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Instructional drawings for teaching about triangles: The case of prospective preschool teachers in India

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This study investigates prospective preschool teachers' ability to identify examples and nonexamples of triangles, as well as the figures they would draw to teach children about triangles. Participants were 52 prospective preschool teachers in India. Results indicated that participants' concept image of triangles was closely related to the prototypical triangle. Most drawings were visually similar to the prototypical triangle and were context-based drawings, rather than abstract drawings.

Keywords: Prospective preschool teachers, India, concept images, triangles, instructional drawings.

Introduction

Children are exposed to geometrical shapes from an early age, well before entering first grade (Clements & Sarama, 2011). At first, children discern shapes using visual reasoning, taking in the whole shape without considering that the shape is made up of separate components (van Hiele, 1999). Visual reasoning begins with nonverbal thinking. Children judge figures by their appearances without the words necessary for describing what they see (Sarama & Clements, 2009). Thus, the examples, as well as nonexamples, presented to children, can impact on children's concept image of different shapes.

This study takes place in India, a multi-lingual country with 22 official languages. The official guidelines document in India for expected mathematics learning outcomes is written in three languages: Hindi, Urdu, and English (NCERT, 2017). Children are often at least bi-lingual, with the language of mathematics adding another layer to this complexity (Bose, 2021). Recently, a national curriculum framework for the foundational stage (ages 3 – 8) was published (NCERT, 2022), recognizing that the early years are critical in a child's educational experience. In considering appropriate content for these early years, the curriculum states:

Concepts formed in the Foundational Stage are largely perceptive (e.g., colour as visually discriminated) and practical (e.g., spoon used as a lever to open a tin cover, money to buy things in a shop) but not theoretical (e.g., colour as a spectrum of light, lever as a simple machine, or money as a medium of exchange). ... Hence content chosen should be sensorially engaging (e.g., activate the child's senses, have aesthetic appeal) and/or be practically relevant in the context of the child's experiences. (NCERT, 2022, p. 138)

Essentially, the curriculum is recognizing the place of concept images (Tall & Vinner, 1981) in the child's cognitive development. In such a context, visual images of mathematical concepts, and how they connect to children's everyday lives become all the more important. In this paper, we examine the images prospective preschool teachers in India would draw for children when teaching them about triangles. We call these, "instructional drawings."

The development of geometrical reasoning

The basic theoretical framework we adopt for this study is Tall and Vinner's (1981) concept image/concept definition theory for the development of mathematical concepts. According to this theory, a concept image is used to describe "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (Tall & Vinner, 1981, p. 152). At a young age, concept images are often formed by the examples and nonexamples children encounter. Within geometry, Tsamir et al. (2008) identified examples and nonexamples of triangles that were intuitively recognized as such by kindergarten children, as well as those that were not intuitively recognized as such. They found that the illusion of threeness, and not necessarily the actuality of threeness, reminded children of the prototypical triangle (Hershkowitz, 1990), causing many to claim that such nonexamples as a rounded-corner "triangle" was a triangle.

Prototypical examples are what originally make up a child's concept image. Hershkowitz (1990) claimed that in addition to the necessary and sufficient (critical) attributes that all examples share, prototypical examples of a shape have special (non-critical) attributes "which are dominant and draw our attention" (p. 73). Regarding triangles, the equilateral triangle oriented with a horizontal base, is considered prototypical (Tsamir et al., 2008). Like all triangles, the equilateral triangle is a closed planar figure with three sides, three vertices, and three angles. These are critical attributes derived from the formal definition of a triangle. Non-critical attributes include the overall size of the figure (large or small) and orientation (horizontal base). Kellogg (1980) suggested that prototypes are formed when certain non-critical attributes of a shape appear frequently in examples and students begin to associate these non-critical attributes with examples of the shape.

In addition to the visual aspects of a concept image, concept images also include the attributes and properties associated with that concept (Tall & Vinner, 1981). While at an early age, children do not necessarily discern the attributes of a concept, they eventually do notice that different shapes have different attributes, although they may not be aware that some attributes are related (van Hiele, 1999). They also may not realize which attributes are critical for identifying a figure and which are not (van Hiele, 1999). For example, studies have found that when a triangle is not oriented with a horizontal side, children may not identify it as a triangle (e.g., Tirosh et al., 2013). Children may also accept curved sides, either concave or convex, when identifying triangles (Tsamir et al. 2008). Eventually, relationships between attributes are perceived and definitions become meaningful.

