^\top J \dot{q} \\ & + \frac{1}{2} \dot{q}^\top \dot{M} \dot{q} - \dot{q}^\top C(q, \dot{q}) \dot{q} - \dot{q}^\top [K \otimes I_n] \dot{q} \\ & - \dot{q}^\top J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} - \dot{q}^\top J(q)^\top [\mathbb{E}_t \otimes I_n] \nabla_{\alpha_t} \tilde{W} \\ = & -\frac{1}{2} \dot{q}^\top \left[\dot{M} - 2C(q, \dot{q}) \right] \dot{q} - \dot{q}^\top [K \otimes I_n] \dot{q}.\end{aligned}$$

But since $\dot{M} - 2C(q, \dot{q})$ is skew-symmetric, we obtain

$$\dot{V}(\alpha_t, \bar{e}_t, \dot{q}) = -\dot{q}^\top [K \otimes I_n] \dot{q} \leq 0. \quad (39)$$

Next, we use Barbashin-Krasovskii's theorem [45], [46]. Hence, we note that on the set $\{\dot{q} \in \mathbb{R}^{nN} : \dot{V} = 0\}$, we have $\dot{q} = 0$, also $\ddot{q} = 0$. In view of (5), it follows that $\dot{x} = 0$ because $\dot{x} = J(q)\dot{q}$. In turn, since all the functions on the right-hand-side of (37) are continuous, we have

$$J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} + J(q)^\top [\mathbb{E}_t \otimes I_n] \nabla_{\alpha_t} \tilde{W} = 0. \quad (40)$$

On the one hand, after (16), we have

$$\nabla_{\alpha_t} \tilde{W} = \nabla_{\bar{e}_t} \tilde{W} - \frac{\partial}{\partial \alpha_t} \left\{ \frac{\partial \tilde{W}}{\partial \alpha_t}(\alpha_t) \right\} \bar{e}_t. \quad (41)$$

On the other hand, because $\alpha = E^\top x - E_s^\top \bar{x}$, then $\dot{\alpha} = [\mathbb{E} \otimes I_n]^\top \dot{x}$ and $\dot{\alpha}_t = [\mathbb{E}_t \otimes I_n]^\top \dot{x}$. Thus, $\dot{\alpha}_t = 0$, which is equivalent to $\alpha_t \equiv \text{const}$ on $\{\dot{V} = 0\}$. In turn, the last term of the right-hand-side of (41) equals to zero. Then, from (40) and using (18), $J(q)^\top [(E_{t_s} + E_t - E_{t_s}) \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} = J(q)^\top [E_t \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} = 0$. Now, since E_t is full rank (because it corresponds to the incidence matrix of a spanning tree) it follows that $\nabla_{\bar{e}_t} \tilde{W} = 0$. We conclude that the only solution that remains in $\{(\bar{e}, \dot{q}) : \dot{V} = 0\}$ for all $t \geq 0$, is the origin, i.e. $\bar{e} = \bar{x}_i - \text{sgn}(a_{ij}) \bar{x}_j = 0$ and $\dot{q} = 0$. Asymptotic stability follows.

Next, we demonstrate inter-agent collision avoidance and connectivity maintenance between agents. From (34), we remark that $\bar{e}_t \in \mathcal{I}_t$ implies $\bar{e} \in \mathcal{I}$, where $\mathcal{I} = \mathcal{I}_r \cap \mathcal{I}_c$ so we

must show that $\mathcal{I}_t = \mathcal{I}_{r_t} \cap \mathcal{I}_{c_t}$ is forward invariant. To that end, we proceed by contradiction. Assume that there exists a $T > 0$ such that $\bar{e}_t(T) \notin \mathcal{I}_t$. It means that $|\bar{e}_{t_k} + \alpha_{t_k}| \rightarrow \Delta_k$, $k \leq M$ or $|\bar{e}_{t_k} + \alpha_{t_k}| \rightarrow R_k$, $k \in \mathcal{E}_m$ for at least one $k \leq M$, which makes $\tilde{W}_k(\alpha_{t_k}, \bar{e}_{t_k}) \rightarrow \infty$, so $V(\alpha_t, \bar{e}_t, \dot{q}) \rightarrow \infty$ as $t \rightarrow T$. However, the latter contradicts the fact that $\dot{V}(\alpha_t, \bar{e}_t, \dot{q}) \leq 0$. Inter-agent collision avoidance and connectivity maintenance follow.

