



**HAL**  
open science

# Analysis of multilayered metamaterial structures with frequency dependent constitutive parameters using wave matrices approach

Said Choukri, Hakim Takhedmit, Otman El Mrabet, Laurent Cirio

## ► To cite this version:

Said Choukri, Hakim Takhedmit, Otman El Mrabet, Laurent Cirio. Analysis of multilayered metamaterial structures with frequency dependent constitutive parameters using wave matrices approach. 2023 Photonics and electromagnetics research symposium (PIERS), Jul 2023, Prague, Czech Republic. pp.842-847, 10.1109/piers59004.2023.10221314 . hal-04425192

**HAL Id: hal-04425192**

**<https://hal.science/hal-04425192>**

Submitted on 6 Feb 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Analysis of Multilayered Metamaterial Structures with Frequency Dependent Constitutive parameters using Wave Matrices Approach

Said Choukri<sup>1,2</sup>, Hakim Takhedmit<sup>1</sup>, Otman El Mrabet<sup>2</sup>, Laurent Cirio<sup>1</sup>

<sup>1</sup>Univ Gustave Eiffel CNRS, ESYCOM F-77454 Marne-la-Valle, France

<sup>2</sup>Abdelmalek Essaadi University, Faculty of Sciences, Departement of Physics, LaSIT lab, BP 2121, Tetouan, Morocco

**Abstract**— Combined with the Kramers-Kronig (KK) extraction method, the wave matrices approach is used in this work to compute the scattering parameters of a metamaterial unit-cell consisting of three stacked layers along the  $z$ - direction. The proposed semi-analytical approach is taking into account the frequency dependence of the constitutive parameters ( $\varepsilon_r(\omega)$  and  $\mu_r(\omega)$ ) of each layer and the loss introduced when the wave traveling through each one. The validity of this method is proved through full wave simulations and analytical calculations, where the comparison between the two results are in good agreement. This semi-analytical approach could also be used to impose some necessary conditions on the structure geometrical parameters in order to maximize and/or minimize the reflection and/or the transmission coefficients.

## 1. INTRODUCTION

Over the past few years, material scientists, physicists and electronic engineers have been increasingly focusing on metamaterials research. Almost, they have been trying to fabricate new and highly performing electronic devices based on meta-structures, such as rectennas [1], filters [2], switches [3], sensors [4] and so on. Most recently in [5], authors have developed flexible absorbing metasurface using textile embroidery. Despite the significant advancements in the fabrication of metasurfaces, some challenges, related to the performances, have proved difficult to overcome. For instance, a metamaterial-based cross-polarization converter characterized by wideband and high efficiency has been proposed in [6]. In order to achieve wideband Polarization Conversion Ratio (PCR), several split-ring resonators, separated by dielectric layers, have been stacked. While in [7], authors suggested a metasurface for energy harvesting with improved RF-to-AC conversion efficiency due to the air spacer introduced between the dielectric substrate and the copper ground plane. In addition, in order to obtain broadband absorption rate, multilayered metallo-dielectric structure which consists of periodically stacked unit-cells along the  $z$  axis of the structure, besides, 12 GHz of the absorption bandwidth has been achieved [8]. Z. Ashoor and Omar M. Ramahi have proposed a rectifying metasurface based on cross-dipole unit-cell, where the RF-to-AC conversion efficiency is improved by using a high permittivity dielectric slab as a superstrate [9]. The majority of these works, however, have mainly been centralized on the objective of performance enhancement by directly stacking dielectrics and metamaterials cells, however, no further analysis or analytical approach providing physical insights into the involved mechanisms.

Authors in [10] proposed an analytical approach, based on Green's functions derivation in spatial-domain, for uniaxial anisotropic multilayered media. Nevertheless, no rigorous analysis of the scattering properties supported the study. In this work, we propose a combined version of the wave matrices approach [11] in multilayered dielectric slabs, and the Kramers-Kronig extraction technique [12] to compute the scattering parameters in multilayered metamaterials structures with frequency dependent constitutive parameters.