Concept images may also include contexts associated with that concept, contextual associations that come from a person's real-life experiences. For example, in Tsamir et al.'s (2008) study, kindergarten children named a triangle-like shape that had jagged sides, as a bonfire, associating this figure with a popular evening activity in the summer. Accordingly, some children claimed that it was not a triangle because it was a bonfire, while others claimed that it was a triangle, even though it was a bonfire. Instead of ignoring such experiences, the *Learning Outcomes for Mathematics* guide (NCERT, 2017) in India explicitly calls for elementary school teachers to integrate "the environmental component with mathematics... teachers are expected to provide learning opportunities while transacting different concepts of mathematics to help children explore and connect with their immediate surroundings (self, family, school etc.)" (p. 53). During first grade, they are also expected to "verbalise the properties of shapes/criterion used by them in sorting/ classifying solids/ shapes" (p. 54).

Previous studies of practicing and preschool teachers' geometrical knowledge

The preschool teacher has an important role in fostering children's mathematical knowledge, including their geometrical knowledge. Yet, preschool teachers do not always receive adequate preparation for teaching this subject (Clements & Sarama, 2011). Several previous studies have investigated various aspects of practicing and prospective teachers' knowledge for promoting children's geometrical knowledge and reasoning. For example, Tsamir et al. (2015) investigated kindergarten teachers' concept images and concept definitions for triangles, circles, and cylinders. Related to the current study, they found that teachers were able to identify examples and nonexamples of triangles, and were able to offer geometrically sufficient definitions for triangles.

Regarding prospective teachers (PTs), one study (Sarama & Clements, 2009) found that nearly two-thirds of prospective elementary teachers identified and categorized shapes based only on their overall physical similarity to prototypes. In another study, prospective preschool teachers were not able to recognize mistakes made by children when identifying triangles (Korkmaz & Şahin, 2020). Additional research (Ulusoy, 2021) has shown that most PTs do not write appropriate definitions for triangles, instead writing necessary but not sufficient conditions and using inaccurate terminology. Furthermore, when asked to draw examples of triangles, they mostly drew what they termed typical triangles, acute triangles with a horizontal base. In the current study, participants were not explicitly requested to draw examples of triangles, but instead were requested to draw shapes they would show children when teaching them about triangles. This request is more in line with the cultural context of India, where the practical, as opposed to the theoretical, is emphasized for young children (NCERT, 2022). Furthermore, they were asked to identify examples and nonexamples of triangles.

The aim of this study is to investigate prospective early childhood teachers' ability to identify examples and nonexamples of triangles and to investigate the instructional drawings PTs in India would present to children when focusing on triangles. We ask: what can we learn about PTs' concept images for triangles from their identifications of examples and nonexamples of triangles and from their instructional drawings for teaching triangles?

Method

Participants were 52 women studying to be preschool teachers in a teacher training program located in Goa, India. All participants had a high school diploma, some learning mathematics until the 10th grade and some learning mathematics through 12th grade, and 39 participants had a college degree. The preschool teacher program was for one year, but did not include specific courses related to teaching mathematics at preschool. The participants speak either in Konkani, Marathi, Kannada, or English. They are expected to teach in English.

Participants were asked to fill out a two-part questionnaire, administered by the second author of this paper. Instructions in the first part were, "Draw shapes that you can show children, to help them learn about triangles. Please number the shapes and explain your choices." In the second part, participants were shown six figures (see Figure 1) and asked, "Is this a triangle? Yes/No." In choosing the figures, both mathematical and psycho-didactical dimensions were considered. That is, we not only considered whether the figure is an example or a nonexample of the triangle, but whether or not the figure, be it an example or nonexample would intuitively be recognized as such.


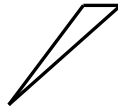




Is this a triangle?	Intuitive	Non-intuitive
Examples	Equilateral Triangle 	Scalene triangle 
Non-examples		Rounded-corner "triangle"  Pizza "triangle"  Open "triangle"  Tall "triangle" 

Figure 1: Is this a triangle?

Participants' identifications were coded as correct or incorrect. Regarding their drawings, several aspects were analyzed. First, we noted if the drawings included triangles, triangle-like shapes, or neither. Triangle-like shapes are shapes visually similar to the prototypical triangle, but are missing at least one critical attribute (similar to the pizza "triangle" in Figure 1). Second, we separated abstract geometrical figures (e.g., triangles, squares, rectangles) from drawings depicting objects related to real life experiences that included a triangle, several triangles, or triangle-like figures, as part of the drawing (e.g., fish, clown hats). Third, we noticed the orientations of the drawings of triangles and triangle-like figures. Finally, we analyzed participants' writings which accompanied drawings.

Results

Frequencies of correct identifications are presented in Table 1. As can be seen, all participants correctly identified the equilateral triangle. The most difficult to identify was the rounded-corner "triangle", and less than half correctly identified the scalene triangle as a triangle.

