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# Adults' evaluations of a child's triangle knowledge

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*This study focuses on adults' geometric knowledge specifically related to triangles. Two questionnaires were filled out by 148 adults. The first questionnaire presented two examples and three nonexamples of triangles. Adults were asked to identify each figure as a triangle or non-triangle and explain their reasoning. The second questionnaire contained the same figures and questions, along with the responses of a fictitious five-year-old boy. Adults were requested to evaluate the boy's identifications and reasoning. Except for one non-triangle, nearly all adults correctly identified the figures and offered geometrically sufficient explanations. Adults tended to accept the young boy's reasoning even though it was not based on critical attributes, claiming that for young children, such reasoning is acceptable.*

*Keywords: Adults' geometric knowledge, examples and nonexamples, triangles, evaluating answers.*

## Introduction

While previous studies have investigated preschool teachers' knowledge for engaging young children with mathematical activities (e.g., Ginsburg, 2016; Tsamir, et al., 2015), young children also spend a great deal of time at home with parents, grandparents, and other responsible adults. Furthermore, the home environment can increase the educational benefits of attending preschool and kindergarten (Anders et al., 2012). This study considers adults that spend time with children out of school, who can also impact on children's early mathematical knowledge. Specifically, research has pointed out the importance of engaging young children with geometrical activities. Children are exposed to geometrical shapes from an early age. Thus, it is essential that proper intervention and guidance are given before intuitions become rooted and difficult to amend. While previous studies investigated parents' reports of their engagement with geometrical activities at home (Zippert et al., 2020), little is known regarding parents' and other adults' geometrical knowledge necessary for providing beneficial support. Our research questions are: Can adults correctly identify examples and nonexamples of triangles and do they offer sufficient geometric reasons for their identification? Are adults able to evaluate a fictitious child's identification of triangles and how will they evaluate that fictitious child's reasoning?

## Background

The acquisition of geometrical concepts includes both visual and attributional reasoning. At the most basic level, children use visual reasoning, taking in the whole shape without considering that the shape is made up of separate components (van Hiele & van Hiele, 1958). Children at this level can name shapes but may not realize which attributes are critical for identifying a figure and which are not (van Hiele and van Hiele 1958). For example, when a triangle is not oriented with a horizontal side, children may not identify it as a triangle (e.g., Burger and Shaughnessy, 1986). Children may also accept curved sides, either concave or convex, when identifying triangles (Tsamir et al., 2008).

Next, children begin to notice that different shapes have different attributes, but the attributes are not perceived as being related. Later, relationships between attributes are perceived and definitions are meaningful. Attributes may be critical or not-critical (Tsamir et al., 2008). In mathematics, critical attributes stem from the concept definition. For example, the critical attributes of a triangle include (a) closed figure, (b) three sides, (c) three vertices, (d) three angles. Non-critical attributes include the overall size of the figure (large or small) and orientation (horizontal base). We aim for children to use only critical attributes as the deciding factor in identifying examples and forming geometrical concepts. Yet, many children revert to the use of non-critical attributes when trying to differentiate between examples and non-examples among similar shapes (Tsamir et al., 2008).

Based on several studies and their own investigation, Tsamir et al. (2008) identified examples and nonexamples of triangles that were intuitively recognized as such by kindergarten children and those that were not intuitively recognized. They found that the illusion of threeness, and not necessarily the actuality of threeness, reminded children of the prototypical triangle (Hershkowitz, 1990), causing many to claim that such nonexamples as a rounded-corner “triangle” was a triangle. They also noted that in Hebrew, the word for triangle and the word for three are very similar, making the attribute of threeness especially significant. Likewise, the ability to name a figure played an important role in the intuitive identification of non-triangles.

In a follow-up study of preschool teachers’ geometric knowledge (Tsamir et al., 2015), it was found that most teachers correctly identified examples and non-examples of triangles. The exceptions were 17% of teachers who claimed that a rounded-corner “triangle” was a triangle and one teacher who incorrectly identified an open “triangle” as a triangle. In that study, teachers were also asked to write a definition for a triangle. The definition of a triangle is a polygon with three sides (or three vertices or three angles). A polygon is a closed figure made up of straight sides. Most definitions (94%) were considered correct. One definition was insufficient as the teacher only wrote that a triangle has three straight lines, which did not guarantee that the shape would be closed. Another teacher wrote that a triangle has three equal sides. Having equal sides is a non-critical attribute of a triangle. In the current study, we investigated the geometric knowledge of adults who were not preschool teachers, and thus did not have professional training.

