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Haptic Sensitivity to Diameter and Tension During String Plucking

Matej Mayet^{1,2}, Jean-Loïc Le Carrou¹, and David Gueorguiev²

Abstract—When playing a plucked string instrument, musicians are placed in a multisensory environment where all the cues perceived allow them to control and adapt their gestures to produce the desired sounds. With the emergence of Digital Musical Instruments (DMIs), the replication of haptic feedback resulting from the interaction between the musician and the instrument has gained in relevancy. A comprehensive understanding of the associated tactile cues is therefore essential to design haptic interactions with DMIs. To investigate the sensitivity of human touch when playing a plucked string instrument, we put in place an experiment in which participants had to discriminate guitar strings based on two of their key physical attributes, diameter and tension, only by plucking them. We found the Weber fraction associated with the 67% Just Noticeable Difference to be 1.53 for string diameter discrimination and 0.4 for string tension discrimination. These results suggest that, during a plucking action, human touch does not perceive as different the typical variations of these two parameters extrinsic to the string.

I. INTRODUCTION

Human somatosensory system allows us to accomplish tasks requiring nearly instantaneous temporal and spatial adjustments during dexterous manipulation [1], [2]. This specificity proves also useful when the accomplishment of the task is attributed to complex, multifactorial, and expressive gestures of the performer, as exemplified in the playing of a musical instrument [3]–[5]. Numerous studies have already endeavored to characterize and replicate the haptic feedback generated by a musical instrument on a large range of novel digital musical instruments [6], [7]. The variety of musical instruments replicated in these studies highlights the importance attached to the haptic connection between the musician and the instrument, whether it is a bowed string [8], keyboard [6], [9], or a plucked string instrument [7], [10]. To characterize this singular and subtle connection, a number of studies delineate the musical instrument as an extension of the body [11], [12]. The instrument becomes a part of musician’s body coordination system, blurring the perceptual boundaries between the self and the instrument. A prime example of such an interaction manifests in the act of plucking a string with the pulp of the finger. There is a direct contact between the instrument and the musician, offering a

vast possibility for nuanced sound production, and furnishing a rich haptic feedback to the performer. As aforementioned, the replication of such an interaction has been undertaken and has demonstrated a beneficial effect from the incorporation of kinesthetic feedback into a multisensory system [10].

To provide the haptic feedback produced by plucking a string with the pulp of the finger, it is essential to grasp the intricacies of the finger/string interaction. Extensive studies have been carried out on this subject, and physical models for both the finger and the string have already been proposed. In their studies, Chadeaux et al. pointed out experimentally the two-phase interaction resulting from plucking a string with the finger pulp [13]. They pointed out that plucking a string starts with the performer pulling the string away from its initial position, the sticking phase, followed by the release of the string that’s sliding on the pulp of the finger before leaving contact with the performer. First, the pressure of the string on the finger during the sticking phase induces a salient kinesthetic feedback, which is an obvious cue to replay. Then, a slipping phase generates frictional forces induced by the fingerpad’s motion on the string raising interrogations on the potential role of frictional sensory cues during string plucking. Still, the sensory salience of haptic cues experienced when plucking a string has not yet been thoroughly explored while a comprehensive understanding of the associated tactile sensitivity appears essential to inform about the level of fidelity required to design such an interaction on DMIs.

With this work, our goal is to attain a more profound comprehension of the haptic perception of plucked strings, specifically in terms of the discrimination of different strings solely based on a modification of their physical attributes. It is likely that parameters intrinsic to the string play a role in its perception when plucked. However, within a specific instrument, such as a classical guitar, only the diameter and tension of the strings will differ between two strings that are closely positioned. The objective would be to determine the just noticeable difference (JND) of both these parameters and to study whether the performer varies the number of plucks based on the difficulty of the task. This study seeks to determine whether the JND for these two parameters implies that the strings of an instrument are distinguished on that basis. Although this study assesses haptic sensitivity in a limited context, without considering all the intricate aspects of musical gestures and the specificity of each plucked string instrument, it provides a first insight about the richness of the tactile cues perceived by humans during string plucking with the fingertip.