## 2. WAVE MATRICES FORMULATION IN BI-LAYERED MEDIA

In this section, the analytical formulation of the wave matrices method for two media is described. For the sake of simplicity, we consider in this work homogeneous and uniform media, so that the medium constitutive parameters are considered constant in all directions. Fig.1 shows two homogeneous media stacked along  $z$ , where each medium has its intrinsic impedance  $\eta(f)$ , permittivity  $\varepsilon(f)$  and permeability  $\mu(f)$ , which are assumed frequency dependent. Normally incident plane waves  $\Psi_1$  and  $\xi_2$  are impinging onto medium 1 and 2, respectively. At the boundary between the two

media, the incident wave  $\Psi_1$  (traveling in the  $+z$  direction) will be separated into its transmitted and reflected parts. Similarly, the same thing happens with the incident wave  $\xi_2$  traveling in the  $-z$  direction.

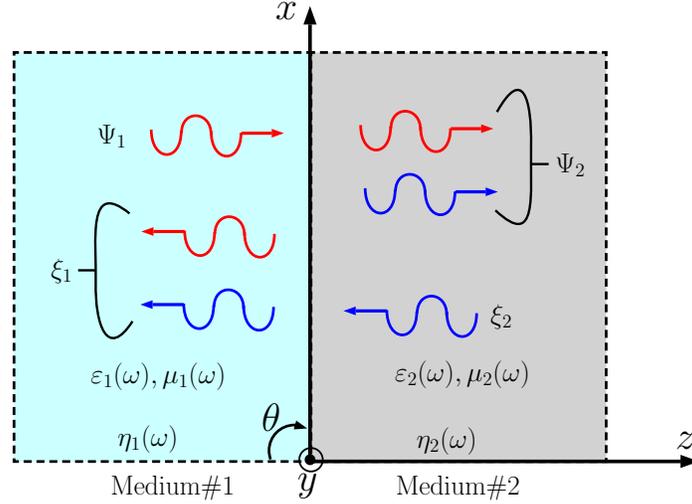


Figure 1: Two-layered media with normal incidence

In Fig.1, the wave  $\Psi_2$  is considered as the sum of the transmitted wave (from medium-1 to medium-2) and the reflected one of the incident wave  $\xi_2$ , while for  $\xi_1$  is the superposition of the transmitted wave (from medium-2 to medium-1) and the reflected part of the incident wave  $\Psi_1$ . The wave  $\xi_1$  can be expressed as follows :

$$\xi_1 = \Gamma_1 \Psi_1 + T_{12} \xi_2 \quad (1)$$

Similarly for the wave  $\Psi_2$  :

$$\Psi_2 = \Gamma_2 \xi_2 + T_{21} \Psi_1 \quad (2)$$

In equations (1) and (2),  $\Gamma_1$  and  $\Gamma_2$  are the reflection coefficients in medium 1 and 2, respectively,  $T_{21}$  is the transmission coefficient from medium-1 to medium-2 and  $T_{12}$  is the transmission coefficient from medium-2 to medium-1. They are expressed as follows :

$$\Gamma_1 = \frac{\eta_2(\omega) - \eta_1(\omega)}{\eta_2(\omega) + \eta_1(\omega)} \quad \text{and} \quad T_{21} = 1 - \Gamma_1 \quad (3)$$

$$\Gamma_2 = \frac{\eta_1(\omega) - \eta_2(\omega)}{\eta_1(\omega) + \eta_2(\omega)} \quad \text{and} \quad T_{12} = 1 - \Gamma_2 \quad (4)$$

Where the intrinsic impedances  $\eta_1(\omega)$  and  $\eta_2(\omega)$  are given by :

$$\eta_1(\omega) = \sqrt{\frac{\mu_1(\omega)}{\varepsilon_1(\omega)}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\mu_{r1}(\omega)}{\varepsilon_{r1}(\omega)}} \quad \text{and} \quad \eta_2(\omega) = \sqrt{\frac{\mu_2(\omega)}{\varepsilon_2(\omega)}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{\mu_{r2}(\omega)}{\varepsilon_{r2}(\omega)}} \quad (5)$$

Where  $\varepsilon_r(\omega) = \varepsilon'_r(\omega) - j\varepsilon''_r(\omega)$  and  $\mu_r(\omega) = \mu'_r(\omega) - j\mu''_r(\omega)$  are the relative permittivity and the relative permeability of the medium.