We now show that the set \mathcal{I} corresponds to the domain of attraction for the closed-loop system (37) by showing that all the solutions starting in \mathcal{I}_t converge to the origin. For any $\epsilon_1 \in (0, R_k)$ and $\epsilon_2 \in (0, \Delta_k)$, consider subsets $\mathcal{I}_{\epsilon_{r_t}} \subset \mathcal{I}_{r_t}$ and $\mathcal{I}_{\epsilon_{c_t}} \subset \mathcal{I}_{c_t}$ defined as $\mathcal{I}_{\epsilon_{r_t}} := \{\bar{e}_t \in \mathbb{R} : |\bar{e}_{t_k} + \alpha_{t_k}| < R_k - \epsilon_1, \forall k \in \mathcal{E}_m\}$ and $\mathcal{I}_{\epsilon_{c_t}} := \{\bar{e}_t \in \mathbb{R} : \Delta_k + \epsilon_2 < |\bar{e}_{t_k} + \alpha_{t_k}|, \forall k \leq M\}$ with $\mathcal{I}_{\epsilon_{r_t}} \cap \mathcal{I}_{\epsilon_{c_t}} = \mathcal{I}_{\epsilon_t}$. From the definition of $\tilde{W}(\alpha_t, \bar{e}_t)$, $V(\alpha_t, \bar{e}_t, \dot{q})$ is positive definite on \mathcal{I}_{ϵ_t} for all $\bar{e}_{t_k} \in \mathcal{I}_{\epsilon_t}$ and $\dot{q} \in \mathbb{R}$ and satisfies $a|\bar{e}_t|^2 + b|\dot{q}|^2 \leq V(\alpha_t, \bar{e}_t, \dot{q}) \leq h(|\bar{e}_t|) + c|\dot{q}|^2$ with $a, b, c > 0$ and $h(\bar{e}_t) := \kappa_2 [\nabla_{\bar{e}_t} \tilde{W}_k]^2$ is a strictly increasing function everywhere in \mathcal{I}_{ϵ_t} . This means $V(\alpha_t, \bar{e}_t, \dot{q}) \rightarrow 0$ as $\bar{e}_t \rightarrow 0$ and $\dot{q} \rightarrow 0$ uniformly in α_t . Therefore, the origin is asymptotically stable for all the trajectories for the closed-loop system starting in \mathcal{I}_{ϵ_t} . As ϵ_1 and ϵ_2 are arbitrarily small, taking $\epsilon_1 \rightarrow 0$ and $\epsilon_2 \rightarrow 0$, we establish asymptotic stability of the origin of (37) for all the trajectories starting in \mathcal{I}_t . Asymptotic stability in the large, on the domain of definition of V , follows. Thus, bipartite formation consensus is achieved with inter-agent collision avoidance and connectivity. \square

B. Asymptotic stability in the presence of disturbance

Now we analyze the system (1) in the presence of disturbances and driven by the control law (24), where χ_i is defined by (21), with $u_i = \dot{q}_i$. We have the following.

Proposition 2: Consider N robot manipulators modeled by (1) and satisfying the Assumptions 1 and 3 in closed-loop with the distributed controller defined by (24), (17), and (21), with $u_i = \dot{q}_i$ and $k_{1i}, k_{2i} > 0$, for all $i \leq N$. Then, for any given formation shape reachable by the end-effectors, the set $\{(\bar{e}, \dot{q}) = (0, 0)\}$ is asymptotically stable for any initial conditions such that, for any $k \leq M$, $\delta_k \in \mathcal{I} = \mathcal{I}_r \cap \mathcal{I}_c$ —see (9)—and $|\alpha_k(0)| > \Delta_k$. In addition, the set \mathcal{I} is forward invariant, i.e., the constraints hold for all $t \geq 0$. \square

Proof: As for Proposition 1 the statement follows if we establish asymptotic stability of the origin in spanning-tree coordinates and forward invariance of \mathcal{I}_t .

First, proceeding as in Section IV-A, we obtain that the closed-loop equations now read

$$\begin{aligned}\ddot{q} = & -M(q)^{-1} \left[C(q, \dot{q})\dot{q} + K_1 J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W}(\alpha_t, \bar{e}_t) \right. \\ & \left. + K_1 J(q)^\top [\mathbb{E}_t \otimes I_n] \nabla_{\alpha_t} \tilde{W}(\alpha_t, \bar{e}_t) + [K_2 \otimes I_n] \dot{q} \right. \\ & \left. - [B(q) \otimes I_n]^\top \chi - d \right],\end{aligned}\quad (42)$$

where $d := \text{col}[d_i]$, $i \leq N$.