## **Method**






The current study was conducted in Israel with a convenience sample of 148 adults, recruited by three researchers from acquaintances in their areas of residence (middle to high socio-economic neighborhoods). Participants were between the ages of 20 and 60, 57% were parents of children between the ages of three and six, and 39% had some other relationship with young children (e.g., grandparents, uncles and aunts). Regarding educational and occupational backgrounds, 88% had an academic degree, and approximately a quarter worked in educational contexts (teachers, principals, aids), but not in preschool. A researcher met with each participant personally and handed them the questionnaire.

Participants were handed two separate questionnaires. The first questionnaire consisted of five figures. For each figure participants were requested to answer the following questions: Is this a triangle? Yes/No. Why? The figures included intuitive and non-intuitive examples and non-examples of triangles (see the figures in Table 1). The equilateral triangle may be considered a prototypical

triangle and thus intuitively recognized as a triangle, accepted immediately without the feeling that justification is required (Tsamir, et al., 2008). The scalene triangle may be considered a non-intuitive example because of its “skinniness”. Whereas the square may be considered an intuitive non-example of a triangle, because children know the name of a square (Sarama & Clements, 2009), the pizza-like “triangle” and the rounded corner “triangle” may be considered non-intuitive nonexamples because of their visual similarity to a prototypical triangle (Tsamir, et al., 2008). Furthermore, we named the pizza-like “triangle” as such, because in Hebrew, a slice of pizza is actually called “a pizza triangle.”

The second questionnaire presented the same five figures, along with a fictitious child’s responses to the same questions participants answered on the first questionnaire. That is, for each figure, Yossi, the fictitious five-year old, identified the figure as a triangle or not and then gave his reason for this response. Yossi’s reasons were based on kindergarten children’s verbal reasons as found in Tsamir et al. (2008). The adults were asked: Is Yossi's identification correct? Is Yossi's explanation acceptable to you? Why? For some of the figures, Yossi correctly identified the figure and for some he incorrectly identified the figure. However, for the purpose of this study, we chose to present only insufficient geometric explanations to see if participants would accept this reasoning (see Table 1).

**Table 1: Yossi's answers**

Is this a triangle?	Intuitive	Non-intuitive	
Examples			
Yossi's answer	<i>"It is a triangle, because we see it is a triangle"</i>	<i>"It is not a triangle, because it is too long"</i>	
Non-examples			
Yossi's answer	<i>"It is not a triangle, because it is a square"</i>	<i>"It is a triangle, because it has 3 lines"</i>	<i>"It is a triangle because it is like a slice of pizza"</i>

## Findings

This section begins by presenting adults’ identifications of the figures presented on the first questionnaire. Not surprisingly, the intuitive example of a triangle (the equilateral triangle) and the intuitive non-example of the triangle (the square) were correctly identified by all participants. Yet less than half of the adults correctly claimed that the rounded-corner “triangle” was not a triangle (see Table 2). For this study, adults’ explanations for their identifications were categorized into sufficient geometrical explanations and other explanations. For example, when explaining why the pizza figure was not a triangle, one sufficient explanation was, “because the line is not straight.” An insufficient explanation was, “this is not how a triangle looks.” In general, most adults gave sufficient geometrical explanations, relating to the critical attributes of a triangle. Noteworthy is the square. Unlike the other

non-triangles, it does not look like a triangle, and it has a different geometric name. Still, 86% of the adults rightly related to the critical attributes of a triangle and wrote that it was not a triangle because “it has four sides” or because “it has to have only three sides.”

**Table 2: Frequency (%) of correct identifications (N=148)**

	Examples (triangles)		Non-Examples (not triangles)		
	Intuitive	Non-Intuitive	Intuitive	Non-Intuitive	
The figures	Equilateral triangle	Scalene triangle	Square	Rounded-corner “triangle”	Pizza “triangle”
Frequencies (%)	148 (100)	142 (96)	148 (100)	65 (44)	132 (89)

Table 3 presents adults’ evaluation of Yossi’s identification. That is, did adults recognize when Yossi correctly or incorrectly identified the figures. Frequencies for evaluations were calculated only for those who they themselves correctly identified the figure. Thus, the value of N is different for each figure. For example, for the scalene triangle, 140 participants identified this figure correctly as a triangle (thus N=140), out of which 134 correctly assessed Yossi’s identification as incorrect. First, we note that most participants correctly evaluated Yossi’s identifications. While it seems surprising that someone who knew that the scalene triangle is indeed a triangle would say that Yossi was correct when he said it was not a triangle, as one adult reasoned, “He’s just a child and therefore doesn’t understand these things.” In other words, they would accept an incorrect identification from a young child because of his age. Regarding the pizza figure, an adult wrote, “I didn’t think of it that way. It really is a pizza triangle.” In other words, that adult thought about Yossi’s reasoning and seemed to change his mind, thinking that perhaps it really is a triangle. Regarding the square, it seemed that those who said Yossi was incorrect, might have been referring to his reasoning, rather than his identification. As one adult wrote, “No. Because it has four angles.”

