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► **To cite this version:**

Marianna Tzekaki, Souzana Papadopoulou. Could pre-schoolers become “good” problem solvers?. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04422826

HAL Id: hal-04422826

<https://hal.science/hal-04422826>

Submitted on 28 Jan 2024

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Could pre-schoolers become “good” problem solvers?

Marianna Tzekaki¹ and Souzana Papadopoulou¹

¹Aristotle University of Thessaloniki, Greece; txekaki@auth.gr

The aim of this study is to investigate how a teaching intervention that familiarizes pre-schoolers in the four-steps process of problem solving improves their problem-solving skills. The problem-solving skills of 20 pre-school children, attending a public kindergarten, were assessed before and after the intervention by task-based interviews on three problems. Results indicates that preschool children can advance their skills and become better problem solvers when they get familiar with PS's four-steps process of better understanding a problem, devising a solution plan, carrying out the plan, and checking the outcome, while working with their peers on challenging problems.

Keywords: Problem solving, pre-schoolers, teaching problem solving

Introduction

Problem solving is one of the most important functions for both mathematical education and everyday life. In general, a problem can be considered as any situation for which a person does not know an immediate solution path, i.e., s/he does not know what to do to reach a solution (Outhred & Sardelich, 2005). Problem solving is a particular activity at the core of mathematical development (English & Sriraman, 2010).

Problem-solving skills, although usually associated to heuristics or techniques or lists of solved problems, mostly involve strategies and metacognitive actions (Schoenfeld, 1992). In this sense, problem-solving teaching is interested in developing skills and ways of thinking and working rather than specific methods. Thus, the aim of this study is to investigate whether a teaching intervention that familiarizes pre-schoolers with the PS's four-steps process develop their problem-solving skills and produces “good” (for their level and way of thinking) problem solvers.

Theoretical background

What is a “good” problem solver?

The four-step process in problem solving is widely known and recognised. Originally introduced by Polya (1962) it was later adopted, with some extensions or modifications, by other researchers (Schoenfeld, 1992; De Corte et al., 2000) but essentially includes the following: (1) Understanding the problem, (2) Devising a plan, (3) Carrying out the plan, (4) Looking back and controlling.

According to this process, a “good”, i.e. a competent problem solver, exhibits a set of skills at each step of solving a problem. In the first step, s/he understands the problem, separates the given from the demanded, excludes unnecessary information, and may reformulate the problem or represent it with drawings or other material. Then, in the second step, s/he develops one or more solution plans, with flexibility and creativity, so that later, in the third step, s/he implements the plans, selecting the best of them and finally reviews the result (Karwowski et al., 2017). At all stages of solving, a skilled solver displays creativity and maintains metacognitive control (Lesh & Zawojewski, 2007).

Many of these qualities look overwhelmingly to a young child. However, these skills need to be developed at an early age, because they require a long-term engagement to create a systematic way

of thinking about problems (English & Sriraman, 2010). Research shows that primary school children develop poor or insufficient problem-solving skills after many years of mathematics instruction, likely due to inadequate early problem solving training (Verschaffel et al., 1999).

Problem solving in early age

Studies suggest that children's systematic problem solving is limited in kindergarten. It is usual that at these ages the educators do not propose appropriate and challenging problems and do not attempt to familiarize pre-schoolers to any problem-solving process (Warfield, 2001).

On the other hand, young children encounter difficulties when dealing with meaningful problems or other mathematical investigations. They have trouble understanding the content, attempt to give immediate or arbitrary answers without organizing a plan, implement solutions that they abandon when they are not effective and rarely check the result (Nortvedt, 2008; Diezmann et al., 2001). In addition, although visualization of a problem is a key element for solving or deepening understanding, young problem solvers do not properly use it. Their problem representation strategies appear to be poor or inadequate, especially without relevant experience (Saundry & Cynthia, 2006). But what do the research findings tell us about the development of "good" solvers at an early age? Apart from heuristics or other techniques, it is generally assumed that problem-solving abilities could be developed in an action-oriented environment and through reflection and discussion (Liljedahl et al., 2016), with challenging problems and appropriate teaching methods (Kaur & Amit, 2016). Pre-schoolers initially use trial and error or guess and check approaches, but with systematic discussion and exchanges, they present progress (Tsamir et al., 2010) in working with problems and reflecting on their own ideas (Artut, 2005). Furthermore, brainstorming and trying out alternatives in the classroom is an effective way to modify young problem solvers' way of thinking (Lesh & Zawojewski, 2007). While there are many teaching suggestions, there are few attempts to design interventions aimed at creating competent problem solvers.

