How do we make an E? Participation theory and theoretical consideration of supportive activities on block play in the family

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Ergi Acar Bayraktar[1] and Birgit Brandt[1]

[1] Technical University Chemnitz, Germany; ergi.acar-bayraktar@zlb.tu-chemnitz.de

Family play situations offer a high potential for the mathematical thinking development of the child. Playing with building blocks can particularly promote spatial geometric ideas. Following, support activities in a block play situation with wooden blocks are analysed from a participatory perspective. It is shown that the supportive activities are realized jointly by the adult and the child. Furthermore, a theoretical extension of the production design is carried out on the basis of the empirical analysis, which is based on theories of interaction and participation.

Keywords: Family, block play, supportive activities, participation theory.

Introduction

Family is central to the child's mathematical development (Acar Bayraktar, 2017; Anderson, 1997; Bjorklund et al., 2004). In concrete play situations, the children can experience different learning situations with mathematical content and thus their mathematical skills and competences are encouraged. Playing with building blocks stimulates children's physical, social, emotional and cognitive growth (Bullock, 1992, p. 16). When children are involved with their family in math play activities with building blocks, they can be supported in their actions by competent family members (Acar Bayraktar, 2017).

In this paper we focus on the questions how family members carry out supportive activities during block play situation (Acar Bayraktar, 2017) and how they realize different statuses of the production design at the same time (Acar Bayraktar & Brandt, 2023). For the empirical analysis, which is based on theories of interaction and participation, we chose a scene of a mother with her daughter from the data collection of the project erStMaL-FaSt (Acar Bayraktar, 2017). Based on Acar Bayraktar (2017), it is reconstructed how the family members realize different statuses/functions of the production design while carrying out supportive activities.

Theoretical Perspective

The interaction theory on mathematics learning processes

Sfard (2008) speaks of “learning-as-participation” and points out that learning arises in specific forms of discourse (ibid., p. 91); the learning process is characterized by an increasingly autonomous participation in mathematical discourse, which she defines as follows: “Mathematical discourse is made special by two main factors: first, by its exceptional reliance on symbolic artifacts as its communication-mediating tools, and second, by the particular meta-rules that regulate this type of communication.” (Sfard, 2001, p. 13). From the perspective of the interaction theory, “learning on an interactional level is understood as participatory progress and learning is often considered to be a form of participation in specific social practices.” (Krummheuer, 2011, p.81).

Production design (Participation theory)

In order to be able to adequately describe learning in classroom interaction processes based on the concept above mentioned, Krummheuer and Brandt (2001) develop a participation model that makes it possible to record different forms of more receptive or more active participation. Based on the idea of decomposing the speaking role, a speaker is initially responsible for the acoustic realization. In addition, a speaker can assume further responsibilities, namely for the syntactic structure with a specific choice of words and form (formulation function) and/or for the content-related (semantic) contribution to the negotiation of taken-as-shared meaning (content function) (Krummheuer, 2011, p. 85). Four different functions can be distinguished according to the degree of participation: If a speaker is responsible in both components (syntactically and semantically) for his or her utterance, then the speaker is called an "author" (Krummheuer, 2011, p.85). If a speaker takes the identical formulation of the parts of a preceding utterance and with them attempts to express his own new idea, the speaker is called a "ghostee" (ibid). If a speaker takes the idea of a preceding utterance and then tries to express this idea with his/her own new formulation, this speaker is called a "spokesman" (ibid). A speaker who neither takes responsibility for nor has originality in the semantic content of their utterances, is call a "relayer" of an utterance (ibid). In addition to these basic statuses, Brandt (2017) distinguishes activities in interaction via their modes of representation. She defines actions as concrete material actions when the individual makes his or her own contributions to negotiation processes "through the concrete handling of the materials" (ibid, p. 109 ff.). She speaks of the modality focus of the activities in the interaction and categorizes them as: **(1) verbally**: expressed by phonetics; **(2) gestural:** expressed with corresponding hand movements that can be interpreted as gestures; **(3) concrete:** involved in concrete material actions (ibid., p. 121).