In order to give a scattering formulation at the boundary between medium-1 and medium-2, Eq.(1) and Eq.(2) can be arranged in the following system of equations :

$$\begin{cases} \Psi_1 = \frac{1}{T_{21}} [\Psi_2 - \Gamma_2 \xi_2] \\ \xi_1 = \frac{1}{T_{21}} [\Gamma_1 \Psi_2 + (T_{12} T_{21} - \Gamma_1 \Gamma_2) \xi_2] \end{cases} \quad (6)$$

We employ a convenient matrix form for the system of equations (Eq.(6)) by defining the following wave matrix  $\mathbf{W}$ :

$$\underbrace{\begin{bmatrix} \Psi_1 \\ \xi_1 \end{bmatrix}}_{\mathbf{Y}} = \frac{1}{T_{21}} \underbrace{\begin{pmatrix} 1 & -\Gamma_2 \\ \Gamma_1 & T_{12}T_{21} - \Gamma_1\Gamma_2 \end{pmatrix}}_{\mathbf{W}} \underbrace{\begin{bmatrix} \Psi_2 \\ \xi_2 \end{bmatrix}}_{\mathbf{X}} \quad (7)$$

Where  $\mathbf{Y}$  and  $\mathbf{X}$  are two vectors representing waves in medium-1 and medium-2, respectively.

### 3. WAVE MATRICES FORMULATION IN MULTILAYERED MEDIUM

Fig.2 depicts  $N$ -stacked layers along the  $z$  axis, each medium is represented by its electric permittivity,  $\varepsilon_i(\omega)$ , magnetic permeability,  $\mu_i(\omega)$ , intrinsic impedance,  $\eta_i(\omega)$  and thickness  $h_i$ . Normally incident plane wave,  $\Psi_1$ , coming from free space, is separated into its transmitted and reflected versions, and this process is produced at each interface between two media due to the impedance mismatching. At the interface between the  $(i-1)^{th}$ -layer and the  $i^{th}$ -layer, the forward and the

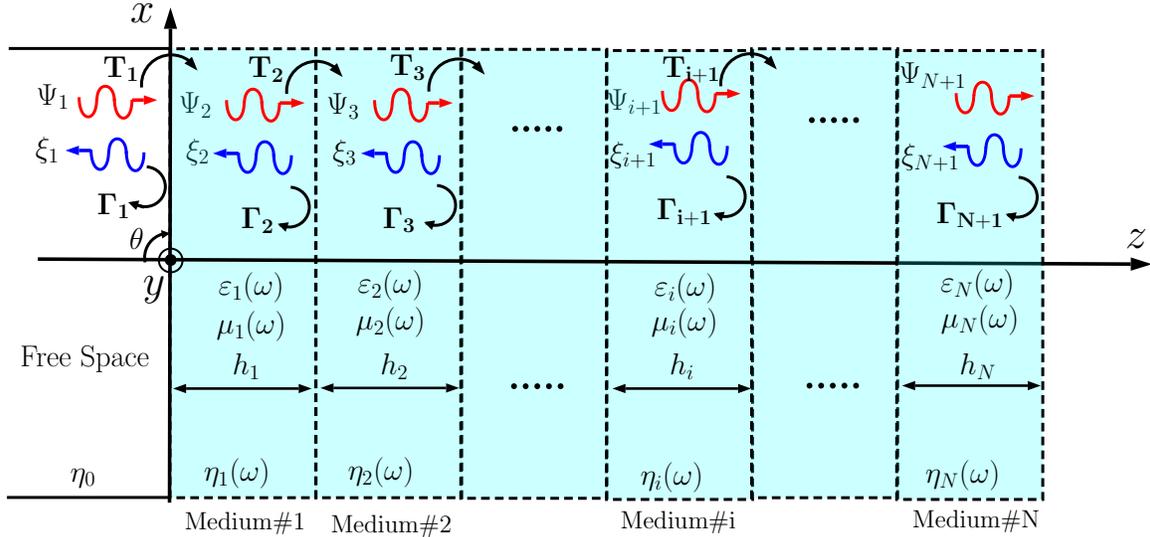


Figure 2: N-layered medium with normal incidences

backward traveling waves  $\Psi_{i+1}$  and  $\xi_{i+1}$  could be expressed by :