Next, we consider the Lyapunov function candidate

$$V(\alpha_t, \bar{e}_t, \dot{q}, \tilde{\chi}) = \tilde{W}(\alpha_t, \bar{e}_t) + \frac{1}{2} [\dot{q}^\top M(q) \dot{q} + \tilde{\chi}^\top \tilde{\chi}]. \quad (43)$$

The derivative of (43) gives

$$\begin{aligned} \dot{V} = & \nabla_{\bar{e}_t} \tilde{W}^\top [E_{t_s} \otimes I_n]^\top J(q) \dot{q} + \nabla_{\alpha_t} \tilde{W}^\top [E_t \otimes I_n]^\top J(q) \dot{q} \\ & + \frac{1}{2} \dot{q}^\top \dot{M} \dot{q} - \dot{q}^\top C(q, \dot{q}) \dot{q} - \dot{q}^\top [K \otimes I_n] \dot{q} \\ & - \dot{q}^\top J(q)^\top \left[[E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} + [E_t \otimes I_n] \nabla_{\alpha_t} \tilde{W} \right] \\ & - \tilde{\chi}^\top [B(q) \otimes I_n] u + \dot{q}^\top [B(q) \otimes I_n]^\top \chi + \dot{q}^\top d, \end{aligned} \quad (44)$$

where we used (22) to obtain

$$\begin{aligned} \dot{V} = & -\tilde{\chi}^\top [B(q) \otimes I_n] u + \frac{1}{2} \dot{q}^\top \left[\dot{M} - 2C(q, \dot{q}) \right] \dot{q} \\ & + \dot{q}^\top [B(q) \otimes I_n]^\top \chi - \dot{q}^\top [K \otimes I_n] \dot{q} \\ & + \dot{q}^\top \left[\tilde{d} - [B(q) \otimes I_n]^\top \chi \right] \\ = & -\dot{q}^\top [K \otimes I_n] \dot{q} \leq 0, \end{aligned}$$

for which we used the skew symmetry of $\dot{M} - 2C(q, \dot{q})$ and $u = \dot{q}$. Next, we use Barbashin-Krasovskii's theorem [45], [46]. We note that on the set $\{\dot{q} \in \mathbb{R}^{nN} : \dot{V} = 0\}$, we have $\dot{q} = 0$ and $\ddot{q} = 0$. In turn, after (42), we have

$$\begin{aligned} K_1 J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} + K_1 J(q)^\top [E_t \otimes I_n] \nabla_{\alpha_t} \tilde{W} \\ - [B(q) \otimes I_n]^\top \chi - d = 0. \end{aligned} \quad (45)$$

As in the Proof of Proposition 1, we have $\dot{x} = 0$ and $\dot{\alpha}_t = 0$ on $\{\dot{V} = 0\}$. Consequently, α_t is constant. Then, from (45) and using (18), we obtain $K_1 J(q)^\top [E_t \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} - [B(q) \otimes I_n]^\top \chi - d = 0$. Replacing the estimation error coordinates in (22b) in the latter equation, we obtain

$$\begin{aligned} K_1 J(q)^\top [E_t \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} - [B(q) \otimes I_n]^\top \chi \\ - \left[\tilde{d} - [B(q) \otimes I_n]^\top \chi \right] = 0. \end{aligned}$$

so $K_1 J(q)^\top [E_t \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} - \tilde{d} = 0$. Then, replacing (23b) in the previous equation, we obtain

$$K_1 J(q)^\top [E_t \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} = [B(q) \otimes I_n]^\top \tilde{\chi}. \quad (46)$$

Differentiating on both sides of the latter, we obtain

$$\begin{aligned} K_1 J(q)^\top [E_t \otimes I_n] \frac{\partial^2 W}{\partial \bar{e}_t^2} \dot{\bar{e}}_t + K_1 \dot{J}(q)^\top [E_t \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} \\ = [B(q) \otimes I_n]^\top \dot{\tilde{\chi}} + [\dot{B}(q) \otimes I_n]^\top \tilde{\chi}. \end{aligned}$$