In general, English and Sriraman (2010), in a summary of studies related to problem solving teaching, suggest that teachers should not provide instructions on how to solve a problem, but let children explore ways to solve it. These same authors argue that there is a paucity of relevant research, particularly for early childhood, a shortcoming to which this research seeks to contribute. Thus, the aim of the present study is to investigate a teaching intervention that familiarizes pre-schoolers in problem-solving process. More specifically it attempts to explore: (1) what are the initial problem-solving skills of pre-school children at each step of solving a problem? and (2) if a teaching intervention that familiarizes the pre-schoolers in the four-step process of problem solving can improve these skills?

Methodology

The intervention was conducted in a public kindergarten in the city of Thessaloniki. Its sample consisted of 20 pre-schoolers (7 boys and 13 girls), between the ages of 5 and 6 years. To investigate their initial and final problem-solving skills, children were examined in three problems before and after the teaching intervention in individual task-based interviews lasting 20 minutes. The problems included (1) shape compositions ("We want to build houses for a village. We have 3 triangles in different colors and also 3 squares. How many different houses can we build?"), (2) area measurement ("Two schools have gardens in their yard (in given rectangular shapes). Which

school has the largest garden?") and (3) quantities' analysis ("A bus has 5 seats on one side and 5 on the other. In how many different ways can five children sit on either side of the bus?"). The problems in the pre and post tests were similar in mathematical content but different in scenarios. For example, the problem of quantities' analysis was presented as following: "Five children go to a restaurant to eat. At the table where they will sit there are 5 chairs on one side and 5 chairs on the other side. In how many ways can the children sit on both sides of the table?". All problems were represented by visualization materials or pictures.

The interview questions assessing children's problem-solving skills were related to understanding the problem, devising a plan, carrying out the plan and looking back (see Results). Following the previous analysis regarding a "good" problem solver (Karwowski et al., 2017), the next aspects were examined in each of the four steps: (1) In understanding: representing the problem with words, material or drawings, referring to the problem's given and demanded. (2) In devising a plan: presenting a plan in words, material or drawing, presenting more than one plans. (3) In carrying out the plan: implementing the plan or a part of it, in words or drawings, or another plan, finding a solution (4) In looking back: finding a way to check the solution.

The teaching intervention to improve these skills has been developed in two directions:

A. Design and use of problems (11 in total) that were relevant to the children in relation to their interests and daily life. The problems were selected to cover all topics of mathematics (space/shapes, measurements, patterns, numbers and operations, probabilities) and were accompanied by visualization material and manipulatives.

B. Organization of activities to approach all the steps of the solution process based on (1) a reference corner in the classroom for the four steps of problem solving, (2) establishing of a playful "routine", (3) use of a song related to the four steps of problem solving, (4) use of picture books for understanding the solution and steps, (5) development of a sharing and discussing environment in the classroom for problems and solutions. The reference corner of the problems and the routine suggested a "solution path" created by flags with simple labels, such as I understand, I have ideas, I solve, I check, on which the children move during their solution process, sometimes singing the corresponding song. These choices were made primarily because children at this young age approach processes and develop skills through playful situations, songs, routines, and appropriate representations (Warfield, 2001). The duration of the intervention was approximately three months, during which a research diary was kept to record children's responses, their reactions, their ideas for solutions, and exchanges for each problem.

Results

Teaching intervention

The intervention began with an initial discussion of what a problem is and how to solve it. The children seemed to have a clear idea of the difficulty of the problem, which they expressed by saying "it makes me sad...", "it's like a giant...", "like a storm...". Regarding how to solve it, some children said, "we ask for help", "we ask someone who knows...", "we ask the computer...", while others said "we try", "we work together", "we do it many times...". At the beginning of the intervention, faced with unknown situations, the children looked for immediate answers, and when they didn't find them, they gave up. Thus, after an initial understanding, an approach of the four-

step process was sought to solve the problems. Depending on the problem, the children's ideas and their difficulties, the teacher systematically tried to make each step more understandable. For this purpose, the "corner of problems", the "solution path" and the related song were introduced to keep the children on the right track at all stages: understanding, having ideas, solving, and checking.

