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An outcome of debating infinitesimals via primary sources: access to the practice of developing mathematics

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Building on the work that engagement with primary historical sources provides students with rich opportunities to reach new understandings about mathematics, we present the case of one university student's (Renaë) experience with a debate activity in a history of calculus course. Our analysis, informed by the theory of mathematical transgressions in which transgressive actions propel students beyond a barrier to outcomes of new upper levels of mathematical understanding revealed several notable outcomes. For this paper, we focus on one outcome of participating in the debate activity: access to practitioner practice, grounded in Lave and Wenger's situated theory of learning. The dimension of access gained by Renaë was insight that mathematics, specifically the concept of infinitesimals, is developed via critique and argumentation. We conjecture that transgressing via the debate activity may foster rich connections from the historical to modern perspective.

Keywords: Situated theory of learning, primary sources, history of calculus, mathematical transgressions, debate activity.

Introduction

In the domain of history of mathematics in mathematics education, learning mathematics via primary sources has been considered a worthwhile practice for decades (Clark et al., 2019; Jankvist, 2014). Although there are multiple approaches for the use of primary sources in the teaching and learning of mathematics, particularly at the upper secondary and tertiary levels (Jahnke et al., 2002), strong empirical evidence on how such approaches actually impact student learning of mathematics is predominantly found in small scale studies. In recent years, research studies drawing upon theoretical frameworks have sought to illuminate complex learning and engagement processes in tertiary contexts in which primary sources have been employed (see Chorlay et al. (2022) for several examples). Among these efforts, the Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS) project has produced curriculum materials in the form of approximately 100 Primary Source Projects (PSPs) for use in tertiary (undergraduate) mathematics classrooms. (See Clark et al. (2022) for an overview of the TRIUMPHS project, including brief descriptions of some of the research resulting from it.) In the research described below, we extend one aspect of the TRIUMPHS research efforts, informed by situated learning theory and mathematical transgressions, for which we present a case for another use of primary sources—motivated by students' experiences with PSPs, but for which we build a debate activity for students' engagement and learning.

Theoretical background

The theoretical framing which informs our analysis stems from psychological theorist Jozef Koziielecki's (1986) "A Transgressive Model of Man." Koziielecki's model has recently entered the mathematics education research literature due to its versatility of interpretation at varying grain-sizes

and its applicability across levels of mathematics learning (e.g., Barnett et al., 2020; Lakoma, 2016; Pieronkiewicz, 2020). At the core, the premise of *transgressionism* is simple: a person moves beyond the barriers of their past achievements to an outcome by engaging in a purposeful action. Transgressionism was further situated in mathematics education with Semadeni's (2015) notion of a *mathematical transgression*. In this sense, an individual crosses "a previously non-traversable limit of own mathematical knowledge or of a previous barrier of deep-rooted convictions" (p. 27). Additionally, the crossing of a barrier must be from a "specific lower level to a new specific upper level" (p. 27). Within this interpretation of transgressionism, there are three discernable components: the *barrier* precluding mathematical achievement, the *transgressive action* propelling one beyond the barrier, and the *outcome* representing the new upper level reached via transgressing through (crossing) the barrier. While we recognize investigation into all three components of mathematical transgressions (barriers, transgressive actions, and outcomes) is crucial in viewing the panorama of a mathematics learning experience, for this paper, we focus only on outcomes.

Outcomes as new upper levels of transgressive actions have appeared in various forms. An outcome can be, say, when children are able to reason with "abstract numbers," without reference to concrete objects, as opposed to their un-transgressed state of only being able to use "concrete numbers," representing physical objects (Semadeni, 2015). New upper levels can be more individualized like those reached after Lakoma's (2016) notion of *affective transgression in learning mathematics*. For instance, teachers may initiate a "corrective emotional experience" by encouraging students to reflect on negative beliefs regarding their mathematical ability. By doing so, students may transgress to a new upper level whereby mathematics is perceived as possible and achievable as opposed to impossible and unachievable in their previous state (Lakoma, 2016).

