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► **To cite this version:**

Mette Amalie Bundgaard. A reconceptualisation of mathematical reasoning competency from a Kindergarten Class perspective. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04422717

HAL Id: hal-04422717

<https://hal.science/hal-04422717>

Submitted on 28 Jan 2024

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A reconceptualisation of mathematical reasoning competency from a Kindergarten Class perspective

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In Denmark, the Kindergarten Class (KC) is a transition year between kindergarten and primary school. However, the extent to which this transition year contributes to coherence in the teaching and learning of mathematics is unclear. This paper theoretically explores how mathematical reasoning competency may be reconceptualised from a KC perspective, to enhance coherence in mathematic learning. The proposed reconceptualisation combines guided play with various aspects of mathematical argumentation framed by reasoning competency. Consequently, the proposed reconceptualisation of mathematical reasoning competency entails exploration, conjecture, and producing and analysing arguments in an inquiry-based learning environment. The reconceptualisation is discussed regarding the limitations of the chosen perspective on reasoning, along with the challenges and potential when implementing the reconceptualisation in KC practice.

Keywords: Mathematical reasoning, argumentation, kindergarten class, guided play, inquiry-based.

Introduction and research question

Mathematical reasoning is an essential part of learning and doing mathematics (Niss, 2003). Consequently, in many curricula, it is a cross-cutting process, rather than pertaining to a specific subject area. This also applies to the Danish primary school mathematics curriculum (Børne- og Undervisningsministeriet, 2019b), where reasoning competency, as it originated in the Danish mathematical competency framework (Niss & Højgaard, 2019), is a learning objective whose importance is equal to that of subject areas such as geometry or algebra. However, this emphasis on mathematical competencies is not present in Danish early childhood education, and the mathematics focus of the institution before entering formal schooling is generally insignificant (Børne- og Socialministeriet, 2018). As early mathematics education should prepare students for the kind of mathematics they will encounter later in their education, mathematical competencies like reasoning competency, should be a focus of early mathematics learning, just as in formal schooling. The lack of coherence in mathematics across the transition from early childhood education to primary school presents a challenge that needs to be explored. In particular, this challenge is significant when it comes to reasoning, especially given the essential role of reasoning in mathematics. This study suggests developing greater coherence in mathematic learning by focusing on reasoning competency in the pedagogical context of KC.

This paper's research question is, what does a reconceptualisation of the reasoning competency from a kindergarten class perspective entail? This research question is investigated while positioning the KC between informal and formal learning and defining the pedagogical approach as that of guided play. In this perspective, this reconceptualisation explores and establishes reasoning competency as a part of mathematical learning in the KC. A discussion of this reconceptualisation in the field of reasoning leads to a focus on the challenges and potential of implementation of the

reconceptualisation in practice. The aim of this paper is to reconceptualise reasoning competency, to promote coherence in mathematical learning across the transition from kindergarten to school, while acknowledging that the kindergarten class is different than formal school. This reconceptualisation will in near future serve as the foundation for a design-based PhD project that investigates and develops reasoning competency in the KC.

Background

When the KC was established in Denmark in 1962, the aim was to prepare six-year-olds for formal education by combining school and kindergarten pedagogy (Vejleskov, 2017), and it is usually located on school premises. In 2003, a more detailed curriculum was introduced for the KC, and in 2009 the KC became mandatory, and was considered an official part of compulsory school, but with its own pedagogical approach and learning objectives. This entailed an increased emphasis on knowledge and learning, mainly concerning language awareness, but in 2014, the curriculum included mathematics, expressed in terms of mathematical awareness (Børne- og Undervisningsministeriet, 2019a). Nevertheless, the KC is still a transitional year between the informal kindergarten and the formal primary school, combining and balancing elements from each institution while maintaining its own pedagogical approach. Next, the differences and similarities between Danish kindergarten and primary school are explored, with the aim of understanding the KC as a pedagogical construct that bridges the other two institutions.