Table 1: Correct identifications of figures as triangles or non-triangles (N=52)

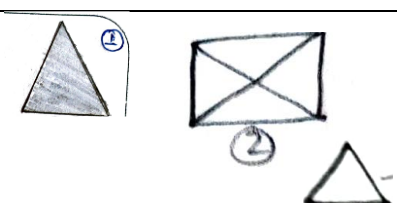
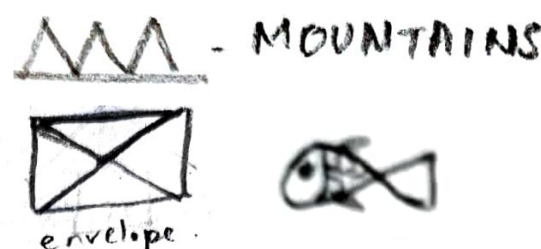
	Equilateral triangle	Scalene triangle	Rounded- corner "triangle"	Open "triangle"	Pizza "triangle"	Tall "triangle"
Frequency	52(100%)	25(48%)	(9)17%	47(90%)	43(83%)	42(81%)

A total of 211 drawings were produced by the participants. Of these, nine figures did not include any triangles, or triangle-like figures. For example, P41 drew a square without diagonals and titled it "square." A square with diagonals was considered to include triangles because the diagonals delineate triangles. P38 drew a circle without any written explanation. Perhaps those participants who did not include any triangles or triangle-like shapes were considering showing children non-examples. However, none of those participants specifically wrote that this was their intention.

Next, we categorized the 202 drawings into abstract figures and context-based drawings that depicted objects related to real life experiences. Some drawings were obviously abstract such as P29 who drew

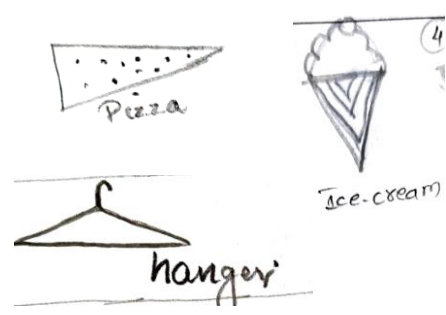

a prototypical triangle with straight lines and wrote underneath “triangle.” Some just drew a triangle and wrote nothing and were also categorized as abstract. Some drew what looked like abstract triangles, but wrote underneath, for example, “samosa” (a common pasty served in India that is triangle-shaped and has a filling). Although this figure looked abstract, because it was accompanied by the word “samosa,” it was coded as context based. Likewise, a rectangle with diagonals, but with nothing written underneath, was categorized as abstract, while a similar drawing with the word “envelope” underneath, was categorized as context based. Examples and frequencies of each category are shown in Table 2. As can be seen, the number of context-based drawings was nearly double the number of abstract drawings. Various contexts could be found in the drawings such as nature (mountains, trees, fish), food (ice-cream cones, pizza, samosa, watermelon, sandwiches), and inanimate objects (houses, stars, hangers, birthday hats, a sign board, and fans).

Table 2: Frequencies of abstract figures and context-based drawings (N=202)

	Abstract drawings	Context-based drawings
		
Frequency (%)	69(34%)	133(66%)

All abstract triangles except one were considered accurate geometric triangles with three straight lines, three pointy vertices, and closed. Thus, the next step was to categorize the context-based drawings into triangles, and triangle-like figures (see Table 3).

Table 3: Frequencies of triangles and triangle-like drawings (N=132)




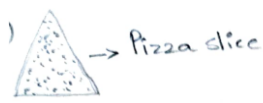



	Accurately drawn triangles	Triangle-like figures
Examples		
Frequency	108(82%)	23(18%)

As seen from the examples in Table 3, the same object (e.g., a slice of pizza or an ice-cream cone) could be drawn such that the triangle part of the drawing met all the necessary conditions (e.g.,

straight lines), or not (e.g., curved lines). Notably, most participants, even when drawing context-based shapes, attempted to draw geometrically accurate triangles.

Next, we noted the orientations of the triangles and triangle-like drawings, both in the abstract and context-based drawings (see Table 4). Considering that context-based drawings as based on real life experiences, it is interesting to note that even those were mostly drawn in a prototypical position.

Table 4: Frequencies of orientations (N=202)

Orientation				Multiple orientations
Examples	 → Pizza slice			
Frequency	120(59%)	30(15%)	17(9%)	35(17%)

Finally, we consider the written statements that accompanied some of the drawings. Eight participants did not include any written statements. Written statements were categorized into four categories (see Table 5): naming and descriptions, mentions of triangle attributes, instructional reasons, and affective issues. Several statements fell into more than one category. Most of the participants' explanations for their drawings included naming the drawing; both context-based and abstract figures were named. Others explained how their drawings include triangles (see Figure 2).