Initially, a lot of time was devoted to the understanding stage, since understanding a problem is a crucial element for its solution, with descriptions, materials, and drawings. The children became familiar with the questions "what do we have?", "what does the problem ask for?" so they eventually asked the questions themselves for each new problem. To avoid hasty understanding or quick answers, the teacher sometimes suggested, "we do not want to solve the problem, we just want to understand and present it". A relevant example is a probability problem involving the questions "which of ice cream, candies or lollipops will come out most often when we spin a wheel?" (that contains one ice cream, two lollipops and three candies). In this problem, the children tried to answer the question "what do we have here?" by first introducing the given elements "ice cream, lollipops, candies... wheel", and to the question "what does the problem ask for", they answered by repeating the question without really understanding it. The teacher attempted to help, by asking them to draw "what would come out the most" and the children still drew (as they explained later) "what they liked best" or "what they saw most on the wheel". In the case of this problem, this limited understanding had to be linked later to the plan "what to do?" to which the children responded, "We turn the wheel to see".

The children also had particular difficulty in devising a solution plan. They first had difficulty understanding what a solution plan meant and then finding ways to express their ideas, though not all of them were successful. Typical is the example of a problem with quantities on a balance where "an orange weighs the same as an apple and a tangerine, and an apple weighs the same as two tangerines. Which way does the scale tip if one orange and one tangerine are on one side of the scale and one apple, and two tangerines are on the other?". The children initially suggested ineffective solutions "let us see which side has more..." or "let us see which side is heavier...", but also other solutions such as "we will take the orange and put one apple and one tangerine..." or "... let us exchange the apple for two tangerines..." or even "... let us exchange the apple and one tangerine for the orange..." which were also supported with pictures.

When solving a problem, the children were given a lot of time to carry out all the solution plans they had proposed in the previous step and see if their ideas worked or not. For example, in the previous problem they tested all their ideas, when the fruits were more or what happened when they exchanged an orange with an apple and a tangerine etc. or in the wheel problem, each child proposed different number of outcomes, thus, the children had to compare all these cases.

A lot of time was also spent on the final step, checking the solutions, in order to understand its particular importance in completing the problem-solving process. For example, the initial visual check in a pattern problem was eventually replaced by an auxiliary template as a safer way of controlling. Similarly, all possible combinations of 3 balls and 3 cones in a problem of ice cream maker, from initially visual became eventually systematic "Let us put cones of the same colour and red, yellow and blue balls on top".

Also, in the wheel problem, where the results were different each time, the children decided to implement their idea of “adding up all the results to see what came out most often”, and correctly concluded that the candies came out most often because there were more on the wheel.

As these brief examples show, during the teaching intervention the children initially had difficulty with all the steps, but gradually became better at understanding the problems, proposed many clever ideas and solution plans that they confirmed by applying them, and gradually learned to control their outcomes. The use of the problem corner and the solution path, as well as the song “I solve my problems, I take my steps” helped them to manage the whole process more easily, but also to recognise each time the steps they had to take to solve a problem.

Pre and post task-based interviews

The improvement in the children’s problem-solving skills was evident in the results of the pre and post task-based interviews in the three problems presented in methodology. These skills were examined in terms of the four-step process and their corresponding analysis concerning the "good" problem solver, presented also in methodology. When comparing the initial and final interviews the following results were found.

In understanding the problems, the interviewer asked, “What is the problem about? You can show it with material or draw it”. Before the intervention, only a few children (9, 0, 8, numbers related to the three problems in the Table 1) showed the problems with material or drawings (3, 1, 2), most of them repeated only a part (9, 3, 15) or all problems verbally (11, 13, 3). Some pre-schoolers (7, 10, 3) were able to explain the given of the problems and correctly identified what was asked, while others could present only the given (7, 3, 10). Finally, some children (2, 4, 7) gave an immediate response to the problems. After the intervention, in the final test, all children verbally stated the problems, clarifying the given and demanded, showing thus improvement of their understanding.

Table 1: Understanding the problem

| 1 st . Understanding | 1rst problem | | 2nd problem | | 3rd problem | |
|---|--------------|------|-------------|------|-------------|------|
| | pre | post | pre | post | pre | post |
| Showing the problem or part of it with material | 9 | | 0 | | 8 | |
| Drawing the problem or part of it | 3 | | 1 | | 2 | |
| Verbally presenting part of the problem | 9 | | 3 | | 15 | |
| Verbally (re) presenting the problem | 11 | 20 | 13 | 20 | 3 | 20 |
| Presenting given and demanded | 7 | 19 | 10 | 20 | 3 | 20 |
| Presenting given but not question | 7 | 1 | 3 | | 10 | |
| Giving a directly answer to the problem | 2 | | 4 | | 7 | |

When organizing a solution plan, the interviewer asked, “Can you think of a way to solve the problem?”. Before the intervention, most children had difficulty doing this. Some of them tried to present a plan using material (5, 6, 4), while others could only verbally present some ideas (15, 11, 9). Only one child presented more than one plan. They mainly presented a way of thinking (12, 4, 9) without defying the problems correctly (5, 0, 8). Few children also gave an immediate answer (2, 11, 3). In the final test, most children organized a solution plan in all three problems, some of them with materials (6, 1, 7), most of them verbally (20, 19, 20) and more than initially some children suggested several solutions (4, 9, 17), depending on the problem. There are still some of them that presented a way of thinking (16, 10, 3) but only one gave a direct answer (Table 2).