Danish kindergarten is a part of the social pedagogy tradition. It emphasizes children's natural learning strategies, such as play, social interaction, activities, and personal investigation, with a parallel focus on children's personal development (OECD, 2006; Ringsmose & Kragh-Müller, 2017). In contrast, the primary school focuses on academic learning in a more formal setting. The guidelines for teaching mathematics in primary school (Børne- og Undervisningsministeriet, 2019b) describes a socio-constructivist perspective, where teaching focuses on dialogue and social interaction. Both institutions view learning through social interaction as fundamental, with the adult as a source of knowledge and inspiration, although this is implemented in different ways. Primary school activities are mostly teacher-initiated and organized according to mathematical learning objectives stated in the curriculum. In kindergarten, daily activities are mainly child-initiated, and the curricula consist of six learning themes that are not defined according to academic subjects, but more broadly, as a theme without specific learning objectives. In that sense, mathematics is not a theme. Still, number sense is mentioned in a theme concerning nature, the outdoors, and science, along with the concept of beginning mathematical awareness (Børne- og Socialministeriet, 2018). The adults' educational background is also different. In kindergarten, adults are pedagogical generalists without any mathematics training. In contrast, primary school teachers are trained as specialists in two to four subjects, one of which may be mathematics, so a mathematics teacher teaches mathematics in first grade.

The main elements from kindergarten and primary school that characterize the KC are as follows:

- The kindergarten class has one subject that is not subdivided into academic subjects.

- KC teachers¹ are trained as pedagogical generalists.
- It is located on school premises, in a classroom.
- There are thirty hours of weekly teaching with an after-school program shared with primary education.
- The children are considered students.
- Each KC has one or two KC teachers.

The first two points are associated with kindergarten, whereas the subsequent four points pertain to primary school. Two additional points also characterize the KC as a compromise between the other two institutions:

- Six broad learning objectives, one of which is the concept of mathematical awareness.
- Play-based learning.

In theory, in the KC, subjects do not exist, and instead, the learning themes from kindergarten are adapted with learning objectives to meet the formal perspective of education. The other element is play-based learning, which combines play from kindergarten pedagogy, and learning from school pedagogy in one pedagogical approach. Guided play is a theoretical framework for this approach.

Guided play as a pedagogical approach in the kindergarten class

In the KC, kindergarten and school pedagogy coexist as a pedagogical approach that combines play and learning, despite their many differences. Play-based learning emerges from an exploration of the relationship between play and learning. Within the social pedagogy tradition, two dominant approaches to play-based learning are play-responsive teaching and guided play. A critical difference between the two is who initiates the play, with the former involving child-initiated play and the latter involving teacher-initiated play that may include learning objectives. In kindergarten, play is child-initiated, but in the KC, the KC teacher's role to initiate activities that enable students to achieve learning objectives, which aligns with the guided play approach. The KC teacher may prompt learning through guided play in one of two ways: "...by carefully preparing the environment beforehand and by scaffolding children's actions as their play unfolds over time" (Weisberg et al., 2015, p. 10). Despite being teacher-initiated, guided play is not teacher-directed. Teacher-directed means that teachers tell the students what to do and is equivalent to the concept of instruction (Weisberg et al., 2015). Instead, guided play is defined as student-directed, as the students engage in self-directed exploration. The students should be active and explore however they wish, but in a circumscribed environment that the KC teacher facilitates. The KC teacher presents opportunities for exploration, and they may also guide the exploration and scaffold the students' actions by proposing discovery strategies. This is a balancing act, as the KC teacher must not direct the students' actions, but only guide them (Weisberg et al., 2015).

Guided play is an approach that develops student's natural learning strategies of play and investigation from a kindergarten perspective, as the students direct play, but simultaneously acknowledges the

¹ The Danish occupational title translates directly as "kindergarten classroom managers," but to avoid misunderstandings in this paper, they will be referred to as KC teachers.

educational setting and learning objectives, as the situation is teacher-initiated with a focus on a specific learning objective. As a pedagogical approach to play-based learning, guided play balances informal and formal settings by being teacher-initiated and student-directed. Guided play meets the need to balance and combine elements of kindergarten and primary school into a pedagogical approach that fits the KC's characteristics. However, these situations are grounded in a socio-constructivist perspective (Weisberg et al., 2013), which emphasizes an inquiry-based environment that promotes student-directed exploration. In such an environment, reasoning is crucial (Yackel & Hanna, 2003), but reasoning in early learning has many definitions, and requires further clarification.

Various perspectives on mathematical reasoning

A brief review of mathematical reasoning research reveals several perspectives on the concept of reasoning, such as the duality of process vs product, types of reasoning as inductive, deductive or abductive, the relationship between understanding and reasoning, taxonomies of reasoning, and reasoning in the exercise and application of mathematics across subject areas. Despite its importance, reasoning is often assumed to be a universally agreed-upon concept. However, the various perspectives highlight its complexity and ambiguity, which also applies to the field of early learning. Thus, recent proceedings from a conference on early childhood mathematics (POEM 4) featured 15 chapters dedicated to various perspectives and understandings of reasoning (Carlsen et al., 2020). In these contributions, reasoning is seldom defined but implicitly present, and includes reasoning as a process (De Simone & Sabena, 2020), types of reasoning (Severina & Meaney, 2020), and reasoning as a mathematical skill, such as multiplicative reasoning (Breive, 2020). Even though there is an increasing focus on reasoning in research, there is a need for further research on mathematical reasoning in early learning (Sternier et al., 2020), and especially on mathematical reasoning that transcends subject areas (Säfström, 2018).