Taking this into account, the participants can fulfil the function of formulation and content, while their actions are realized verbally, gesturally or concretely. An author is therefore free to choose a mode for the realization or to combine them as desired. In the status of the spokesman, it is possible to change the mode of realization. In order to bring an already expressed idea into a new **formulation**, at least one mode must be varied; others can be adopted or omitted. However, in the status of the ghostee it is initially possible for all implementation modes to be adopted. However, it is also conceivable that the traductive content (spokesman) is made recognizable by an additional gesture while the verbal expression remains the same. One form of implementation must therefore remain constant here, while others can be added or varied. This extended model is shown in Table 1.

Table 1: Extended production design (Acar Bayraktar & Brandt, 2023, p.27)

	Speaking person				
	realisation	formulation			content
		verbally	concrete	gestural	
author	+	+	+	+	+
spokesmann	+	+ / -	+ / -	+ / -	-
ghostee	+	- / +	- / +	- / +	+
relayer	+	-	-	-	-

Taking familial interaction research into account, the question arises in which form of the participation of children and family members takes place and in particular which functions are realized with regard to the production design. So far, there have not been any really systematic studies

in which responsibility and originality in helping to shape an interaction process in a family context have been realized. The present work represents a contribution to the processing of this specific research need.

The interactionist perspective on learning processes in the family context

Family play situations offer potential for stimulating and expanding the child's mathematical thinking development. In this sense, families accompany their children in their early mathematical learning processes and offer a parallel support system to the institutions of kindergarten. In social-constructivist theory, the term support system describes the constitutive contribution of the social system to the cognitive development of the individual (for more see Acar Bayraktar, 2017). In this sense interaction processes are characterized in different ways by each family but learning situations are accomplished no matter which language and cultural attitudes are used by the families (Acar Bayraktar, 2017, p. 430–442). Each family member adapts themselves to such a pattern of interaction, by making appropriate changes in their definition of the situation to a commonly shared interpretation (ibid). They then use this patterned process of negotiation as their support system, and the changes in their definitions of the situation are an expression of their cognitive achievement of adaptation (ibid).

Supportive activities of family members during block play

While playing with wooden blocks, all family members establish supportive activities together and mutually supportive effects occur. The types of negotiation realized in this way, which can be understood as supportive activities, can be assigned to the following categories (Acar Bayraktar, 2017, p. 436; cf. Bjorklund et al., 2004):

(1) Prompt: Assistance in finding an answer, without specifying a strategy (e.g., “How should you put the blocks on top of it?”); **(2) Prompt after error:** Request for checking and improvement (e.g., “Are you sure?”, “Be careful!”); **(3) Affirmation:** Approving statement (e.g., “That's exactly how it should be!”); (**4) Disaffirmation:** Rejection (e.g., “No, not like that!”); (**5) Provide solution:** If the answer is given, a solution is offered; (**6) Motivation:** Positive or negative motivation through encouragement, criticism, etc. (e.g., “You're doing great!”; “You lost.”); **(7) Conclusion:** Give an overview of the current situation (e.g., “It's Dad's turn!”, “You have won 3 points!”); (**8) Demonstration:** Demonstrating an idea at the action level with material as a model for imitation, representation of the individual steps (e.g., the replica of the character takes place at the action level); (**9) Instruction:** Suggested strategy (e.g., “Maybe you should put the block in the middle, right?”); (**10) Re-representation:** Repetitions of ideas and structures verbally and non-verbally, also with a change of representation mode (e.g., the number of wooden blocks is represented with the fingers).

The analyzes of familial situations in a block play have shown that family members sometimes integrate supportive activities directly into the building activities without accompanying these enactive actions verbally, e.g. during demonstrations or re-representations. Following Latour's actor-network-theory (Latour, 2005), Fetzer and Tiedemann (2015) speak of object actors in connection with material-related actions, with the help of which learners can clarify their ideas in shared learning situations. With objects as actors, abstract ideas can thus be discussed in a collective exchange. Regarding this background, in this paper we would like to show in more detail in which function and

in which modality mode family members are involved in the interaction process while realizing supportive activities.