**Table 3: Frequency (%) of correct Yossi's identifies triangles**

	Examples (triangles)		Non-Examples (not triangles)		
	Intuitive	Non-Intuitive	Intuitive	Non-Intuitive	
The figures	Equilateral triangle (N=148)	Scalene triangle (N=142)	Square (N=148)	Rounded-corner “triangle” (N=65)	Pizza “trinagle” (N=132)
Yossi's identification	correct	incorrect	correct	incorrect	
Frequencies (%)	148 (100)	136 (96)	143 (97)	65 (100)	123 (93)

Adults were also asked to evaluate Yossi’s reasons for his identification. For this analysis we consider adults who they themselves correctly identified the figure shown as an example or non-example, and

also correctly evaluated Yossi's identification. Recall that Yossi's reasons for his identifications were geometrically insufficient. Recall also that most adults gave sufficient geometric reasons for their own identifications. From Table 4 we see that except for the scalene triangle, adults tended to accept Yossi's reasoning.

**Table 4: Frequency (%) of Yossi's reasons**

	Examples (triangles)		Non-Examples (not triangles)		
	Intuitive	Non-Intuitive	Intuitive	Non-Intuitive	
The figures	Equilateral triangle (N=148)	Scalene triangle (N=136)	Square (N=143)	Rounded-corner "triangle" (N=65)	Pizza "triangle" (N=123)
Yossi's reasons	<i>We see it is a triangle</i>	<i>It is too long</i>	<i>It is a square</i>	<i>It has 3 lines</i>	<i>It is like a slice of pizza</i>
Unaccepted (%)	51 (34)	67 (49)	12 (8)	14 (22)	31 (25)
Accepted Yossi's reasons (%)	71 (48)	43 (32)	118 (83)	41 (63)	61 (50)
No answers / Unclear answers	26 (18)	26 (19)	13 (9)	10 (15)	31 (25)

Adults' explanations for their decision to accept Yossi's reasoning were categorized separately by two of the above authors, using a bottom-up approach (Mayring, 2014) through several iterative cycles of data categorization and refinement, until central themes arose. Four major categories were found, at which point a third author conducted a deductive analysis to validate the categorization of responses.

1. Yossi's age. Several adults related to the fact that Yossi was young. For example, regarding Yossi's explanation for why the equilateral triangle is a triangle, one wrote, "For a young child the explanation is acceptable." Regarding the square, which Yossi corrected identified as a non-triangle, one adult wrote, "From a boy in kindergarten it's very nice that he can give an explanation according to what it is not." For the scalene triangle, which Yossi incorrectly identified, another adult wrote, "He (Yossi) is still young and doesn't understand about the lengths of the sides." For the rounded corner "triangle," someone wrote, "Although it doesn't have three angles, for a six-year-old child, in my opinion, it's enough." This adult knew that this figure was not a triangle, evaluated Yossi's identification as incorrect, but would still accept his explanation because the boy is young.

2. Reasons including critical attributes. In this category were adults who accepted Yossi's reasoning, but also related to critical attributes of a triangle. For example, regarding the equilateral triangle one adult wrote, "It's acceptable, but you can be more accurate with an explanation and say it has three

sides.” For the square, another adult wrote, “It’s a correct identification and a nice answer. Four sides and four angles.” For the pizza “triangle”, “I would accept it because it really looks like a triangle. But a triangle cannot have a rounded side.”

3. Affective references. Several adults remarked that Yossi’s reasoning was creative, nice, or ‘cool’. For example, regarding the scalene triangle, an adult wrote, “Every answer would be acceptable in my eyes because it’s difficult for children to explain abstract ideas.” This adult is expressing her reluctance to say to a child that he is wrong, empathizing with a child’s difficulties.

4. Visual references. In this category, were adults who accepted Yossi’s explanations because those explanations related to how the figure looked. As one adult wrote for the equilateral triangle, “That is how you identify a triangle, with your eyes.” For the square, one adult wrote, “He’s right. He remembers the shape of the figure.” For the rounded-corners triangle, “It’s acceptable because the general shape really looks like a triangle.”

The frequencies of each of the above four categories are given in Table 5. Frequencies are given only for those who themselves identified the figure correctly, correctly evaluated Yossi’s identification, and accepted his reasoning. As can be seen, most adults accepted Yossi’s explanations because they believed the reasoning was age appropriate. Interestingly, for the square, most adults accepted Yossi’s reasoning and did not feel the need to explain their acceptance. This coincides with the finding in Table 4 that Yossi’s reasoning for the square was the most accepted out of all the figures.