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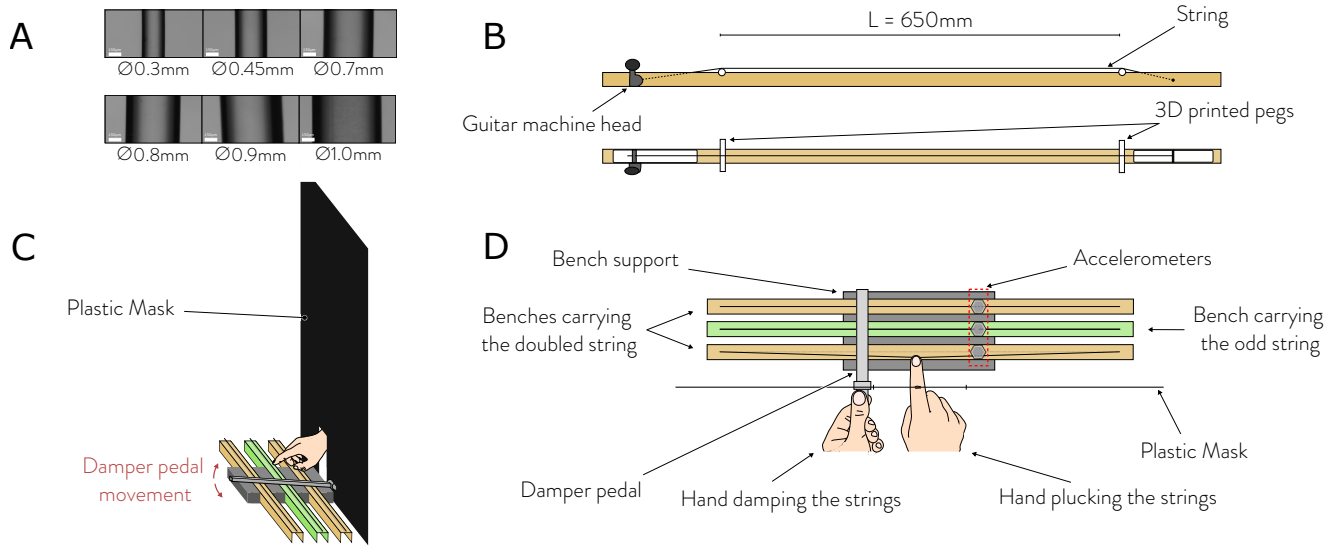


Fig. 1. (A) Microscope view of the different strings used in the study with corresponding diameters. (B) Side view of a bench carrying a string (top), upper view of the bench (bottom). (C) Isometric view of the experimental setup. (D) Upper view of the experimental setup.

II. MATERIALS AND METHODS

In the first place, we wanted to determine the sensitivity of haptic perception to a variation of string's diameter and tension outside a specific musical context. 10 healthy volunteers participated in this experiment, 7 males and 3 females, with ages ranging from 20 to 53, achieving the entirety of the experiment in about 1 hour and 45 minutes including a 5 minutes pause in the middle. All participants gave written informed consent. This study was performed in accordance with the Declaration of Helsinki.

A. Experimental Setup

We built a setup that enabled us to present 3 strings to the participants, two identical ones and a different one either in its diameter or tension, depending on the parameter studied in the given experiment. A special care was taken to prevent participants from discriminating the strings through vision or hearing instead of the tactile cues generated by plucking. (Fig.1C). To study the impact of diameter variation, we utilized a sample of strings consisting of five models made from clear fluorocarbon, with each corresponding to a specific stimulus level (Table I). The surface of each string was smooth, devoid of any wound, and no coating was applied (Fig.1A). Regarding tension, a single model of clear fluorocarbon string was employed, as tension was the sole variable. Similar to the other strings, its surface was smooth without any wound, and no coating was added. Each string was mounted on an U shaped aluminium bench, fixed to one end and tightened with guitar machine heads (MLK6LN, Kluson) at the other end. The boundaries of the string were set using two 3D printed pegs fixed to the bench and lifting the string up (Fig.1B). Since we performed a 3-Alternative Forced Choice (3AFC) task (see Section II-C), each bench was created twice, and the reference benches were tripled for self-comparison, resulting in the construction of a total

of 22 benches. Before each experimental session, all 22 benches were tightened to the desired tension (see Section II-C) using a chromatic tuner (SAT500, Seiko Instruments).