As $\dot{\bar{e}}_t = E_{t_s}^\top J \dot{q} = 0$, $\dot{J}(q) = \frac{\partial J(q)}{\partial q} \dot{q} = 0$ and $\dot{B}(q) = \frac{\partial B(q)}{\partial q} \dot{q} = 0$, we have $[B(q) \otimes I_n]^\top \dot{\tilde{\chi}} = 0$. Then, replacing (21) in the latter, we obtain $[B(q) \otimes I_n]^\top [[A \otimes I_n] \tilde{\chi} - [B(q) \otimes I_n] u] = [[B(q)^\top A] \otimes I_n] \tilde{\chi} = 0$, since $u = \dot{q} = 0$. Differentiating the latter again, we have

$$\begin{cases} [[B(q)^\top A] \otimes I_n] \dot{\tilde{\chi}} = [[B(q)^\top A^2] \otimes I_n] \tilde{\chi} = 0 \\ [[B(q)^\top A^2] \otimes I_n] \dot{\tilde{\chi}} = [[B(q)^\top A^3] \otimes I_n] \tilde{\chi} = 0 \\ \vdots \\ [[B(q)^\top A^{l_i-1}] \otimes I_n] \dot{\tilde{\chi}} = [[B(q)^\top A^{l_i}] \otimes I_n] \tilde{\chi} = 0. \end{cases} \quad (47)$$

Next, let

$$p(\lambda) = \lambda^{l_i} + c_{l_i-1} \lambda^{l_i-1} + \dots + c_1 \lambda + c_0 \quad (48)$$

denote the characteristic polynomial of A . On the one hand, after the Cayley-Hamilton Theorem $p(A) = 0$. Therefore, $\frac{1}{c_0} B(q)^\top [p(A) \otimes I_n] \tilde{\chi} = 0$, that is,

$$\begin{aligned} \frac{1}{c_0} [B(q)^\top (A^{l_i} + c_{l_i-1} A^{l_i-1} + \dots + c_1 A) \otimes I_n] \tilde{\chi} \\ + \frac{1}{c_0} B(q)^\top [c_0 \otimes I_n] \tilde{\chi} = 0, \\ - \frac{1}{c_0} [B(q)^\top [A^{l_i} + c_{l_i-1} A^{l_i-1} + \dots + c_1 A] \otimes I_n] \tilde{\chi} \\ = [B(q) \otimes I_n]^\top \tilde{\chi} = 0. \end{aligned} \quad (49)$$

On the other hand, the equations in (47) continue to hold if the left-hand sides are multiplied by the coefficients c_p with $p \leq l_i$ and remark that $c_{l_i} = 1$. Therefore,

$$[B(q)^\top [A^{l_i} + c_{l_i-1} A^{l_i-1} + \dots + c_2 A^2 + c_1 A] \otimes I_n] \tilde{\chi} = 0.$$

From the latter and (49), we conclude that $[B(q) \otimes I_n]^\top \tilde{\chi} = 0$. In turn, from (46) we have $J(q)^\top [E_t \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} = 0$. Since E_t is full rank (because it corresponds to the incidence matrix of a spanning tree), $\nabla_{\bar{e}_t} \tilde{W}$ vanishes only at $\bar{e}_t = 0$. We conclude that the only solution that remains in $\{(\bar{e}_t, \dot{q}) : \dot{V} = 0\}$ for all $t \geq 0$, is the origin, i.e., $\bar{e} = \bar{x}_i - \text{sgn}(a_{ij}) \bar{x}_j = 0$ and $\dot{q} = 0$. Asymptotic stability follows.

Next, we prove inter-agent collision avoidance and connectivity maintenance or equivalently forward invariance of the set \mathcal{I} . From (34), we remark that $\bar{e}_t \in \mathcal{I}_t$ implies $\bar{e} \in \mathcal{I}$, where $\mathcal{I} = \mathcal{I}_r \cap \mathcal{I}_c$. Then, we proceed by contradiction to show that \mathcal{I}_t is forward invariant. Assume that there exist a $T > 0$ such that $\bar{e}_t(T) \notin \mathcal{I}_t$. It means that $|\bar{e}_{t_k} + \alpha_{t_k}| \rightarrow \Delta_k$, $k \leq M$ or $|\bar{e}_{t_k} + \alpha_{t_k}| \rightarrow R_k$, $k \in \mathcal{E}_m$ for at least one $k \leq M$, which makes $\tilde{W}_k(\alpha_{t_k}, \bar{e}_{t_k}) \rightarrow \infty$, so $V(\alpha_t, \bar{e}_t, \dot{q}, \tilde{\chi}) \rightarrow \infty$ as $t \rightarrow T$. However, the latter contradicts the fact that $\dot{V}(\alpha_t, \bar{e}_t, \dot{q}, \tilde{\chi}) \leq 0$. Inter-agent collision avoidance and connectivity maintenance follow.