Reasoning competency is one of the eight competencies outlined in the Danish mathematical competency framework (Niss & Højgaard, 2019), with a perspective on reasoning that transcends subject areas. Niss (2003) defines each competency as subjective, as it concerns an individual's actions and behaviour, and is “someone’s.” However, a competency also has a social side, as it is culturally conditioned and situational (Højgaard, 2022), which is independent of age and social context. There are two key aspects to enacting reasoning competency: producing and analyzing arguments (Niss & Højgaard, 2019). When enacting reasoning competency, it is not restricted to one subject area, but a mathematical aspect is necessary. Therefore, reasoning competency is only activated and exercised with mathematical content and do not entail general logic unless mathematics is included. This perspective on reasoning has been explored only for formal learning, and not for informal learning. As a learning objective of Danish primary schools, reasoning competency is enacted in an environment where dialog and exploration are central, and independent of subject areas (Børne- og Undervisningsministeriet, 2019b). Thus, reasoning competency is relevant to early learning in the KC, as it is already interpreted within a socio-constructivist perspective and supports the continuity of students’ mathematical learning. The learning objective of mathematics in the KC (mathematical awareness) also emphasizes the enactment of mathematics independent of subject areas, as in the case with the reasoning competency. However, for reasoning competency to be reconceptualised from the KC perspective, it is necessary to clarify how the main aspects of reasoning

competency—analysing and producing arguments—may be understood in the context of a KC. At the same time, it needs to be viewed from the perspective of guided play as the main pedagogical approach.

Reconceptualising reasoning competency from a KC perspective

Reasoning competency needs to be modified to embrace both formal and informal learning. Sumpter and Hedefalk (2018) acknowledge that mathematical reasoning exists in both formal and informal settings. They studied both informal and formal learning situations in kindergartens. Here, they found three types of arguments when there is play in social interactions: 1) predictive arguments, as arguments regarding which strategy a student will use; 2) verifying arguments, as arguments regarding which strategy a student used; 3) evaluative arguments, as arguments for the conclusion. These types of arguments concern the productive part of reasoning competency. The students' actions when they produce these arguments are student-directed and may occur in planned and informal situations. However, as the KC is an inquiry-based classroom, not only do students produce arguments for sharing, but the arguments also have a purpose when they are part of social learning in a classroom. Therefore, in a setting that combines informal and formal learning, it is not enough to look at the type of argument involved, but also the purpose of the argument produced. From a formal perspective, Yackel (2001) views mathematical argumentation in an inquiry-based environment, where the purpose of an argument distinguishes explanation from justification. Explaining refers to actions that the student will encounter or has encountered, to clarify and make sense of mathematics (Yackel, 2001). Justification refers to validation (Whitenack & Yackel, 2002). In this sense, producing one of the three types of arguments is also an explanation or a justification.

At first glance, both theories strongly underpin the productive side of an argument, with no attention given to the analytical part of reasoning competency. However, in a socio-constructivist classroom, arguments are challenged and judged until they make sense and are accepted by the classroom community (Yackel, 2001). For one to judge an argument, whether in a formal or informal setting, one must be by analysing it, and if necessary, challenge it until it is an acceptable argument. Thus, to analyse an argument, students need to be in an environment where thinking and arguments are shared, but also to understand when an explanation or justification is acceptable. Here, it is not enough for the argument to be logical: it must have roots in and contain mathematics. This is a key element of both Yackel's (2001) and Sumpter and Hedefalk's (2018) work, and as stated previously, also of reasoning competency. The KC teacher needs to facilitate an inquiry-based classroom where students can explore, and arguments are produced but also analysed, and afterwards accepted by peers. With a guided play approach, the KC teacher needs to guide the students not only towards a learning objective, but also towards an acceptable mathematical argument and the development of an explorative environment that produces and analyses the type of arguments that emerge. Thus, focusing on enacting reasoning competency combines formal and informal learning that involves a duality of individual and social aspects. A socio-constructivist view of teaching and learning accommodates guided play and the duality of reasoning competency emphasizing producing and analysing various types of mathematic-based arguments. However, from a guided play perspective, personal explorations are a crucial element. Therefore, in the KC, producing and analysing arguments cannot stand alone as key elements, so the reconceptualisation of reasoning competency includes the

exploration that precedes argumentation. Conjecture will also be an element of this reconceptualisation, and links exploration and argumentation. Conjecture may evolve from exploration which need argumentation.