During the experiment, the benches were placed in a 3-slot support (Fig.1C) that was made out of viscoelastic resin, making it easy to replace the benches in between the trials, while holding them tightly during the exploration. An opaque plastic mask was placed between the support carrying the stringed benches and the participant, thus suppressing the visual modality. There was a 150mm wide opening in the mask, centered on the strings, that allowed the participant to pluck the strings with the hand. A damper pedal was used in between the plucks to muffle the vibrations of the strings, limiting a potential transmission of vibrotactile and auditory cues to the participant after plucking the string. Participants also had to be isolated acoustically to prevent them from using sound frequency to identify the odd string. Thereby, they were asked to wear earplugs throughout the entire experiment, in addition to listening to Brownian noise through a headset [14]. We also aimed at determining how many plucks were performed by the participant for each trial. To that end, 3 piezoelectric accelerometers (M352C65, PCB Piezotronics) were fixed on the resin support, one placed under each of the 3 benches (Fig.1D), continuously measuring the acceleration of the waves produced by the plucked strings. The signals were digitalized at 51.2kHz by a 24-bit acquisition card (NI-9234, National Instruments).

B. Experimental Procedure

The participant was seated in front of the plastic mask during the experiment. The method of exploration was explained and the participant performed a 5 minutes training session consisting in a free exploration to acquaint with the setup and the rules of the experiment. The rules defining the proper method of exploration were as follow: the par-

ticipants should pluck the string with the index finger of their dominant hand. Only the pulp of the finger should be used to pluck, and stroking along the string is not permitted. Once participants assessed the action of plucking a string and were able to perform fluid movements to get from one string to another, the main acquisition started. The experimenter placed the benched strings on the support following the order determined by the randomization program, after which the participant could start the exploration. The participant had 20 seconds to pluck each of the presented strings according to the exploration rules. They could choose in which order and the number of times they would pluck each of the strings. Once they plucked a string, they were asked to stop its vibration afterwards by releasing the damping pedal in order to avoid the transmission of auditory cues to the participant, which could happen if they chose to pluck the same string twice successively. Once the participant found the odd string or once time was out, they had to indicate on a keyboard the number of the string they think was different: they could choose from 1 to 3, corresponding to the order of placement of each string in front of them.

C. Varying one parameter at a time: Mersenne's Law

Since our study focuses on determining the JND of the diameter and the tension of a plucked string, these physical attributes had to be the only modified variables. Mersenne's equation (Eq. 1) links all the properties of the string to the fundamental frequency at which it vibrates [15] :

$$f_0 = \frac{1}{2L} \sqrt{\frac{T}{\mu}} \quad (1) \quad \mu = \frac{\pi}{4} \rho d^2 \quad (2)$$

With L being the length of the string, T the tension at which the string is tightened, μ the linear density of the string, d the diameter of the string and ρ the density of the string.

- When we varied the diameter, the square of which is proportional to linear density (Eq. 2), it only changed the fundamental frequency f_0 since the length $L = 650mm$, tension $T = 30N$ and density $\rho = 1984 \pm 28 \text{ kg/m}^3$ of the strings were fixed.
- When we varied the tension, it only changed the fundamental frequency f_0 since the length $L = 650mm$, diameter $d = 1mm$ and density $\rho = 1991 \text{ kg/m}^3$ of the strings were fixed.

With Equation 1, knowing the values of all the parameters, we were able to bring each of the strings to the desired tension by calculating the corresponding frequency, and tightening the string until we reached that frequency.