We now show that the set \mathcal{I} corresponds to the domain of attraction for the closed-loop system (37) by showing that all the solutions starting in \mathcal{I}_t converge to the origin. For any $\epsilon_1 \in (0, R_k)$ and $\epsilon_2 \in (0, \Delta_k)$, consider subsets $\mathcal{I}_{\epsilon_{r_t}} \subset \mathcal{I}_{r_t}$ and $\mathcal{I}_{\epsilon_{c_t}} \subset \mathcal{I}_{c_t}$ defined as $\mathcal{I}_{\epsilon_{r_t}} := \{\bar{e}_t \in \mathbb{R} : |\bar{e}_{t_k} + \alpha_{t_k}| < R_k - \epsilon_1, \forall k \in \mathcal{E}_m\}$ and $\mathcal{I}_{\epsilon_{c_t}} := \{\bar{e}_{t_k} \in \mathbb{R} : \Delta_k + \epsilon_2 < |\bar{e}_{t_k} + \alpha_{t_k}|, \forall k \leq M\}$ with $\mathcal{I}_{\epsilon_{r_t}} \cap \mathcal{I}_{\epsilon_{c_t}} = \mathcal{I}_{\epsilon_t}$. From the definition of the $\tilde{W}(\alpha_{t_k}, \bar{e}_{t_k})$, $V(\alpha_t, \bar{e}_t, \dot{q}, \tilde{\chi})$ is positive definite for all $\bar{e}_{t_k} \in \mathcal{I}_{\epsilon_t}$, $\dot{q} \in \mathbb{R}$ and $\tilde{\chi} \in \mathbb{R}$ and satisfies $a|\bar{e}_t|^2 + b|\dot{q}|^2 + c|\tilde{\chi}|^2 \leq V(\alpha_t, \bar{e}_t, \dot{q}, \tilde{\chi}) \leq h(|\bar{e}_t|) + d|\dot{q}|^2 + e|\tilde{\chi}|^2$ with $a, b, c, d, e > 0$ and $h(\bar{e}_t) := \kappa_2 [\nabla_{\bar{e}_t} \tilde{W}_k]^2$ is strictly increasing everywhere in \mathcal{I}_{ϵ_t} . This means $V(\alpha_t, \bar{e}_t, \dot{q}, \tilde{\chi}) \rightarrow 0$ as $\bar{e}_t \rightarrow 0$ and $\dot{q} \rightarrow 0$ uniformly in α_t . Therefore, the origin is asymptotically stable for all the trajectories for the closed-loop system starting in \mathcal{I}_{ϵ_t} . As ϵ_1 and ϵ_2 are arbitrarily small, taking $\epsilon_1 \rightarrow 0$ and $\epsilon_2 \rightarrow 0$, we establish asymptotic stability of the origin of (37) for all the trajectories starting in \mathcal{I}_t . Asymptotic stability in the large, on the domain of definition of V , follows. Thus, bipartite formation consensus is achieved with inter-agent collision avoidance and connectivity. \square

V. SIMULATION RESULTS

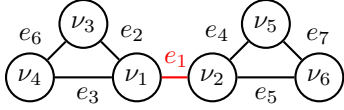


Fig. 1. An undirected signed network of 6 robot manipulators. The black lines represent cooperative edges, and the red line is the competitive edge.

We provide a numerical example to show the performance of our control laws, first the one in (17) in the absence of disturbance and then the one in (24) in the presence of disturbance. For that, we consider a system of $N = 6$ two-link robot manipulators interconnected over a structurally balanced undirected signed network, modeled by a graph as the one depicted in Figure 1. For the corresponding graph, we define the orientation of the seven edges as $e_1 = \nu_1 + \nu_2$, $e_2 = \nu_1 - \nu_3$, $e_3 = \nu_1 - \nu_4$, $e_4 = \nu_2 - \nu_5$, $e_5 = \nu_2 - \nu_6$, $e_6 = \nu_3 - \nu_4$, and $e_7 = \nu_5 - \nu_6$. The set of nodes may be split into two disjoint subgroups as $\mathcal{V}_1 = \{\nu_1, \nu_3, \nu_4\}$ and $\mathcal{V}_2 = \{\nu_2, \nu_5, \nu_6\}$, so the network is structurally balanced. From (28), the edges $e_i, i \leq 5$ correspond to the edges of the spanning tree, and the remaining edges, e_6 and e_7 , correspond to the cycles. The corresponding incidence matrix is given by