Similar to the above examples, instructor-initiated activities may serve as a vehicle to propel students across a barrier to a new upper level with regard to their mathematics learning. Specifically, we view participation in the debate activity (described as part of the Methods section) as a transgressive action itself. Through the lens of situated theory of learning (Lave & Wenger, 1991), participation in the practice of a community enables newcomers (i.e., mathematics students entering the mathematical community) to become more knowledgeable about what it means to be a member of the related community of practice (Lave, 2019). Activities in which students participate as part of [entering] the community of practice may not be restricted to mere "classroom activities" that end with filling out a sheet of paper; rather, the activities may involve a full absorption into the culture of practice (Lave & Wenger, 1991).

Therefore, we choose to focus on one construct that resulted from "both absorbing and being absorbed in" (Lave & Wenger, 1991, p. 95) the mathematical community via participation in the debate activity: *access to practitioner practice*. For newcomers to become full participants of a particular community of practice, it is essential that they have access to not only a general body of activities of old-timers in that mathematical community, but also opportunities to participate in related activities. Subsequently, newcomers may cultivate an understanding of the activities of the practitioners, develop their practice, as well as gain a better understanding of themselves with respect to the activities of the community. Thus, we use *access to practitioner practice* to capture a new level of

understanding in which students feel they have insight in the activities of old-timers like that of mathematical innovation and the associated thought processes.

Research goal

In our work, we have used a lens of mathematical transgressions to capture students' experiences with learning undergraduate mathematics via PSPs (Watford, 2022). More recently, we have expanded our focus from PSPs to more broadly considering students' engagement with primary historical sources embedded as part of other learning activities. As part of our previous work, we investigated the potential of transgressive actions to foster students' participation in the practice of mathematicians (Clark et al., 2022). Furthermore, we connect these with the situated theory of learning (Lave & Wenger, 1991), attending to the outcome of access and—for the purposes of this paper—participation as a transgressive action. For the research presented here, we highlight the situated learning outcome that resulted from students' engagement in the debate activity, particularly because it sheds light on a new dimension of the outcome which we did not observe in our previous work. The research question guiding our investigation is:

How might students gain access to mathematical practice by participating in a debate activity incorporating the use of primary historical sources?

Methods

Context, participants, and data sources

The context for the research we share in this paper is a one-semester course entitled *Calculus and Its History*, which has Calculus II¹ as the prerequisite and is designated as an upper division writing course for undergraduate mathematics majors (that is, those studying mathematics at university but who do not intend to teach K–12 mathematics) at Gauss University². As part of the university's requirement for an upper division writing course, *Calculus and Its History* includes several writing assignments that comprise a substantial portion of the course. Such assignments require students to read and reflect on peer-reviewed academic sources related to the historical development of the calculus. Course assignments also incorporate “doing mathematics” in addition to writing about mathematics, such as written responses to PSPs. The course culminates with a portfolio capstone project designed to demonstrate students' inquiry into the historical and mathematical development of a calculus-related concept. In the portfolio, students research a selected topic, present and analyze historical mathematical problems³ related to the topic, and reflect on how their own understanding has been informed by their research.

Students typically take the course in either their third or fourth year of study (in a four-year program) and they may be majoring in one of four offerings: pure mathematics, applied and computational

¹ In the US, Calculus II is the second semester of a three-semester calculus sequence (single-variable, focus on integration).

² All names relevant to the research context used in this paper are pseudonyms.

³ At least one primary historical source must be used.

mathematics, biomathematics, or actuarial science. The student of focus for this paper, Renae, was of special interest because she was majoring in applied and computational mathematics. From our previous course implementations and due to the background experience of applied and computational mathematics students, we expected the discourse related to ideas more closely associated with pure mathematics and its development to be less readily accessible than discourse pertaining to applications. Therefore, we purposefully selected Renae from consenting participants for this first exploration resulting from our research.