D. Psychophysical method

We designed a 3-Alternative Forced Choice (3AFC) discrimination task by presenting three strings to the participant,

two being identical and one being different. The psychophysical method used for this 3AFC task is the Method of Constant Stimuli, with a set of 5 stimulus levels for each parameter: 1 stimulus of reference to which we compared the other 4 stimuli and itself as a catch trial (Table I). Thereby, we performed 5 comparisons for the diameter and 5 comparisons for the tension, with 8 repetitions for each one, leading to a total of 80 trials for the whole experiment. Given the large duration of the study, only one stimulus of reference was selected for the diameter and for the tension. A complete randomization of the trials was performed to mitigate any potential bias.

E. Choice of stimuli levels

The values of stimuli levels were chosen in order to find the JND of both the diameter and tension by fitting respectively two psychometric functions to participant's performance for each stimulus level. The diameter and tension values for both reference and comparison strings were chosen to encompass as much as possible the range of values typically found in classical guitar strings. We also had to choose a range of values that would likely contain the JND we aim at determining without being too spread out away from it. Pilot sessions were performed to that end. The stimulus level corresponds to the difference in diameter or tension between the comparison string and the reference. It is important to underline that the diameter variation relative to the reference string is an increase, while the tension variation relative to the reference string is a decrease. This distinction holds significance to appropriately contextualize the respective Weber fractions. All stimuli levels and corresponding comparisons are detailed in Table I.

	Variable's Value		Difference	
	Reference	Comparison	Units	Percentage
	0.3	0.3	0	0
String's Diameter (in mm)	0.3	0.45	0.15	+50
	0.3	0.7	0.4	+133
	0.3	0.8	0.5	+167
	0.3	0.9	0.6	+200
	115	115	0	0
String's Tension (in N)	115	92	-23	-20
	115	65	-50	-43
	115	58	-57	-50
	115	41	-74	-64

TABLE I

VALUES TAKEN BY THE VARIABLES FOR DIAMETER AND TENSION DISCRIMINATION, WITH THE CORRESPONDING DIFFERENCES BETWEEN THE REFERENCE AND COMPARISON STRINGS

F. Psychometric and Statistical Analyses

Given that the discrimination task is a 3AFC, the chance level is established at 33%, while the discrimination threshold, or JND, is computed at 67% [16]. The 67% JND is defined as the value at which a diameter or tension difference

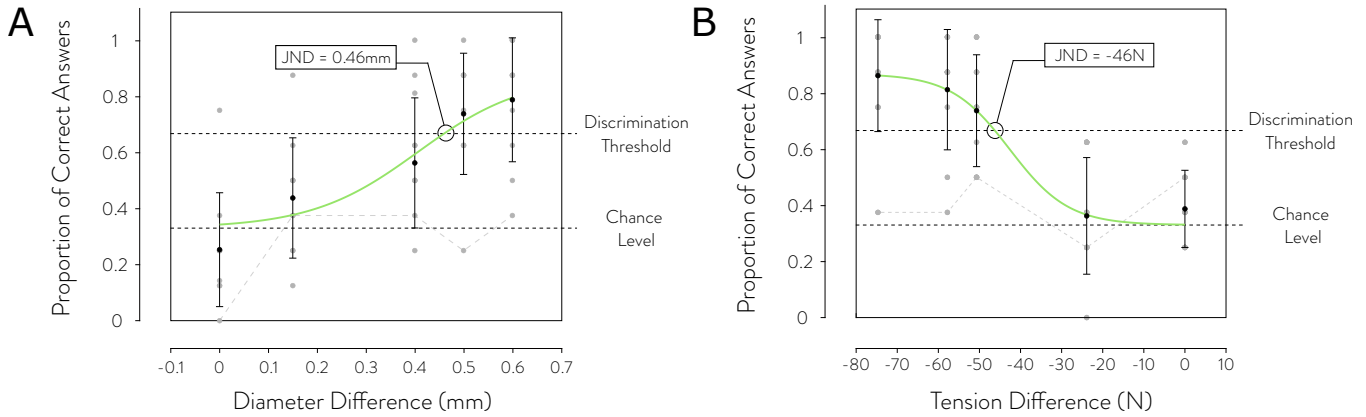


Fig. 2. (A) Psychometric curve for string diameter discrimination. Each black point denotes the average proportion of correctly detected odd strings for each diameter difference across all participants. The grey points represent the individual proportions of correct detection for each stimulus level, the ones linked by dotted lines represents one participant’s odd performance mentioned in Section 3. Error bars represent the mean and standard deviation (S.D.) of 80 repetition for each variation. (B) Psychometric curve for string tension discrimination. Same legend but for the tension discrimination task.

