



HAL
open science

Prediction of mathematic performance in primary school by cross-domain and domain-specific precursor skills

Jakob Kelz

► **To cite this version:**

Jakob Kelz. Prediction of mathematic performance in primary school by cross-domain and domain-specific precursor skills. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04422746

HAL Id: hal-04422746

<https://hal.science/hal-04422746>

Submitted on 28 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Prediction of mathematic performance in primary school by cross-domain and domain-specific precursor skills

Jakob Kelz

University of Klagenfurt, Institut für Didaktik der Mathematik, Austria; jakob.kelz@aau.at

The predictive power of mathematics performance by precursor skills is analysed here at the time of primary level. Domain-specific and cross-domain precursor skills, collected from 62 students, explain about 45% of overall mathematics performance at the end of third grade. This percentage is consistent with the referenced literature. Cross-domain precursor skills (intelligence) predict significantly better than domain-specific ones (e.g., quantity-number knowledge). When considering gender differences within prediction, boys were measured to have higher predictive power than girls. This tendency has not yet been confirmed within the literature. Of practical relevance seems to be the fact that precursor skills are already developed at kindergarten age, represent the basis for mathematics performance at primary level, and thus should be developed as early as possible.

Keywords: Prediction, math performance, primary level, precursor skills.

Introduction

Children develop important mathematical precursor skills even before they enter school. Consequently, primary mathematics education is not the beginning of mathematical performance development (e.g., Purpura, et al., 2013), which is supported by scientifically based and tested developmental models. The term mathematical precursor skills encompasses skills that emerge before school entry and provide the foundation for general mathematical understanding and acquisition in school (Krajewski & Schneider, 2006). Domain-specific (e.g., prior mathematical knowledge) and cross-domain (e.g., intelligence) precursor skills can be distinguished. The study by Aunola et al. (2004) shows that the gap in mathematical performance development is wide open even before school entry. As mathematical competencies are steadily built in a cumulative learning process (Weißhaupt et al. 2006), the consequences of insufficiently developed specific mathematical precursor skills can be devastating (Shanley, et al., 2017). This is because specific mathematical precursor skills developed in preschool represent a highly significant predictor of further successful mathematical learning (see, Gallit et al., 2018; Mazzocco & Thompson, 2005; Rittle-Johnson, et al., 2017).

In the present study, the predictive power of domain-specific and cross-domain precursor skills is measured. Furthermore, the predictive power will be analysed separately for girls and boys.

Theoretical background

The following discussion of precursor skills is classified according to cross-domain and domain-specific. Cross-domain precursor skills are relevant to both language and mathematics, while domain-specific precursor skills are relevant only to mathematics. Domain-specific precursor skills are also associated to the domains of cognition, quantity knowledge, number knowledge, and application.

Cross-domain precursor skills

Cross-domain precursor skills include working memory, language comprehension, processing speed, visual-spatial thinking, fluid reasoning and total IQ value. Working memory refers to the capacity to store, process, and manipulate information in the short term (Shah & Miyake, 2005). Language

comprehension tasks mainly target the understanding of prepositional relations, such as on, under, beside, and between. Procession speed could be defined as the time it takes a person to do a mental task. Visual-spatial thinking means that mental images and/or spaces are built up in the mind. Fluid reasoning refers to the ability to discover underlying concepts or conceptual relationships between visually presented objects and to use inferential reasoning to identify and apply rules. The intelligence quotient (IQ) is a parameter determined by means of an intelligence test for the evaluation of the intellectual performance.

Domain-specific precursor skills

The domain-specific precursor skills spatial location, spatial relations, seriation, seriality, compare and classify are specifically relevant to mathematics performance. According to Franke (2007), the perception of spatial location describes the recognition of the space-location relationship of an object to the viewpoint of the observer. Spatial relations are elaborated with the learning of terms such as on, beside, behind, between, inside, outside, under, over, through, in front of, to the right, to the left, and next to. A difference to spatial location is that at least two objects are related. Seriation is a cognitive precursor skill that refers to the ability to order or rank elements according to a characteristic (e.g., size, length, weight). Seriality is the antecedent ability to recognize and continue patterns. According to Lenart et al. (2014), the ability to compare is also a prerequisite for ordering objects according to certain features (seriation) and grouping them together based on certain features (classification).