$$\Psi_{i+1} = \Psi_i e^{-j\beta_i h_i} \quad (8)$$

$$\xi_{i+1} = \xi_i e^{j\beta_i h_i} \quad (9)$$

Where  $\beta_i = \frac{\omega}{c} \sqrt{\mu_{ri}(\omega)\varepsilon_{ri}(\omega)}$  is the propagation constant for medium- $i$ , for conductive media (lossy case) it could be expressed by :

$$\beta_i = j \frac{\omega}{c} \sqrt{\mu_{ri}(\omega)\varepsilon_{ri}(\omega)} \sqrt{1 - j \frac{\sigma}{\omega\varepsilon_i(\omega)}} \quad (10)$$

In our study, each medium can be considered as a dielectric material, conductive material or metamaterial slab with frequency dependent constitutive parameters. So that the metamaterial constitutive parameters are extracted using the Kramers Kronig technique and they are expressed as follows [12]:

$$\varepsilon(\omega) = \frac{n(\omega)}{\eta_{eff}(\omega)} \quad \text{and} \quad \mu(\omega) = n(\omega)\eta_{eff}(\omega) \quad (11)$$

Where  $n$  is the refractive index and  $\eta_{eff}$  is the effective impedance of the metamaterial unit-cell extracted from electromagnetic simulation and given by :

$$\eta_{eff}(\omega) = \pm \sqrt{\frac{(1 + S_{11}(\omega))^2 - S_{21}^2(\omega)}{(1 - S_{11}(\omega))^2 - S_{21}^2(\omega)}} \quad (12)$$

the relationship between incident plane waves  $\Psi_1$  and  $\xi_1$  and those traveling the  $N$ -layers can be given by cascading  $N$  wave matrices. However, the wave matrices corresponding to  $N$ -layered medium is obtained as follows [11]:

$$\underbrace{\begin{bmatrix} \Psi_1 \\ \xi_1 \end{bmatrix}}_{\mathbf{Y}} = \prod_{i=1}^{N+1} \frac{1}{T_i} \begin{pmatrix} e^{\beta_i h_i} & \Gamma_i e^{-\beta_i h_i} \\ \Gamma_i e^{\beta_i h_i} & e^{-\beta_i h_i} \end{pmatrix} \underbrace{\begin{bmatrix} \Psi_{N+1} \\ \xi_{N+1} \end{bmatrix}}_{\mathbf{X}} \equiv \underbrace{\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}}_{\mathbf{W}} \underbrace{\begin{bmatrix} \Psi_{N+1} \\ \xi_{N+1} \end{bmatrix}}_{\mathbf{X}} \quad (13)$$

Where  $\mathbf{W}$  is the  $2 \times 2$  resultant matrix, which has the coefficients  $A_{ij} \in \mathbb{C}$ . If the thickness  $h_N$  of the last layer extends to  $\infty$  (case of the free space or good conductor), however, it is convenient to assume that the plane wave  $\xi_{N+1} = 0$ . Then, the total transmission and reflection coefficients of the equivalent medium, constituted by the periodic arrangement of the  $N$  layers (ref Fig.3), are given by :