Analysis of interaction and participation

Research methodology of this paper fulfills the analysis of interaction and participation, which are actually based on theories of interaction and participation mentioned above (Krummheuer & Brandt, 2001). In other words, this paper lays out the execution of the analysis of interaction and participation, which means that while the processes of interaction between a family member and child are reconstructed, interpretations of their actions within the processes of interaction are also reflected. For further reading and/or steps taken in analysis of interaction and participation, see Krummheuer and Brandt (2001) or Acar Bayraktar (2017).

The chosen scene from the project erStMaL-FaSt

erStMaL-FaSt is part of the longitudinal study erStMaL, which was carried out in 12 day-care centers in Frankfurt am Main (Germany) with 178 participating children (Acar Bayraktar, 2017, p. 23–55). The data consists of video-recording and transcripts of the recordings. Four play situations were developed for the family study and the participating families were videotaped once a year while they played a game together (ibid). A total of nine families, partially with a bilingual background, took part (ibid). This article focuses on an unusual block play situation of a bilingual family, in which adult person has insufficient spatial-geometrical abilities for negotiating with the child about geometrical meanings (Acar Bayraktar, 2017, p. 317–320). In the selected play, the participants are supposed to build a three-dimensional version of a geometrical shape depicted on a card with wooden blocks. The play situation offers family members different opportunities to perform the relations between 2D and 3D representations.

The chosen scene: “How do we make an E?”

Aleyna (4;8 years) and her mother take part in the chosen scene, which was collected in the first observation phase. They have already played three rounds and open the fourth round. It is the mother's turn; she looks at the card and begins to build the 3D representation shown on the card. She asks Aleyna for help. Both take wooden blocks out of the box. After various counting activities and addition tasks, they piled blocks on top of each other. A body is created in which five blocks are stacked on top of each other. In Figure 1, you can see created body. This corpus can be interpreted as the base of the figure on the card and is subsequently referred to as the “fiver base”.

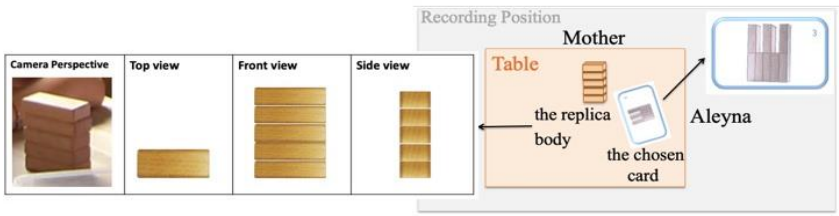

Figure 1: The recording position, the chosen card and the replica body (“fiver base”)

In the sequence analyzed here, the card is in the reading direction in front of Aleyna, the mother sees the card rotated by 90°. The mother identifies a part of the figure as an E and asks her daughter “How

do we make an E?". Thereby she offers Aleyna a figurative interpretation of the configuration from her own perspective. Thereafter, unlike the chosen card, they built a corpus, which actually is not identical with the figure on the chosen card. In Figure 2, their rebuilding actions are shown. Although they try to rebuild the corpus (see Figure 2), their attempts lead to a confrontation between the three-dimensional body and the two-dimensional figure on the playing card. In particular, in the course of the interaction by rebuilding process, there are different ideas about how to position K2, which is the second taken block from the pile on the table to build an "E" during mother-daughter interaction (see Figure 2). This difference of opinion is expressed by a mutual shifting of the position of this block <232-248> (see Figure 2), however, neither addressing verbally the relationship between position of building blocks and the replica body ("fiver base"), nor relating the building block K2 explicitly with a block drawn on the chosen card.