between the reference string and the comparison string was correctly identified in 67% of the trials. The JND for string diameter discrimination was computed by fitting a Logistic function to the proportion of correct responses averaged among all the participants for each diameter stimulus level, using the maximum likelihood criterion. The same method was used to find string’s tension JND. The psychometric fittings were conducted using *PAL_PFML_Fit* function and *@PAL_Logistic* model, provided by the Palamedes Toolbox [17]. Normality of the analysed samples in Section III-C was verified using the D’Agostino and Pearson normality test on GraphPad Prism software.

III. RESULTS

A. String Diameter Just Noticeable Difference

For the psychophysical task of discriminating two strings with different diameters, we fitted a psychometric curve (in green) to the experimental data points (Fig.2A). The 67% JND was found to be 0.46mm in string’s diameter. In other words, on average, an increase of 0.46mm in string’s diameter was necessary for participants to be able to distinguish it from the reference 67% of the times. The Weber fraction was calculated for the determined JND relatively to reference’s diameter, and was found to be 1.53. The slope of the fitted curve and the lapse rate were estimated at 9.17 and 0.12, respectively. Among the 10 participants, it is worth noting that one participant exhibited proportions of correct answers that didn’t exceeded 0.375 for all five diameter differences, oscillating around chance level (see dotted line Fig.2A). This participant showed no peculiar behavior for tension discrimination.

B. String Tension Just Noticeable Difference

For the psychophysical task of discriminating two strings with different tensions, we fitted a psychometric curve (in green) to the experimental data points (Fig.2B). The 67% JND was found to be -46N in string’s tension. In other words, on average, a decrease of 46N in string’s tension

was necessary for participants to be able to distinguish it from the reference 67% of the times. The Weber fraction was calculated for the determined JND relatively to reference’s tension, and was found to be 0.4. The slope of the fitted curve and the lapse rate were estimated at 0.14 and 0.13, respectively. Among the 10 participants, one participant exhibited proportions of correct answers that didn’t exceeded 0.5 for all five tension differences, oscillating around chance level (see dotted line Fig.2B). This participant showed no odd behavior for diameter discrimination

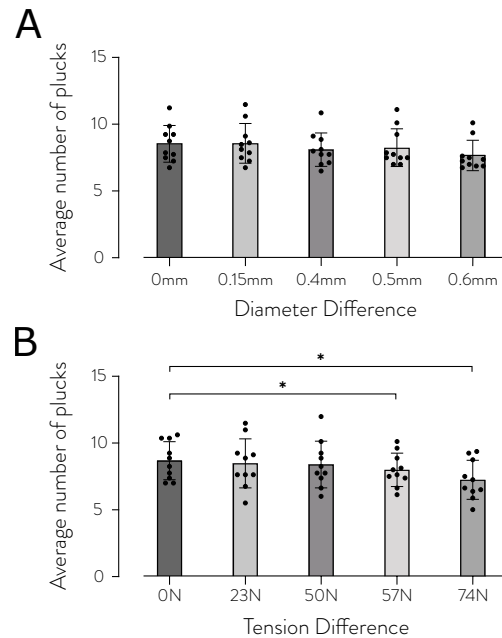


Fig. 3. (A) Average number of plucks for each diameter difference. The bars represent the number of plucks averaged among all the participants, and the error bars represent the mean and standard deviation (S.D.). Each point represent the average number of plucks performed by one participant for each diameter difference. (B) Average number of plucks for each tension difference. Same graph but for tension differences