$$T_{tot} = \frac{\Psi_{N+1}}{\Psi_1} = \frac{1}{A_{11}} \quad (14)$$

$$\Gamma_{tot} = \frac{\xi_{N+1}}{\Psi_1} = \frac{A_{21}}{A_{11}} \quad (15)$$

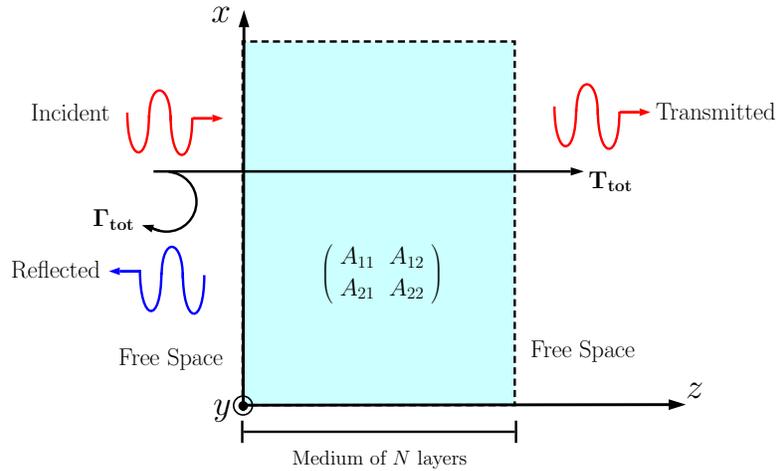


Figure 3: Schematic representation of the scattering coefficients of  $N$  layers equivalent medium

#### 4. APPLICATION TO AN HYBRID DIELECTRICS-METAMATERIALS SLAB : SCATTERING PARAMETERS CALCULATION

To illustrate the validity of the proposed method, the structure shown in Fig.4 has been considered for electromagnetic simulations. The structure is made of three layers stacked along the  $z$  axis, where the first one is a metamaterial unit-cell with the thickness  $h_1 = 1.6 \text{ mm}$  and frequency dependent constitutive parameters  $\epsilon_{r1}(\omega)$  and  $\mu_{r1}(\omega)$ , the second layer is a teflon dielectric slab that has the thickness of  $h_2 = 15 \text{ mm}$  and constitutive parameters  $\epsilon_{r2}$  and  $\mu_{r2}$ , while the lower layer is a low loss foam with the height  $h_3 = 40 \text{ mm}$  and constitutive parameters  $\epsilon_{r3}$  and  $\mu_{r3}$ . Note that the second and third layers have their constitutive parameters considered constant over the simulation frequency range.

First, the upper unit-cell has been characterized, using presimulation, in order to extract its constituent parameters using the Kramers-Kronig extraction technique [12]. Second, the structure depicted in Fig.4 has been analysed using the wave matrices method, considering the frequency dependent constitutive parameters of the metamaterials unit-cell, to calculate the total reflection and transmission coefficients given by Eq.14 and Eq.15. Finally, the same structure has been simulated using CST MWS studio with periodic boundary conditions in the  $x$  and  $y$  directions with two Floquet ports in the  $z$  direction for excitation purposes. The comparison between full-wave simulated and analytically calculated transmission and reflection coefficients is reported in Fig.5.

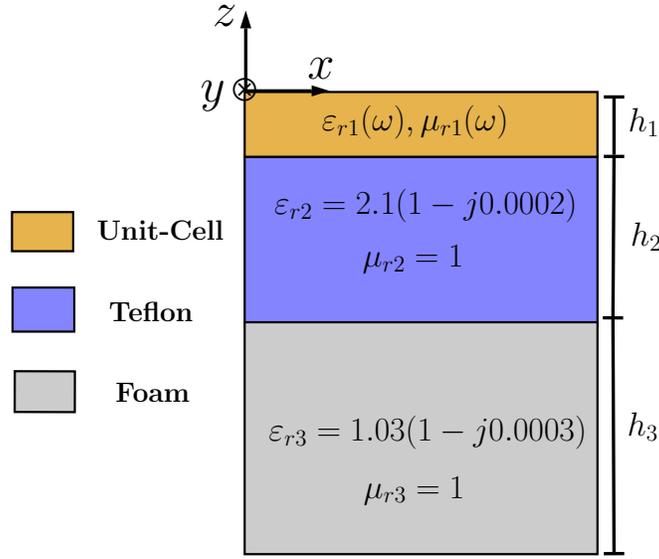


Figure 4: Schematic representation of the scattering coefficients of  $N$  layers equivalent medium