Table 2: Devising a plan

| 2 nd . Devising a plan | 1 st problem | | 2 nd problem | | 3 rd problem | |
|--|-------------------------|------|-------------------------|------|-------------------------|------|
| | pre | post | pre | post | pre | post |
| Presenting a plan with material | 5 | 6 | 6 | 1 | 4 | 7 |
| Verbally presenting a plan | 15 | 20 | 11 | 19 | 9 | 20 |
| Presenting more than one plan | 1 | 4 | 1 | 9 | | 17 |
| Presenting a way of thinking for solving | 12 | 16 | 4 | 10 | 9 | 3 |
| Presenting a way of thinking for solving, without properly defying the problem | 5 | | | | 8 | |
| Presenting a way of thinking and giving a solution | 2 | | 6 | | | |
| Giving a direct answer to the problem | | | 5 | 1 | 3 | |

When carrying out the plan, the interviewer asked, “Can you carry out your solution?”. Initially, several children (9, 9, 3 in the respective problems) implemented what they had mentioned in their plan, while some pre-schoolers (6, 1, 5) implemented their plan or part of it, but also found other ways with the help of manipulatives. Finally, after the intervention, several children (12, 0, 9), especially in the first and third problem, were supported by the material and found more solutions but relatively few children (3, 12, 7) carried out what they had mentioned in their initial solution plan in all three problems. Some children (4, 2, 0) implemented part of it but also found other ways in the first and third problem (Table 3).

Table 3: Carrying out a plan

| 3 rd . Carrying out a plan | 1 st problem | | 2 nd problem | | 3 rd problem | |
|--|-------------------------|------|-------------------------|------|-------------------------|------|
| | pre | post | pre | post | pre | post |
| Implement the plan or part with material | | 12 | | | | 9 |
| Implement the plan | 9 | 3 | 9 | 12 | 3 | 7 |
| Implement the plan or part, but find other | 6 | 4 | 1 | 2 | 5 | |

In looking back, the interviewer asked, “Look what you did, is it right?”. Before the intervention, most children (12, 19, 20) were unable to do any checking in all three problems. However, in the final test, the results were better (18, 3, 8), as most children (18) found ways to look back in the first problem, but only 3 in the second and 8 children in the third problem respectively.

Discussion

Regarding the first research question on initial problem-solving skills, the present study shows that preschool children have difficulties in each step of problem solving (as shown in the Tables), associated with a general lack of concentration and patience for solving as presented in the teaching intervention, which is consistent with the findings of Artut (2015). Specifically:

In the understanding step, most children in this study did not initially use any material or other way to present the problems. They were able to verbally repeat a problem, but mainly stated what was given rather than what was asked, sometimes gave a direct answer, as also shown by Diezmann et al. (2001). At the stage of devising a plan, children often looked confused about what were being asked, simply presenting a way of thinking. Arriving at the solution step, children often did not have plans and attempted again to find a solution (Outhred et al., 2005). Depending on the problem, the material helped them find an answer. For this step, Tsamir et al. (2010) also reported that the pre-schoolers discover problem-solving methods in the implementation stage. Finally, in the checking step, the majority of children could do nothing. According to many researchers, the solution of a problem is completed by its solution (De Corte et al., 2000).

In relation to the second research question, the comparison of the results of the initial and final tests of problem-solving skills suggests that, after the teaching intervention, the pre-schoolers improved their abilities to follow all the steps and work on each one, as also shown by Tsamir et al. (2010) in their research with children working on problems with multiple outcomes. More specifically, most of the children in the present study improved their understanding of the problems, by identifying given and demanded, developing and implementing solution plans and finally finding ways for checking their results (depending on the problem). These elements were the result of systematically familiarizing children with the stages of solving a problem and a teaching management that encouraged and supported development of ideas, exchanges, and discussion.

Can pre-schoolers, ultimately, become “good” problem solvers? The duration of the teaching intervention and the number of problems realized were limited to confirm such an answer, however it is clear that the children in this study became better problem solvers, a fact that could allow them to develop into skilled problem solvers later.

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