$$E_s = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 \end{bmatrix}.$$

Each manipulator is modeled by the Euler-Lagrange equations in (1), with inertia and Coriolis matrices given by

$$M_i(q_i) = \begin{bmatrix} \alpha_i + 2\beta_i \cos(q_{2_i}) & \delta_i + \beta_i \cos(q_{2_i}) \\ \delta_i + \beta_i \cos(q_{2_i}) & \delta_i \end{bmatrix},$$

$$C_i(q_i, \dot{q}_i) = \delta_i \begin{bmatrix} -\sin(q_{2_i})\dot{q}_{2_i} & -\sin(q_{2_i})(\dot{q}_{1_i} + \dot{q}_{2_i}) \\ -\sin(q_{2_i})\dot{q}_{1_i} & 0 \end{bmatrix},$$

where $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2 (m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i = l_{2_i}^2 m_{2_i}$ with l_{1_i}, l_{2_i} and m_{1_i}, m_{2_i} are the length and the mass of links 1 and 2. The physical parameters are $m_1 = 1.2\text{kg}$, $m_2 = 1\text{kg}$, and $l_1 = l_2 = 1\text{m}$ for all $i \leq N$. The kinematic model for each manipulator is given by

$$x_i = \begin{bmatrix} l_1 \cos(q_{1_i}) + l_2 \cos(q_{1_i} + q_{2_i}) \\ l_1 \sin(q_{1_i}) + l_2 \sin(q_{1_i} + q_{2_i}) \end{bmatrix} + x_{i_0},$$

and the Jacobian matrix

$$J_i(q_i) = \begin{bmatrix} -l_1 \sin(q_{1_i}) - l_2 \sin(q_{1_i} + q_{2_i}) & -l_2 \sin(q_{1_i} + q_{2_i}) \\ l_1 \cos(q_{1_i}) + l_2 \cos(q_{1_i} + q_{2_i}) & l_2 \cos(q_{1_i} + q_{2_i}) \end{bmatrix}.$$

First, consider the system (1), where $d_i = 0$ for all $i \leq N$, with the bipartite formation control law (17), where $k_{1_i} = 15$, $k_{2_i} = 10$ for all $i \leq N$ and the barrier-Lyapunov function in (16), with $B_{r_k}(s) = \ln\left(\frac{R_k^2}{R_k^2 - |s|^2}\right)$, $B_{c_k}(s) = \ln\left(\frac{|s|^2}{|s|^2 - \Delta_k^2}\right)$. The bases of six robot manipulators are located at $x_{1_0} = [1, 0.5]^\top$, $x_{2_0} = [2, -0.5]^\top$, $x_{3_0} = [-0.5, -1]^\top$, $x_{4_0} = [-3, 0.5]^\top$, $x_{5_0} = [-1, -2]^\top$, $x_{6_0} = [2, -1]^\top$. The initial conditions for each agent are $q_1(0) = [\pi, \pi/3]^\top$, $q_2(0) = [2\pi/3, \pi/3]^\top$, $q_3(0) = [\pi, \pi/3]^\top$, $q_4(0) = [0, \pi/2]^\top$, $q_5(0) = [\pi, \pi/3]^\top$, $q_6(0) = [0, -\pi/3]^\top$, $\dot{q}_1(0) = \dot{q}_2(0) =$

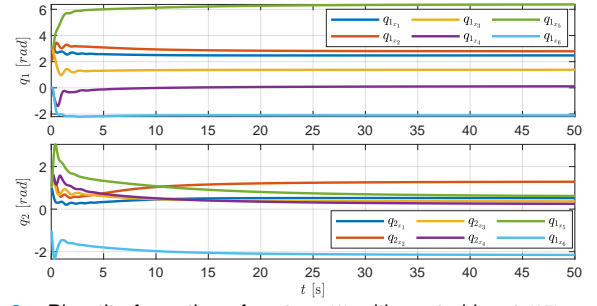


Fig. 2. Bipartite formation of system (1) with control input (17) on joint trajectories.