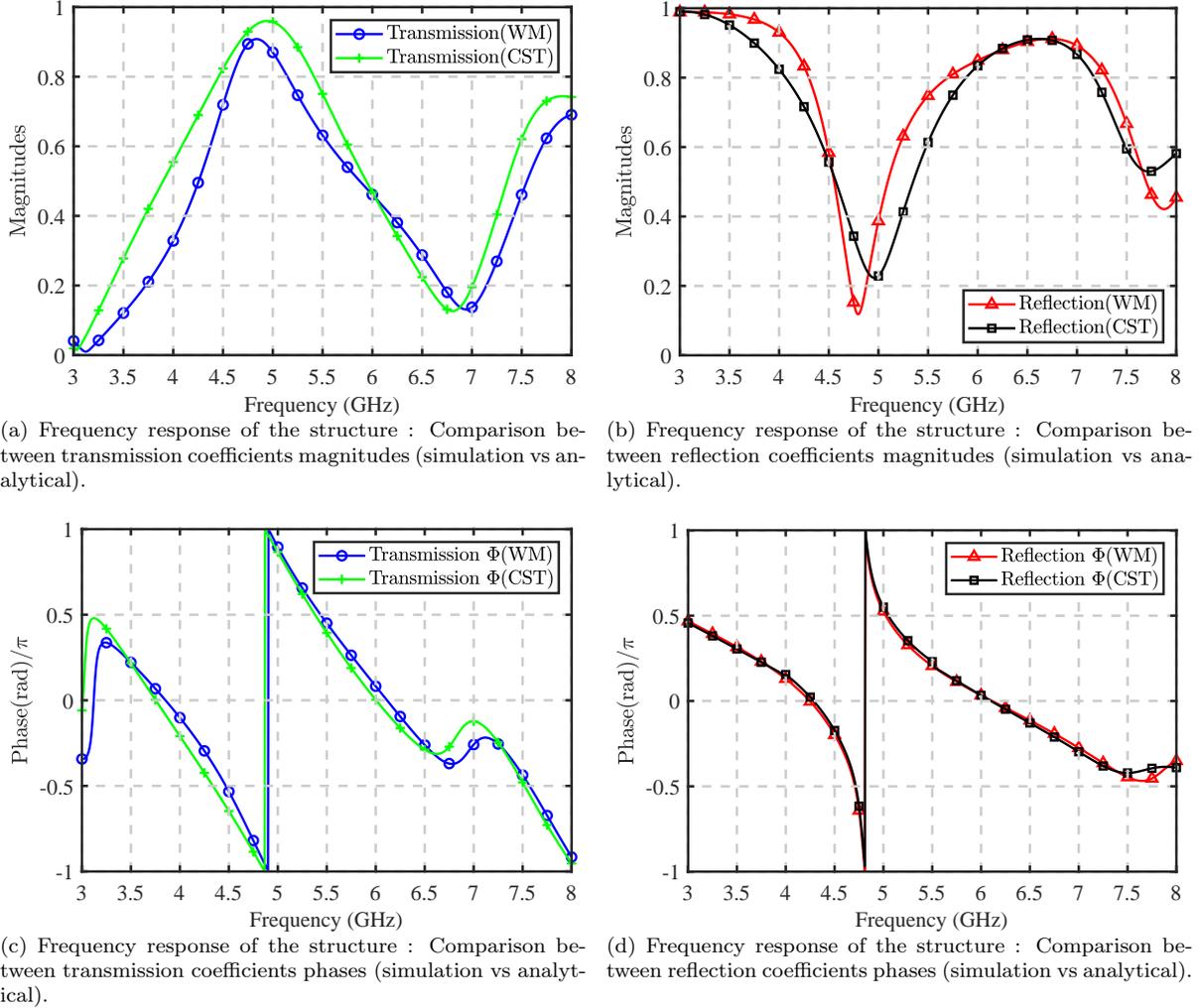


Figure 5: The scattering parameters of the metamaterial unit-cell constituted of three stacked layers

As can be seen, there is a very good agreement between both results in terms of computed reflection and transmission phases, while for the magnitudes still there is a slight difference, as can

be observed from Fig.5(a) and Fig.5(b), which may be referred to the uncertainties of the extraction technique being used.