Other domain-specific precursor skills can be assigned to quantity knowledge. One-to-one association is a mapping which finds its mathematical specification in the notion of a bijective mapping from a quantity A into a quantity B. For a set to be counted, exactly one unique number word is assigned to each element. Comparing quantities requires a combination of comparison and quantity knowledge. The precursor skill seriation of quantities describes the ability to order quantities according to a characteristic (e.g., cardinality) or to recognize arrangement rules of quantities. Phonological awareness of quantities is the ability to recognize syllables in words and to associate their number with the quantity that has the same number of elements.

In this paragraph, domain-specific precursor skills are described that can be associated to number knowledge. Recognizing Arabic numbers, quantity-number mapping, comparison of numbers, ordinal number aspect and cardinal number aspect can be listed as important precursor skills. According to the triple-code model of Dehaene (1992), in order to recognize Arabic numerals, the module of visual Arabic representation is required. The mapping of quantities and numbers, mentioned by Lenart et al. (2014), is subject to the precursor skill of quantity-number literacy mentioned earlier. The ability to compare numbers also corresponds to the analogue size representation module in Dehaene's (1992) triple-code model. According to Schneider et al. (2016), the ordinal aspect of numbers, i.e., the knowledge that a number indicates rank in an ordered set, develops from the cognitive precursor skill of seriation. Understanding the cardinal aspect of numbers, i.e., knowing that a number describes the magnitude of a quantity, stems from the cognitive precursor skill of classifying.

Mathematics performance in primary education

According to Prenzel et al. (2008), five levels of mathematical performance can be distinguished, each of which is built on another:

1. numerical and conceptual basic knowledge (routine procedures) without reference to application (level 1)
2. basic skills in dealing with the decimal system, plane geometry and quantities (level 2)
3. secure calculation in curricular range and simple modelling (level 3)
4. mastery of basic arithmetic using decimal structure and conceptual modelling (level 4)
5. problem solving in mathematical contexts (level 5) (Prenzel et al., 2008, p. 114).

Prediction of mathematics performance by precursor skills

The importance of precursor skills for later mathematical abilities is discussed in more detail in this section. In general, several studies have already provided evidence that precursor skills, such as prior knowledge of numbers and quantities, have a significant influence on later mathematical performance (Kolkman et al., 2013; Krajewski & Schneider 2006; Weißhaupt et al. 2006).

Comparing cross-domain and domain-specific skills, there is a consensus in the literature that domain-specific precursor skills are more predictive. Here, it should be noted that cross-domain precursor skills such as intelligence and working memory have an indirect influence on later mathematical performance via number-quantity knowledge (Gallit et al. 2018). The range of prediction of precursor skills was measured by Watts et al. (2018) in a longitudinal study up to age 15 and by Niklas and Schneider (2012) up to age 23.

Depending on the underlying model, different abilities are defined as precursor skills. Nevertheless, all precursor skills are based on quantity-number precognition, which is the foundation for further finer definitions. The precursor skills defined by Moraske et al. (2018), digit knowledge, digit identification, counting skills, quantity recognition, and quantity decomposition, provide an example of the finer definitions of precursor skills based on quantity-number precognition. Moraske et al. (2018) demonstrated a predictor effect for these precursor skills. Specifically relevant to this work is the prediction of precursor skills by the end of the elementary school years. Krajewski and Schneider's (2006) findings emphasize that number and quantity skills predict mathematics performance best at the end of primary school.

In the study by Gallit et al. (2018), precursor skills were able to explain 42% of the mathematical performance assessed later. This is consistent with the studies presented previously, which measured predictive power between 30% and 50%. Moreover, Gallit et al. (2018) confirm math-specific prior knowledge as the most predictive predictor, while general cognitive precursor skills act indirectly through number-set knowledge. Conclusively, they summarize that mathematical precursor skills substantially evolve and are closely related to each other.

The study

The basic idea of the project, later titled "Längsschnittanalyse mathematischer Geschlechtsdisparitäten bei 6- bis 10-jährigen Grundschulkindern" ("Longitudinal analysis of mathematical gender disparities in 6- to 10-year-old elementary school children"), was developed by Karl-Heinz Graß and Jakob Kelz in 2016. This topic was chosen not only because of the 2015 PISA results, in which male students in

Austria performed significantly better than female students in mathematics, but mainly because of the inconsistent findings in the literature regarding the gender gap in primary education. Indeed, it is not clear whether this gap exists or when it develops. One idea is to analyse precursor skills along with their prediction on mathematics performance, which now leads to the following research questions.