As participants in the research study, students agreed to be interviewed, as well as providing access to the work they completed as the normal part of the course. Thus, there were several data sources, including interviews (recordings and transcripts), student work samples, field notes, and recordings of class sessions. For this paper, we focus on analysis of the post-debate interviews. The protocol for the interviews was informed by our field notes and analysis of the recording of the debate activity. Interview questions included prompts about challenges in preparing for or participating in the debate activity as well as if students now have or can do something that they did not have or could do prior to engaging in the debate activity. In our analysis of the interviews, we identified outcomes evidenced by students remarking on how they are *now* able to do something or have some new state of being after engaging in a particular (transgressive) action. We also identified possible barriers precluding realization of the outcome and transgressive actions that carried students beyond the barriers to the outcome. Outcomes were then subsumed under more general codes, such as *identity*, *access to practitioner practice*, and *learning*, informed by situated theory of learning (STL). For the purpose of this paper, we focus only on the case of Renae and her outcome of having *access to practitioner practice*.

The debate activity: Berkeley, Leibniz, and Newton

The focus of our contribution is based on our analysis of two 75-minute class sessions that took place in Week 11 of the Fall 2022 course, in which students were placed in three groups representing Leibniz, Newton, and Berkeley, with the goal to participate in a debate of sorts. That is, students were tasked with using a variety of resources to construct an argument to defend their group's assigned person's conceptions of infinitesimals, as well as to prepare for possible response to the arguments of the other two historical figures. On day 1, students completed (with their group) a sequence of tasks⁴ to remind them of the use of infinitesimals in both Leibniz' (1646–1716) and Newton's (1642–1726) work previously studied in the course, as well as to provide a short excerpt of Berkeley's (1685–1753) commentary on these conceptions, which he characterized as “ghosts of departed quantities” (Berkeley, 1734, Section XXXV). After completing the pre-debate tasks, students continued to work for the remainder of the class session to draft their argument for the debate. To assist with preparing their arguments, each group was given a “Debating Infinitesimals” handout, which contained historical excerpts (e.g., excerpts taken from Katz & Sherry, 2013; Kitcher, 1973) pertinent to Berkeley, Leibniz, or Newton. By the end of day 1, each group provided an outline of the

⁴ These tasks were taken from the Primary Source Project (PSP), *An Introduction to a Rigorous Definition of Derivative* (Ruch, 2017).

argument they constructed. On day 2, the debate took place in the following way. Groups were given 10 minutes at the start of class to regroup to finalize their main argument details. Each group (Newton, Leibniz, Berkeley) presented their main argument for approximately five minutes. Then, an open discussion took place (approximately 30 minutes), in which the groups rebutted and responded to the points in each other's main argument. Finally, the entire class engaged in discussion (approximately 20 minutes), prompted by the instructor focusing on a point raised during the debate. She asked the question, "Do you, as emerging mathematicians, think that elegance has a place in a discussion around mathematics? (And why or why not?)" And: "How would you apply the notion of elegance to the mathematics (or person) that you focused on for this debate activity?"

In light of transgressions and STL lenses we bring to the study of learning mathematics via primary historical sources, we consider students' participation in the debate activity that we designed and implemented as serving as the transgressive action we identify as *participation in the practice of mathematicians*.

Outcomes of the debate activity

Within a transgressions lens, outcomes are situated as a new level or state of having or being in which students reported having some insight, knowledge, or perspective which was not present or was less developed before having engaged in the debate. The outcome we present here is associated with a general theme of *access to practitioner practice*, informed by Lave and Wenger's (1991) situated theory of learning. Our focus for this paper is to highlight the experience of Renae (positioned as Leibniz) and to showcase this outcome which resulted from transgressing via participating in the debate activity. At the time she took the course, Renae was a third-year university student, majoring in applied and computational mathematics.