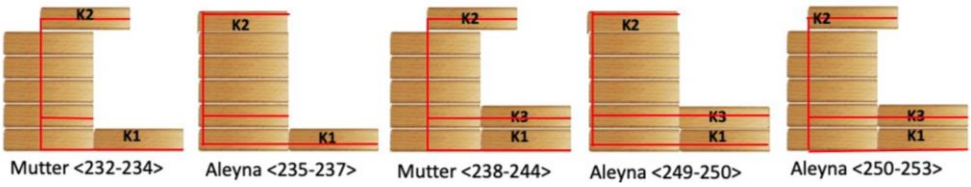

Figure 2: Changes in the position of block K2 (front view) over the course of the interaction and the end version of built corpus (Final replica<250-253>)

In the end of the chosen scene, Aleyna notices that the corpus they recreated is not spatially consistent with the figure on the card. She says "it must be like this - you did it wrong" by pointing at the part of 3 blocks, namely the "E", on the chosen card with her right index finger <271-274>. Although Aleyna at the age of 4.8 is only at the beginning of a process of her spatial geometric development, she can establish the relationship between a three-dimensional corpus and its projections and rotations (Newcombe & Huttenlocher, 2003, p. 179 ff.). Despite her mother's incorrect explanations, she can recognize the problem, shows the right place of the three wooden blocks on the card and verbalizes this, whereby her verbal expression, however, without gestures, does not reveal any concrete spatial-positional relationship. She benefits from her mother's framing in such a way that she can recognize the incorrectness of the built corpus and also gain spatial geometrical experiences. As a result, she also develops increasing autonomy and steps in several times to correct the joint construction work. In doing so, she contradicts her mother and produces her own ideas especially in dealing with the spatial geometrical analysis. Overall, they negotiate critically and realize the collective argumentation process. It turns out that both mother and daughter act primarily in the status of the author, which also expresses Aleyna's autonomous participation status. It also becomes clear that the negotiation process about spatial geometrical issues is realized not only verbally but also gestural and through concrete material actions.

Theoretical extension

At play, Aleyna and her mother carry out collective solutions in order to achieve a correct replica. Such a process also enables them to adopt different participation structures and statuses. The argumentative exchange is not only implemented *verbally*, but also with *gestures* and especially with *concrete realizations*. While the production design in Krummheuer and Brandt (2001) considers only "acoustic realizations" for the "loudspeaker function", it becomes clear that non-verbal, gestural and

concrete realizations play an important role in collective processing (p.121). In the following, we apply this modification to the block play situation presented below.

Production design of supportive activities in the mother-child interaction of the family Ak

With modified production design, the following will take a closer look at the supportive activities of Aleyna and her mother. In doing so, we concentrate on the supportive activities that combine at least two realization modes.