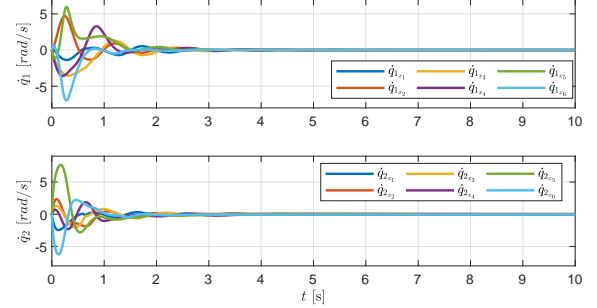


Fig. 3. Bipartite formation of system (1) with control input (17) on joint velocities.

$\dot{q}_3(0) = \dot{q}_4(0) = \dot{q}_5(0) = \dot{q}_6(0) = [0, 0]^\top$, with $q = [q_1, q_2]^\top$ and $\dot{q} = [\dot{q}_1, \dot{q}_2]^\top$ and the relative displacements of the end-effectors are $b_1 = [0, 0.3]^\top$, $b_2 = [-0.3, 0]^\top$, $b_3 = [0.3, 0]^\top$, $b_4 = [-0.3, 0]^\top$, $b_5 = [0, -0.3]^\top$, $b_6 = [0.3, 0]^\top$, with $b = [b_x, b_y]^\top$. The constraint sets are $\Delta_k = 0.1$ for all $k \leq M$ and $R_k = 7$ for all $k \in \mathcal{E}_m$.

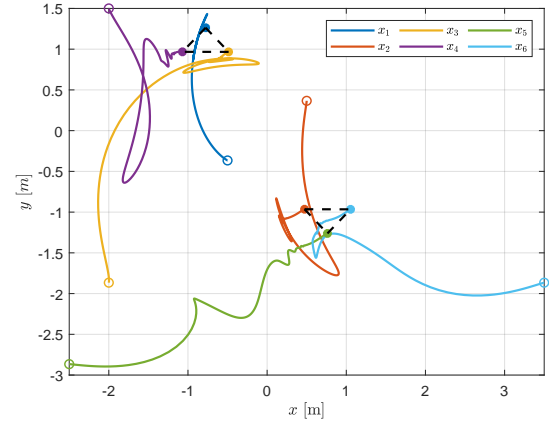


Fig. 4. Evolution of the manipulators' end-effector from the initial positions (o) to the final positions (*). Each group of end-effectors forms a triangle around the symmetric consensus points.

The joint positions and velocities are depicted in Figures 2 and 3, respectively, and all velocities converge to zero. The paths of each end-effector up to bipartite formation are depicted in Figure 4, and their final configuration is depicted in Figure 6⁴. Moreover, it is clear from Figure 5 that collision avoidance and connectivity maintenance among the manipulators' end-effectors are both guaranteed.

⁴A video of the simulation is available at: <http://tinyurl.com/simulationRM>.

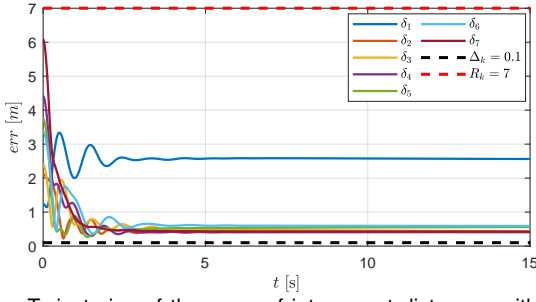


Fig. 5. Trajectories of the norm of inter-agent distances with control input (17). The black dashed line is the minimum distance constraint, and the red dashed line is the maximum distance constraint for end-effectors.

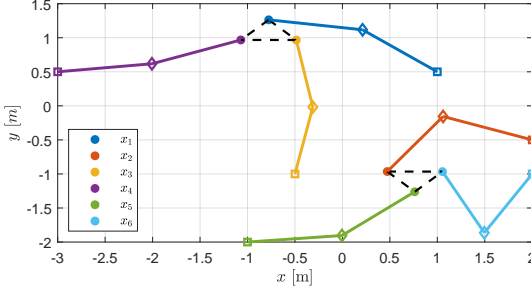


Fig. 6. Final positions of the manipulators and their end-effector.