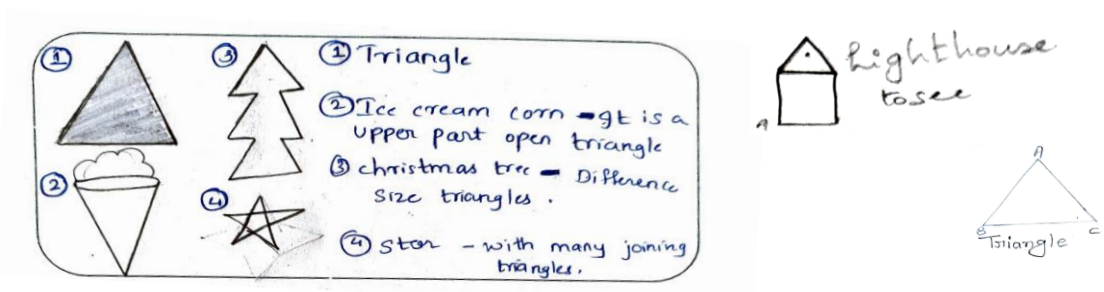


Figure 2: Descriptions of drawings

Some participants listed attributes of a triangle, some critical (e.g., three sides), and some non-critical (slanting side). For example, P46 drew five drawings (two abstract and three context-based) and wrote, “Two slanting lines joined at the corner and one straight line down the tip of one slanting line to another slanting line. Ex. Pizza, samosa.” P34 drew a triangle, labeled its vertices A, B, and C, and wrote: “3 sides – AB, BC, AC. 3 angles - $\angle ABC$, $\angle BCA$, $\angle CAB$. 3 vertices – A, B, C.” Some participants gave instructional reasons for their drawings, the most common being that the illustrations would be familiar to children. For example, P36 drew a star and wrote, “All children learn the poem *twinkle-twinkle little stars* and make drawings of stars from 2 triangles. It is easy and identifiable for kids.” P9 drew three abstract triangles with different orientations and a star and wrote, “I am using those types of shapes for teaching the children because they will understand better.” Finally, some participants mentioned affective reasons for their drawings, such as P2 who drew a triangle and labeled it a samosa, and wrote, “They love such kind of things, ... and keep [them] in

mind.” The reference to children loving samosas is an affective reason, while acknowledging that this will help them remember (“keep in mind”) the shape, is an instructional reason.

Table 5: Frequencies of types of explanations (N=44)

	Names of the drawings	Attributes of a triangle	Instructional reasons	Affective reasons
Frequency (%)	33(75%)	9(20%)	11(25%)	4(9%)

Summary and discussion

Like previous studies of prospective preschool teachers’ identification of triangles (e.g., Sarama & Clements, 2009), in the current study, PTs’ concept images of triangles included mostly shapes visually similar to an equilateral triangle with a horizontal base. Noticeably, few participants identified the scalene triangle as a triangle. It would thus seem that perhaps participants were only using visual reasoning (van Hiele (1999) level one). However, PTs did notice attributes. Specifically, when focusing on the nonexamples, most PTs knew that an open shape or a shape with a curved line cannot be a triangle. It was the attribute of vertices that caused participants to incorrectly identify the rounded-corner “triangle” as a triangle. Perhaps participants did not notice that the corners were rounded or perhaps they did not know that having pointy vertices is a critical attribute (Hershkowitz, 1990). It might simply be that participants did not know that vertices are not the same as corners.

Regarding participants’ instructional drawings, most were visually similar to the prototypical triangle (Tsamir, et al., 2008), lending further evidence that participants’ concept image was strongly tied to the equilateral triangle. The problem is that using these drawings in instruction can instill in young children a strong concept image of a prototypical triangle, difficult to undo in later years (Kellog, 1980). On the positive side, in line with the spirit of curriculum guidelines in India (NCERT, 2022), participants were able to make use of everyday contexts to connect the shape of a triangle to various aspects of a child’s life, such as food, animals, and the nature surrounding them. Furthermore, most participants drew context-based drawings as accurate triangles, thus acknowledging the difference between what we see in everyday life and the abstractness of geometrical shapes.

Aside from the few participants who drew non-triangular shapes, none drew what may be considered intuitive nonexamples, such as circles. Regarding non-intuitive nonexamples (Tsamir et al, 2008), none of the triangle-like figures drawn by participants (see Table 3) were accompanied by written statements claiming that they were intended to be nonexamples. Thus, another important issue to discuss with prospective preschool teachers is how nonexamples can be used to teach children about critical and non-critical attributes. In general, participants’ written explanations were mostly descriptions, and did not include explicit reasons for why they chose to draw those specific figures. This may be related to the Indian context, where English is the language used in school, but not necessarily in the home. Yet, while respecting the cultural context of the country that encourages preschool teachers to draw children’s attention to geometrical shapes in one’s surroundings, teachers should also begin to introduce children to the mathematical language that will accompany them during their school years. Thus, further research is needed to investigate prospective preschool teachers’ ability to define triangles, as well as other geometrical shapes.

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