From the point of view of production design, mother and daughter carry out their participation in all activities presented here in the status of author, except for Aleyna, who *prompts after an error* in the status of the spokesman. The positioning of K2 is disputed until the end, Aleyna pursues her own spatial geometric idea, which she repeatedly introduces and contradicts her mother. This also makes it clear here that the controversial moments, namely the positioning of the block K2, are negotiated via several modes of representation. In addition, spatial geometric ideas during building blocks are *demonstrated* by *concrete material actions* in accompanying *verbal statements*. In particular, each player confirms their actions with the word "so" and rejects the position of blocks with the word "not so" <228-246>; those are only becoming comprehensible in the context of the *concrete action*. Similarly, *instructions* on spatial geometrical strategies are expressed *verbally context-bound* and by pointing at a block on the card or in the replica with the right index finger. The spatial geometric idea is to be located in *gestures*. In contrast to the *concrete material action,* the gesture gives an *instruction* of a possible concrete building activity, but without performing it as in *demonstration* (for imitation). Both *demonstration and instruction* may be associated with a supportive activity *prompt after error*. It is here explicitly formulated verbally but at the same time either an alternative solution offer is *demonstrated* by the *concrete action* or an *instruction* of an alternative way for the correction is given by the *gesture*. Through realizing multimodal supportive activities, the mother initiates the end of the play turn by saying "correeccctt!!" while clapping hands <247>. Her *affirmation* is pronounced very stretched out and together with the gesture it gives an emotional effect that marks the support activity *motivation* <248>. Thereon Aleyna pushes the block K2 <249-253>, so that at the end of this *concrete material action* it returns to the configuration generated by the mother as before (see Figure 2). This can be seen as an alternative solution, that Aleyna realizes a *demonstration* while implicitly *providing a solution* to fix the incorrectness of the built corpus. Thereupon she says "but, but, something is missing" <253-257>. Therewith Aleyna reconstructs the spatial geometric idea of her mother, in conjunction with her subsequent verbal utterance she distances herself from this configuration, so that the unit of action fulfills a character of spokesman in which can be seen as a *providing solution*. However, this (initially) purely non-verbal activity is ignored by the mother through the supportive activity *conclusion*. During the verbal announcement of the next play turn, the mother takes chosen card and cleaning blocks away <258-260>. Thereby she ends the current play situation while the accompanying *concrete action* lends a certain finality to this *conclusion*. *Gesture* and *concrete action* thus reinforce the effectiveness of verbal realizations. Finally, Aleyna also makes herself "heard" again with a multimodal request. She says "Just give me your card" by stretching out her hand in front of her mother <261-265> and after taking the card from her mother's hand adds "it must be like this, you did it wrong" by pointing at the part with 3 blocks on the card <266-273>. It is precisely the

expansive gestures that seem to lend emphasis to her request and thus enable her to carry out another supportive activity with her final evaluation of the move, which is also perceived by the mother.

Conclusion

The collective recreation of the interaction process with building blocks in this chosen scene is affected by numerous supportive activities into different states of production design of both mother and child. In multimodal activities, the different forms of implementation take on different functions with regard to the supportive aspect. While they are realizing one supportive activity, they also perform one formulation style of the production design. Through realizing multimodal supportive activities, the negotiation process leads them to the emergence of another supportive activity at the same time (cf. Bjorklund et al., 2004). Regarding the analysis above this result is given in Table 2.

Table 2: Occurrence of supportive activities during negotiation of shared-meanings

realized supportive activity	accompanying Formulation		realized supportive activity	accompanying Formulation		accomplished supportive activity
demonstration	*gesture*	+	*instruction*	*verbal statements*	→	*providing solution*
demonstration	*concrete material actions*	+	*instruction*	*verbal statements*	→	*providing solution*
demonstration	*concrete material actions*	+	*instruction*	*verbal statements*	→	*prompt after error*
demonstration	*concrete material actions*	+	*instruction*	*gesture + verbal*	→	*providing solution*
affirmation	*verbal*	+	*motivation*	*gesture*	→	*conclusion*
providing solution	*gesture+ verbal*	+	*disaffirmation*	*verbal*	→	*conclusion*

Thereupon, it became clear that spatial geometric ideas particularly in this play situation are more likely to be expressed through gestures and concrete material actions. Language in this area remains context-bound and assumes accompanying functions. In a collective interaction process, object actors – wooden blocks – are in connection with material-related actions and help family members clarify their ideas in shared meanings (cf. Fetzer & Tiedemann, 2015). Furthermore, gestures and concrete material actions reinforce the effectiveness of verbal realizations (cf. Krummheuer & Brandt, 2001). Both participants act in the status of the author, while they contradict each other and produce their own ideas while realizing different supportive activities with different formulations.

In conclusion, it can be stated that in a dyadic mother-child interaction, both participants support each other and carry out various supportive activities autonomously in the course of the interaction sequence. Thus, the supportive activities are realized not only by the mother as an adult – as supposed expert – but also by Aleyna as a child. For their supportive activities in the collective reproduction process, they use all three modes of representation for the formulation function. This shows that the different forms of modes can take on quite different (content-related) functions from a supportive perspective.

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