In a second run of simulations, we consider the system (1), where $d_i \neq 0$ and with the robust bipartite formation control law in (24). We take the same initial conditions as before. Let $k_{1_i} = 200$ and $k_{2_i} = 300$ for all $i \leq N$. The matrices in (3) of the exosystem generating the disturbance are given as

$$S_{M_i} = S_{E_i} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad C_{M_i} = C_{E_i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The matrices of the internal model in (21) are given as

$$A_{M_i} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad A_{E_i} = \begin{bmatrix} 0 & \pi/2 \\ -\pi/2 & 0 \end{bmatrix}, \\ B_{M_i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_{E_i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

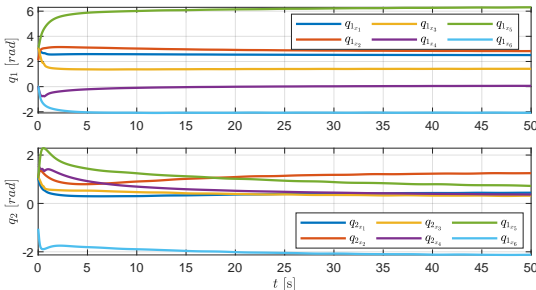


Fig. 7. Bipartite formation of system (1) with control input (24) on joint trajectories.

The joint positions and velocities are depicted in Figures 7 and 8, respectively, and all velocities converge to zero. The paths of each end-effector up to bipartite formation are depicted in Figure 9. Their final configuration is the same as in Figure 6. Moreover, it is clear from Figure 10 that collision avoidance and connectivity maintenance are guaranteed among the manipulators' end-effectors.

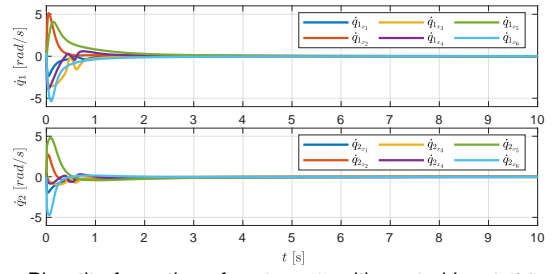


Fig. 8. Bipartite formation of system (1) with control input (24) on joint velocities.

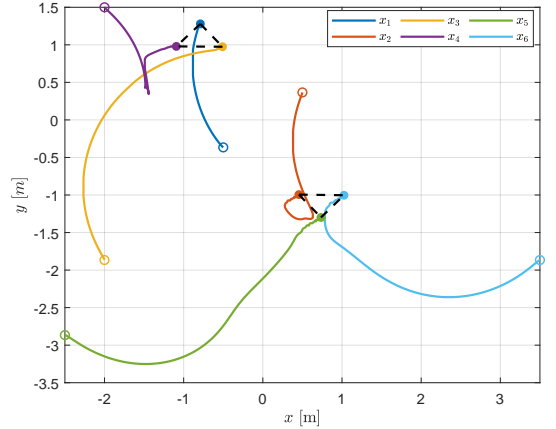


Fig. 9. Evolution of the manipulators' end-effector from the initial positions (o) to the final positions (*). Each group of end-effectors forms a triangle around the symmetric consensus points.

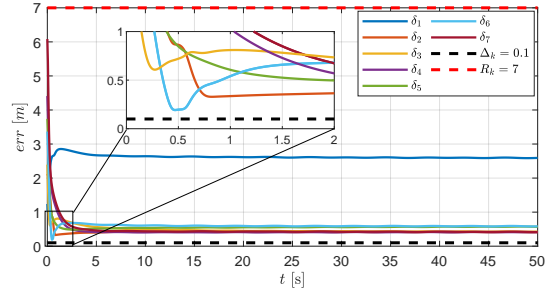


Fig. 10. Trajectories of the norm of inter-agent distances with control input (24). The black dashed line is the minimum distance constraint, and the red dashed line is the maximum distance constraint for end-effectors.

VI. CONCLUSIONS

We addressed the problem of constrained bipartite formation of cooperative-competitive robot manipulators' end-effectors, modeled by Euler-Lagrange equations. We considered a structurally balanced and undirected signed graph. First, we presented a bipartite formation control law based on the gradient of a barrier-Lyapunov function that guarantees that end-effectors do not collide and stay within their sensing regions. Then, in order to deal with perturbed robot manipulators, we robustified our controller with an internal model-based approach to reject disturbances. We established the asymptotic stability of the bipartite formation manifold both in the absence and the presence of disturbance. Further research aims to extend these results to directed signed networks.

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