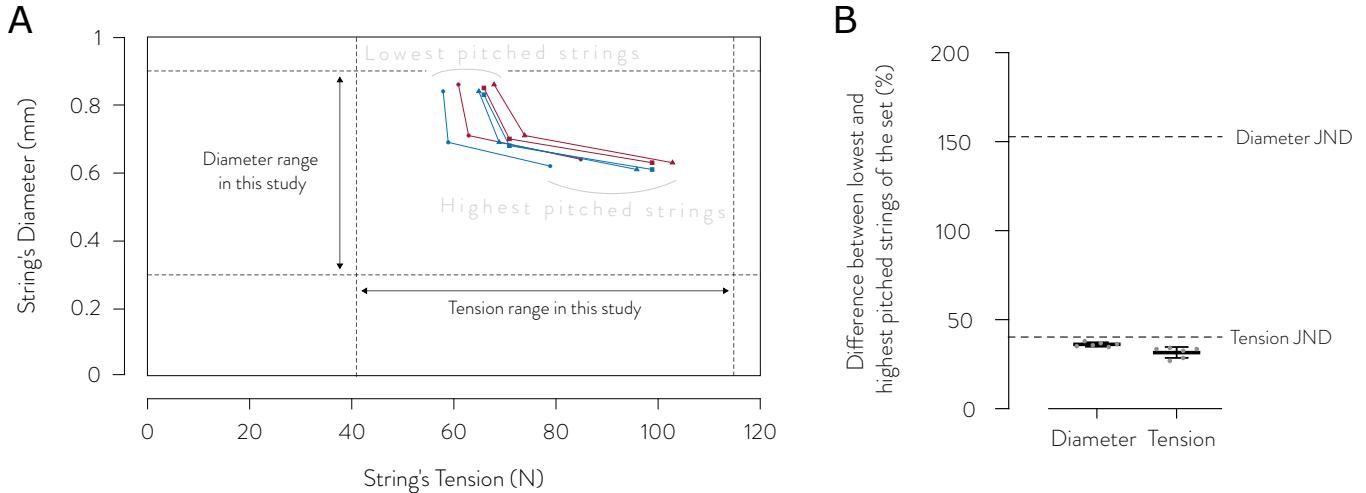


Fig. 4. (A) Diameters and tensions of fluorocarbon strings from 6 commercially available string sets. Each point indicates the diameter and tension of one of the 3 fluorocarbon strings belonging to a string set, colored lines link points corresponding to strings of the same set. The grey brackets indicate G3 and E1 strings from each set. (B) Average difference in diameter and tension between the highest and the lowest pitched fluorocarbon string of each set, relative to the highest pitched one. The grey points represent the individual differences for each set, and the error bars represent the mean and standard deviation (S.D.) of the differences for the 6 sets. The dotted line indicate both tension and diameter JNDs found in this paper, respectively 40% and 153%.

C. Influence of Diameter and Tension Difference on the Number of Plucks

Since we measured the number of plucks performed by the participants during each trial, we were able to find whether it was related to the task's difficulty. We define task's difficulty as the magnitude of the diameter or tension difference between the reference string and the comparison string. The smaller the difference the more challenging it is for the participant to identify the odd string.

We averaged the number of plucks of each participant for each stimulus intensity, and computed the mean among all participants for diameter and tension variations as shown in Figure 3A and Figure 3B, respectively. A Repeated-Measures one-way ANOVA with Geisser-Greenhouse correction showed no differences in the mean number of plucks between stimuli levels for the diameter ($F(1.457, 13.12) = 2.856, p = 0.1046$).

Interestingly, a Repeated-Measures one-way ANOVA with Geisser-Greenhouse correction showed a significant difference in the mean number of plucks between stimuli levels for the tension ($F(2.186, 19.68) = 5.194, p = 0.0136$) and a post-hoc Dunnett's test revealed a significant difference in the mean number of plucks between a 0N difference and a 58N difference ($p = 0.0341$), as well as between a 0N difference and a 75N difference ($p = 0.0196$).

IV. DISCUSSION

In this study we determined the Weber fraction for two important parameters of strings: the diameter and tension with values of 1.53 and 0.4, respectively. The preponderant difference in these ratios tend to indicate that discriminating two strings solely based on their diameter is a more difficult task to accomplish than doing so only based on their tension. This assumption might explain why significant differences

were found in the number of plucks between the smallest and biggest tension differences, and why it was not the case for diameter variations. Given that discriminating string's diameter is a more challenging task, the participants might have been more prone to use the entirety of the exploration time they were given to answer correctly (Fig. 3).