Questions and hypotheses

Question 1: How strong is the predictive power of cross-domain and domain-specific precursor skills with regard to primary mathematics performance? *Hypothesis 1:* There are studies that show a predictor effect for domain-specific and cross-domain precursor skills with respect to later mathematics performance (Kolkman et al. 2013; Krajewski & Schneider 2006; Weißhaupt et al. 2006). Regarding the predictor effect, according to Schneider et al. (2016), domain-specific precursor skills are assumed to achieve greater predictive power than cross-domain precursor skills.

Question 2: Are there gender differences in the predictive power of cross-domain and domain-specific precursor skills? *Hypothesis 2:* No gender differences are assumed. For both genders, it is hypothesized that both domain-specific and cross-domain precursor skills predict mathematical performance.

Method

Sample

The sample was drawn from 4 Styrian elementary schools in the surrounding area of Graz. An attempt was made to select schools that together, with regard to urban-rural, gender and migration, yielded a representative sample of Styrian elementary schools. There were 62 students in the first grades of these schools. According to Kelz (2017), the average age at school entry was 6;9 years ($SD = 0;5$). In the course of the longitudinal analysis over the three measurement points, only complete data sets were included in the calculations. These were 32 girls and 30 boys.

Survey instruments

The Eggenberger Rechentest (ERT 0+) is a diagnostic tool for dyscalculia disposition from the end of kindergarten age to the middle of the 1st grade, which differentiates particularly well in the lower third of mathematical performance. Although the difficulty level is set low, this survey instrument was chosen because of its specific design for the school entry phase. The test was administered strictly according to instructions from Lenart et al. (2014) and took an average of two hours to complete. The test consists of 71 items, of which four to six items each are assigned to 17 different precursor skills. Lenart et al. (2014) categorize these scales into the factors: cognitive basic skills, quantity knowledge, number knowledge, and application of mathematical skills. A test task (precursor skill spatial location) of the ERT 0+ is listed below:

Tick the right-hand box of these boxes in the middle row!



The Deutscher Mathematiktest (DEMAT 3+) was used in the third grade. In contrast to the ERT 0+, the DEMAT 3+ is based on the German curriculum, which, however, has highly similar content with

regard to the Austrian curriculum. The DEMAT 3+ tests the three areas of arithmetic, factual calculation, and geometry.

The Wechsler Intelligence Scale for Children - Fifth Edition (WISC-V) is a single test procedure designed to assess the cognitive abilities of children or adolescents between the ages of 6 and 16. The cognitive domains tested are language comprehension, fluid reasoning, visual-spatial processing, working memory, processing speed and general intellectual level. The test procedure was carried out in elementary schools by trained test persons and subsequently evaluated according to David Wechsler's manual.

Measurement schedule

The sample was drawn prior to school entry. In October 2016, in the students' first year of school, the ERT0+ was used to survey domain-specific precursor skills. The WISC-V was used to survey cross-domain precursor skills during the primary years between 2016 and 2018. The DEMAT3+ was used to survey mathematics performance during the school year 2018/2019. The sample was tested again at the end of the third school level, as this represents the "Grundstufe 2" (third and fourth grade of primary school) and the fourth school level was not possible for organisational reasons. However, the sample will be retested at the end of the ninth grade.

Statistical evaluations

The questions are analysed using multiple linear regression. Here, the dependent variable is DEMAT3+ mathematics performance. The independent variables that attempt to explain the dependent are domain-specific (ERT0+) and cross-domain precursor skills (WISC-V). When the linear model is calculated, the result will be a corrected R-squared. This indicates what percentage of the dependent variable is explained by the independent variables. The table of coefficients is concerned with whether the independent variables are significant with respect to the significance level. In the case of the T-value, it is crucial that it is different from zero, otherwise no influence of the independent variables can be assumed. According to the research question, the multiple linear regression is first calculated for the total sample and then for girls and boys.

Results

Predictive power of precursor skills

A multiple linear regression model was used to estimate the predictive power of precursor skills. According to this model, math performance scores are predicted by precursor skills at 44.9%. When considering gender differences, variance explanations are higher for boys (37.5%) than for girls (29.4%). Looking at the cross-domain precursor skills, the variance resolution is 42.8% for the total sample, 34.8% for girls, and 47.2% for boys. The domain-specific precursor skills lag significantly behind the cross-domain ones in predicting mathematics performance. For the total sample, the corrected R-squared is 15.6%, for girls 9.2%, and for boys 11.7%.