## 5. CONCLUSION

In summary, the wave matrices method has been adopted to calculate the total transmission and reflection coefficients of multilayered metamaterial structure. Using presimulation, the constitutive parameters of the elementary cell have been calculated, then investigated by this method. The proposed semi-analytic approach is efficient and valid for both lossy and lossless cases. However, if the constitutive parameters of each layer are known, by measurements or simulations, the scattering parameters can be determined certainly using the proposed method.

## ACKNOWLEDGMENT

This work was supported by the Hubert Curien program (PHC), Project TOUBKAL TBK/20/100 N° 43761TJ.

## REFERENCES

1. K. T. Chandrasekaran, K. Agarwal, Nasimuddin, A. Alphones, R. Mittra and M. F. Karim, "Compact Dual-Band Metamaterial-Based High-Efficiency Rectenna: An Application for Ambient Electromagnetic Energy Harvesting," in *IEEE Antennas and Propagation Magazine*, vol. 62, no. 3, pp. 18-29, June 2020, doi: 10.1109/MAP.2020.2982091.
2. X. Guan et al., "Miniaturized High Temperature Superconducting Bandpass Filter Based on D-CRLH Resonators," in *IEEE Transactions on Applied Superconductivity*, vol. 29, no. 5, pp. 1-4, Aug. 2019, Art no. 1501504.
3. A. Karvounis, V. Vogler-Neuling, F. Richter and R. Grange, "Electro-Optic Metamaterial Switch Based on BaTiO<sub>3</sub> Nanoparticles," *2020 Conference on Lasers and Electro-Optics (CLEO)*, San Jose, CA, USA, 2020, pp. 1-2.
4. L. Dong, L. -F. Wang and Q. -A. Huang, "Applying Metamaterial-Based Repeater in LC Passive Wireless Sensors to Enhance Readout," in *IEEE Sensors Journal*, vol. 18, no. 4, pp. 1755-1760, 15 Feb.15, 2018.
5. Yalan Yang, Jianping Wang, Chaoyun Song, Rui Pei, Jayakrishnan M. Purushothama, Youran Zhang, "Electromagnetic shielding using flexible embroidery metamaterial absorbers: Design, analysis and experiments," *Materials and Design*, Vol. 222, 111079, 2022.
6. Jiang, Yannan, Mianji Li, Jiao Wang, Xialin Huang, and Shitian Zhang. 2023. "A Metamaterial-Based Cross-Polarization Converter Characterized by Wideband and High Efficiency" *Crystals MDPI* 13, no. 1: 17. <https://doi.org/10.3390/cryst13010017>
7. Aldhaeabi, Maged A., and Thamer S. Almoneef. 2021. "Double-Sided Metasurface Array for a Dual-Band and Polarization-Independent Microwave-Energy-Harvesting System" *Materials MDPI* 14, no. 21: 6242. <https://doi.org/10.3390/ma14216242>
8. Han Xiong, Jin-Song Hong, Chao-Ming Luo, and Lin-Lin Zhong, "An ultrathin and broadband metamaterial absorber using multi-layer structures", *Journal of Applied Physics* 114, 064109 (2013) <https://doi.org/10.1063/1.4818318>
9. A. Z. Ashoor and O. M. Ramahi, "Polarization-Independent Cross-Dipole Energy Harvesting Surface," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 67, no. 3, pp. 1130-1137, March 2019
10. P. -P. Ding, C. -W. Qiu, S. Zouhdi and S. P. Yeo, "Rigorous Derivation and Fast Solution of Spatial-Domain Green's Functions for Uniaxial Anisotropic Multilayers Using Modified Fast Hankel Transform Method," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 60, no. 2, pp. 205-217, Feb. 2012.
11. R. E. Collin., *Field Theory of Guided Waves*, McGraw Hill, Second Edition, New York, 1990.
12. Z. Szab, G. -H. Park, R. Hedge and E. -P. Li, A Unique Extraction of Metamaterial Parameters Based on KramersKronig Relationship, in *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, no. 10, pp. 2646-2653, Oct. 2010.