Thus, a proposed reconceptualisation of reasoning competency in the KC requires students to engage in these actions: exploring, conjecturing, and producing arguments that are characterized by their various types and purposes, but also challenging, judging, or accepting other students' arguments as a part of the inquiry-based classroom.

Discussion

The concept of reasoning is complex and has various interpretations throughout all levels of mathematics education. This paper adopts a distinct stance on reasoning and employs the competency framework (Niss & Højgaard, 2019) as its foundation. Consequently, my reconceptualisation of reasoning competency operates within the confines of this framework, and do not consider other understandings of reasoning. In some ways, this limitation also imposes a broader perspective on mathematical reasoning that is not restricted to reasoning about a specific subject area, as is often seen in early childhood mathematics (e.g., Breive, 2020), but still bears a clear relationship to mathematics. This reconceptualisation transcends all subject areas, accommodating both individual and social constructs. However, this broader perspective still has limitations, as it may be limited to a specific pedagogical approach within a particular context, which is not necessarily true of other perspectives on reasoning.

Although the implementation of the reconceptualisation of reasoning competency has potential, it is also necessary to discuss its limitations. In theory, the KC is defined by a specific pedagogical approach, but in Denmark, as in the rest of the Nordic countries, the movement towards schoolification is a concern. In Danish KCs, guided play is often undermined by the common practice of dividing teaching into subjects and lessons and relying on textbooks as primary teaching material. Consequently, often the teaching is not only teacher-instructed, but also teacher- or textbook-directed, and there are few opportunities for students to direct their own explorations. It is possible to integrate guided play into the more formalized aspects of this practice, but this demands a shift from teacher-directed action to student-directed exploration. But this is not enough. In guided play there is an emphasis on personal exploration, instead of exploration as social interaction, which is necessary for the students to meet the analytical part of reasoning competency. Although guided play includes a social perspective, the social interaction focuses on teacher–student relations, as the KC teacher is the knowledgeable other who guides the students. There is no elaboration on social interactions between students in the guided play approach. But in an inquiry-based classroom, sharing thinking and arguments is essential to negotiating an acceptable mathematical argument and what defines the argument's purpose. If there is no sharing, but only individual exploration guided by the KC teacher, how can the students analyse one another's arguments? This underpins the importance of the KC teacher's role as a guide and as a facilitator of the classroom environment, to promote reasoning competency.

The KC teacher's role in implementing the reconceptualisation of reasoning competency is crucial, as guiding students towards producing and analysing arguments is a key element. At the same time,

students' language development and limited prior exposure to mathematical thinking and knowledge may influence their production of arguments, as these must be founded in mathematics. Laine et al. (2018) state that when guided, young children can develop mathematical thinking, which should also apply to students' reasoning in the KC. However, self-directed actions do not necessarily involve producing or analysing mathematical arguments. This suggests an important focus on the KC teacher's guiding the students towards mathematical argumentation and an investigation of which questions a KC teacher should ask. Yet, this requires knowledge of mathematical content and processes, and of how to ask questions that promote arguments (Sumpter & Hedefalk, 2018). Therefore, KC teachers' limited mathematics education presents a crucial challenge to the implementation of the reconceptualisation, particularly regarding the KC teacher's role in an inquiry-based environment and the negotiation of an acceptable mathematical argument. In the reconceptualisation of the reasoning competency, the KC teacher's role is not elaborated. Still, in guided play and an inquiry-based environment, the KC teacher is essential for initiating explorations, facilitating and guiding the students in their exploration, conjecture and argumentation, and establishing a classroom where this is possible. Therefore, implementing this reconceptualisation in a Danish kindergarten class will have implications for teaching and learning, for students and KC teachers, while contributing to the continuity of mathematical reasoning.

Nevertheless, this reconceptualisation fills a local gap, and is not a general understanding of reasoning, as it is now limited to the KC's specific pedagogical context. At the same time, it fulfils the ambition of creating coherence in the curriculum. Therefore, this reconceptualisation balances formal and informal perspectives on reasoning by combining theories from both perspectives, and is, for now, a theoretical contribution that will be the subject of further empirical investigation and development when it is implemented in the KC. In general, there is potential for adapting the reconceptualisation of reasoning competency into KC practices, but more empirical investigation is needed.

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