The table of coefficients makes it clear that for the total sample as well as for the girls and boys, the domain-specific precursor skills have no significant influence. As a result, the influence of the cross-domain antecedent skills is statistically verified to be greater than that of the domain-specific precursor skills. The T-values are different from zero and thus confirm the influence of the significant coefficients.

Table 1: Coefficients including T-value and significance value

Sample	Coefficients	T	Significance
Total sample			
Cross-domain	0.136	5.548	0.001
Domain-specific	0.049	0.466	0.642
Girls			
Cross-domain	0.113	3.590	0.001
Domain-specific	0.137	0.866	0.394
Boys			
Cross-domain	0.165	4.264	0.001
Domain-specific	-0.014	-0.094	0.926

Discussion and Conclusions

The present study demonstrates a predictor effect of precursor skills and is in line with the findings of Kolkman et al. (2013) and Weißhaupt et al. (2006). However, it contradicts Schneider et al. (2016), as cross-domain precursor skills have greater predictive power than domain-specific precursor skills. Therefore, Hypothesis 1 can only be conditionally accepted further. This could be due to the ERT0+, which differentiates less well in the upper third of performance than the WISC-V intelligence test and thus explains less well the variance of the DEMAT3+.

The present results imply that the predictive power is stronger for boys than for girls. This is evident in both cross-domain and domain-specific precursor skills. This finding requires confirmation and further investigation, so in the present case the results should be viewed as a tendency only. As in the overall sample, it appears that cross-domain antecedents predict better than domain-specific ones. This is the reason for the complete rejection of hypothesis 2.

In sum, it appears that mathematics performance in the third grade can be predicted by precursor skills. The present Austrian study also falls within the 30-50% range that has emerged from the international literature. The importance of intelligence for the predictor effect was confirmed. It remains unclear whether that effect influences performance directly or indirectly through domain-specific precursor skills. The tendency for boys' performance to be more predictive remains worth investigating for follow-up studies.

The high predictor effect of precursor skills (44.9%) on mathematics performance is very assertive. At kindergarten age and before school entry, the foundation is already laid for mathematics performance at the primary level. An early promotion of those precursor skills would have a positive effect on mathematics performance. This applies internationally and to any form of preschool that promotes precursor skills before school entry (Watts et al. 2018). At least a good third of later mathematics performance can be promoted in this way. This applies to both girls and boys. Ideas for promoting precursor skills in kindergarten would be, in addition to the classics dice games, number

rhymes and building blocks, clearly declared material for early mathematical development. This material is also accompanied by various test procedures to check precursor skills. In this way, almost all precursor skills could be promoted to enable the acquisition of mathematical competences at primary level.

Limitations and outlook

The greatest limitation results from the regionality of the sample. Therefore, the findings of this study are to be seen primarily as tendencies. Another limitation of this study results from the ERT0+. The ERT0+ measures domain-specific precursor skills, but not in a sufficiently differentiated way. This explains why domain-specific precursor skills lag behind cross-domain skills in this study. In the continuation of the project, mathematics performance will be followed up in terms of a longitudinal design in order to get closer to the goal of elucidating the relationship between mathematics performance and gender. Furthermore, in order to give more weight to the purely quantitative study, the variable lessons will be qualitatively investigated in the future.