Despite successfully accomplishing the tasks required for the experiment, two participants performed poorly either in string diameter or tension discrimination tasks. It would be interesting to investigate whether this surprising performance comes from a reduced sensitivity to a specific parameter or from an exploration strategy that is more suitable to specific cues.

The particular difficulty of discriminating the diameter of two strings can also be underlined by the substantial gap between the JND determined in this paper (1.53) and haptic related JNDs previously determined in the literature. As elements of comparison, we can mention Weber fractions estimated at 0.19 and 0.18 for a normal force change detection task [18], or ratios estimated at 0.16 [19] and 0.11 [20] for tangential force change detection tasks. Regarding the JND for string tension computed in this paper, it falls within a similar range as the previously mentioned JNDs, albeit somewhat larger. This discrepancy can potentially be attributed to the singular dynamic interaction of plucking a string, as well as the small contact area that has been shown to decrease the JND for normal force, which is proportional to tension in our case [21].

During a string pluck, there is a large duration difference between the sticking (static) and slipping (dynamic) phases of the fingertip on the string. The duration of these phases were respectively estimated at 206 ms and 2 ms for harp strings [13]. While we can assume that participants evaluate string's tension mainly when pulling the string (during the

sticking phase), the evaluation of diameter could potentially occur during both sticking and slipping phases. Therefore, the short duration of the dynamic motion of the fingertip on the string could limit the spatial and temporal sensitivity, which is more accurate during dynamic touch [22].

Finally, we decided to examine the concrete implications of the estimated JNDs to commercially available guitar strings, providing a meaningful context for our findings. We chose 6 sets of classical guitar strings coming from 3 different brands. These sets are composed only of 3 fluorocarbon strings: E1, B2 and G3, going from high to low in terms of the sound frequency they produce. Figure 4A shows that all the strings of the sample take values of diameter and tension comprised in the range of values we chose for the JNDs estimations in this study. It also shows a consistent pattern of increasing diameter and decreasing tension from the highest pitched string (E1) to the lowest pitched string (G3) within the fluorocarbon string set. From these data, we quantified the percentage of variation in both diameter and tension between the highest pitched and the lowest pitched fluorocarbon string of each set, relative to the highest pitched one, as shown in Figure 4B. On this graph, we also indicated the diameter and tension JNDs, and we can see that none of the sets exhibit a difference large enough in any of the parameters to surpass the respective JNDs. It lets us suppose that string's tension and diameter within a set do not change significantly enough for the musician to perceive a difference by touch. However, it's important to contextualize and emphasize that the JNDs were not computed within a musical context. Larger differences exist between the notes when the guitar's frets are used and it would be interesting to consider the nuanced aspects of a performer's gesture in order to draw definitive conclusions on this matter.

V. CONCLUSION

Ultimately, we were able to assess the sensitivity of human touch to variations in string diameter and tension during a plucking task. We calculated the Weber fraction for both string diameter and tension, yielding values of 1.53 and 0.4, respectively. Interestingly, those values are quite large compared to ratios estimated for haptic-related JNDs in force discrimination tasks. We also determined that the average number of plucks increased when string's tensions were more difficult to discriminate, which was not the case for the diameter. This might be due to the greater difficulty associated with perceiving differences in the diameter as shown by the substantial difference between the JNDs for both parameters.

In future studies, we will aim at determining how much human touch can be sensitive to intrinsic attributes of the string such as it's wound, the material it is made of or the coating applied on it. Furthermore, integrating sensors that could offer a thorough understanding of the dynamics between the string and fingertip during this interaction appears essential to understand the underlying physical cues. Our goal is to gain a comprehensive understanding of human

haptic perception during interaction with musical strings, which might ultimately lead to the development of a digital plucked string instrument that closely replicates the most impactful haptic sensations induced when playing its classical counterpart.

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