**Table 5: Frequency (%) of adults’ explanations which accepted Yossi’s reasons**

	<b>Examples (triangles)</b>		Non-Examples (not triangles)		
	Intuitive	Non-Intuitive	Intuitive	Non-Intuitive	
The figures	Equilateral triangle (N=71)	Scalene triangle (N=43)	Square (N=118)	Rounded-corner “triangle” (N=41)	Pizza “triangle” (N=61)
Yossi’s reasons	<i>We see it is a triangle</i>	<i>It is too long</i>	<i>It is a square</i>	<i>It has 3 lines</i>	<i>It is like a slice of pizza</i>
Age-related	43 (61)	32 (74)	25 (21)	20 (49)	32 (52)
Critical attributes	2 (3)	0 (0)	13 (11)	6 (15)	3 (5)
Emotional	2 (3)	1 (2)	7 (6)	0 (0)	4 (7)
Visual	5 (7)	2 (5)	12 (10)	6 (15)	13 (21)
No explanation	19 (27)	8 (19)	61 (52)	9 (22)	9 (15)

## Discussion

Our first aim was to investigate if adults could identify examples and nonexamples of triangles and their reasons for their identifications. As shown in the results, nearly all adults recognized intuitive and non-intuitive examples and nonexamples of a triangle. Like the preschool teachers in Tsamir et al.'s (2015) study, and the kindergarten children in Tsamir et al.'s (2008) study, the one figure which caused confusion was the rounded-corner "triangle." If one takes in the whole shape of this figure, it is very similar to the prototypical triangle. Furthermore, this figure is identical to the hazard or warning symbol, such as the sign used on the road, which in Hebrew is called a "warning triangle." Thus, adults may either not notice the rounded corners, or even disregard them.

Interestingly, not all adults who correctly identified a figure, correctly assessed Yossi's identification. From adults' reasons of their evaluation, we learned that it was not necessarily that they incorrectly evaluated Yossi's identification, but that they would accept this incorrect identification, because the child is young. This is a bit worrisome. If a child says that the pizza "triangle" is a triangle, and an adult responds, "nice," or "yes," then that adult is reinforcing visual reasoning that takes in the whole shape (van Hiele & van Hiel, 1958). Both the rounded-corner "triangle" and the pizza "triangle" are shapes seen in an everyday context when children are not necessarily in school. These shapes may also appear in children's books and puzzles found in the home. Thus, one implication of this study is the necessity to raise adults' awareness to the difference between mathematical language and everyday language. An adult might say, "We call this slice of pizza a triangle, but feel the crust, it is rounded, so it is not really a triangle." Such a response recognizes the visual similarity but also points out a critical difference. According to research, with guidance, children as young as five are capable of discerning attributes of a figure, even if they do not necessarily make connections between those attributes (Sarama & Clements, 2009; van Hiele & van Hiel, 1958).

The square is a different case. Recall that the vast majority of participants related to critical attributes when explaining why it is not a triangle. Yet, when it came to Yossi, whose reasoning did not include references to critical attributes, adults were willing to accept his explanation, and of the 114 adults who accepted Yossi's reasoning, only 13 adults wrote that they would add to this explanation a reference to critical attributes. So, should we accept Yossi's explanation? What if Yossi claims that a square is not a rectangle because it is a square? Regarding the equilateral triangle, adults accepted Yossi's reasoning that he sees it is a triangle. But it is this type of reasoning that led Yossi to say that the scalene triangle is not a triangle. Findings also indicated that most adults accepted Yossi's reasoning because of his young age. This infers that if the child was older, they would not accept the insufficient reasoning. This is curious and requires additional research. Do adults believe that young children are incapable of recognizing critical attributes?

Another implication of this study is the need for preschool teachers to know what might be happening at home or after school. As noted previously, the home environment can increase the educational benefits of attending preschool and kindergarten (Anders et al., 2012), but not if the home is contradicting what is taught in school. Shapes are all around us, and if the parents are calling a certain shape a triangle when the teacher is saying it is not a triangle, confusion can follow. Because the participants in this study were not preschool teachers, they might not be fully aware of their role as children's first mathematics teachers. Thus, another implication of the study is for mathematics educators to raise parents' awareness of children's abilities to learn about shapes and reason about



shapes. Workshops for interested parents and other caregivers could also offer suggestions for playful activities that have the potential to increase children's recognition of critical and non-critical attributes. With the recent experience of a world-wide pandemic, when many children were not in school, we realize more than ever the importance of increasing the potential of adults to promote early mathematics.

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