References

- Aunola, K., Leskinen, E., Lerkkanen, M., & Nurmi, Jari-Erik (2004). Developmental Dynamics of Math Performance From Preschool to Grade 2. *Journal of Educational Psychology*, 96(4), 699–713. <https://doi.org/10.1348/000709905X51608>
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44(1), 1–42. [https://doi.org/10.1016/0010-0277\(92\)90049-N](https://doi.org/10.1016/0010-0277(92)90049-N)
- Franke, M. (2007). *Didaktik der Geometrie in der Grundschule* [Didactics of geometry in elementary school]. Akademischer Verlag.
- Gallit, F., Wyschkon, A., Poltz, N., Moraske, S., Kucian, K., & Aster, M. (2018). Henne oder Ei. Reziprozität mathematischer Vorläufer und Vorhersage des Rechnens [Hen or egg. Reciprocity of mathematical precursors and prediction of arithmetic]. *Lernen und Lernstörungen*, 7(2), 81–92. <https://doi.org/10.1024/2235-0977/A000205>
- Kelz, J. (2017): *Mathematik und Geschlechtsdisparitäten - Eine Analyse in der Schuleingangsphase* [Mathematics and gender disparities - An analysis in the school entry phase]. Karl-Franzens-Universität Graz.
- Kolkman, M. E., Kroesbergen, E. H., & Leseman, P. P. (2013). Early numerical development and the role of non-symbolic and symbolic skills. *Learning and Instruction*, 25, 95–103.
- Krajewski, K., & Schneider, W. (2006). Mathematische Vorläuferfertigkeiten im Vorschulalter und ihre Vorhersagekraft für die Mathematikleistungen bis zum Ende der Grundschulzeit [Mathematical precursor skills at preschool age and their predictive power for math performance until the end of elementary school]. *Psychologie in Erziehung und Unterricht*, 53(4), 246–262.
- Lenart, F., Schaupp, H., & Holzer, N. (2014). *Eggenberger Rechentest ERT 0+ : Diagnostikum für Dyskalkulie-Disposition für das Ende des Kindergartenalters bis Mitte der 1. Schulstufe* [Diagnostic tool for dyscalculia disposition for the end of kindergarten age to the middle of 1st grade]. Huber.

- Mazzocco, M., & Thompson, R. (2005). Kindergarten predictors of math learning disability. *Learning Disabilities Research & Practice*, 20, 142–155. <https://doi.org/10.1111/j.1540-5826.2005.00129.x>
- Moraske, S., Penrose, A., Wyschkon, A., Kohn, J., Rauscher, L., Aster, M., & Esser, G. (2018). Prävention von Rechenstörungen- Kurz- und mittelfristige Effekte einer Förderung der mathematischen Kompetenzen bei Risikokindern im Vorschulalter [Prevention of dyscalculia - short- and medium-term effects of promoting math skills in at-risk preschool children]. *Kindheit und Entwicklung*, 27(1), 31–42. <https://doi.org/10.1026/0942-5403/a000242>
- Niklas, F., & Schneider, W. (2012). Die Anfänge geschlechtsspezifischer Leistungsunterschiede in mathematischen und schriftsprachlichen Kompetenzen [The beginnings of gender-specific performance differences in mathematical and literacy skills]. *Zeitschrift für Entwicklungspsychologie und Pädagogische Psychologie*, 44(3), 123–138. <https://doi.org/10.1026/0049-8637/A000064>
- Prenzel, M., Gogolin, I., & Krüger, H. (2008). *Kompetenzdiagnostik: Zeitschrift für Erziehungswissenschaft* [Competence diagnostics: Journal of Educational Science]. VS Verlag für Sozialwissenschaften / GWV Fachverlage
- Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal*, 50, 178–209. <https://doi.org/10.3102/0002831212465332>
- Rittle-Johnson, B., Hofer, K. G., Fyfe, E. R., & Farran, D. C. (2017). Early Math Trajectories: Low-Income Children's Mathematics Knowledge From Ages 4 to 11. *Child Development*, 88(5), 1727–1742. <https://doi.org/10.1111/cdev.12662>
- Schneider, W., Küspert, P., & Krajewski, K. (2016). *Die Entwicklung mathematischer Kompetenzen* [The development of mathematical skills]. <http://www.utb-studi-e-book.de/9783838546162>.
- Shah, P., Miyake, A. (Ed.) (2005). *The Cambridge handbook of visuospatial thinking*. Cambridge University Press.
- Shanley, L., Clarke, B., Doabler, C. T., Kurtz Nelson, E., & Fien, H. (2017). Early number skills gains and mathematics performance: Intervening to establish successful early mathematics trajectories. *Journal of Special Education*, 51(3), 177–188. <https://doi.org/10.1177/0022466917720455>
- Watts, T.W., Duncan, G.J., Clements, D.H. and Sarama, J. (2018). What Is the Long-Run Impact of Learning Mathematics During Preschool? *Child Development*, 89, 539–555. <https://doi.org/10.1111/cdev.12713>
- Weißhaupt, S., Peucker, S., & Wirtz, M. (2006). Diagnose mathematischen Vorwissens im Vorschulalter und Vorhersage von Rechenleistungen und Rechenschwierigkeiten in der Grundschule [Diagnosis of prior mathematical knowledge at preschool age and prediction of arithmetic performance and arithmetic difficulties in elementary school]. *Psychologie in Erziehung und Unterricht*, 53, 236–245.