Access to practitioner practice

In our previous research, in which we drew upon the combined lenses of STL and transgressions, we coded different dimensions of our outcome of interest (access to practitioner practice). For example, in data collected as part of the TRIUMPHS project, we found one dimension of the outcome in that students remarked on having "insight" into expert mathematical thought, particularly about the process of mathematical innovation. Students also reported being "in-tuned" with historical mathematicians and their thought processes. However, data from our previous work did not provide evidence that students saw mathematical innovation as a product of peer-critique.

Renae expressed how, by engaging in the debate, she believed she gained access to a rather particular aspect of mathematical practice—and one which we consider yet another dimension of access to practitioner practice: developing mathematics through critique and argumentation. Moreover, Renae noted that she gained a new perspective on mathematical practice in which she now sees how mathematical argumentation and critique is essential to develop mathematical concepts, particularly infinitesimals. In the following excerpt, Renae admitted that before the debate, she was completely unaware that critics of mathematics (e.g., Berkeley) existed:

I didn't know Berkeley existed before the debate. So now I can talk about Berkeley and about how he hated Leibniz and Newton, and all of their stuff. I don't know why, but as I was going through

the course, it didn't occur to me that there would be people out there that would be like, 'why are you doing this?' You know, like, 'why are you talking about differentials? Why are you talking about areas under the curve? Why are you talking about tangents? What's the point of this? This is useless.'

Renaë described her previous "lower level" (Semadeni, 2015) specifically with relation to the debate. We conjecture this previous state was bounded by a limited exposure to expert mathematical practices such as criticizing mathematical claims or substantiating claims to colleagues. By Renaë's own acknowledgement, being exposed to the history of mathematics throughout much of the semester, did not transgress her to this reported outcome. Instead, it was through participating in the debate in which Renaë was tasked with promoting and defending mathematical concepts and ideas that she was able to transgress the barrier. Renaë also expanded on her previous thoughts that mathematics was impervious to criticism. She described how, prior to the debate activity, she thought resistance to new concepts was isolated to other domains, such as science:

I didn't think that that would be something related to calculus. You know, I thought that was more of, like a science thing. But I feel like, of course, people wouldn't want math to expand for some reason, I'm not sure why, but... we like the simplicity of things, you know. And we're scared of change.

In general, Renaë gained access to the expert practice of developing the mathematics discipline via critique and argumentation. However, Renaë further remarked how engaging in the debate allowed her access to expert practice with regard to the development of the specific concept of infinitesimals:

I can kind of see why people debated infinitesimals in mathematics, but then I can also see from a different viewpoint why people tried so hard to defend it to be in mathematics because it's an odd concept, but I feel like it's really important... Even if I fully don't understand it now, I feel like I have a better understanding. I can talk about it better, especially from the viewpoint of life.

It is remarkable that Renaë was able to see first-hand and participate in the development of mathematics by arguing for the concept of infinitesimals. The practice of not only recognizing the existence of differing mathematical viewpoints but also developing the need to understand multiple mathematical perspectives resulted from Renaë's participation in the debate activity. We conjecture that Renaë gained access to an aspect of mathematical practice that many university students may not experience in their mathematical coursework. Because of her participation in the debate activity, i.e., the lively exchange of differing mathematical perspectives and the task of defending them, Renaë now sees new value in substantiating and defending mathematical arguments as essential for developing the mathematics discipline.

Identifying a new dimension of access to practitioner practice

Throughout the course, students remarked on historical notions of rigor in many investigations focused on the development of the calculus. The prevailing theme of juxtaposing mathematical rigor of today with the rigor as the calculus was being developed appeared as a central theme in the debate activity as well with students challenging historical standards of rigor and making connections with their perception of elegance (as in, "can a proof be elegant?") and in the present-day mathematics.

With more traditional mathematics lectures in which students merely listen to an instructor, students are often not afforded the opportunity of substantiating their claims with varying degrees of mathematical rigor. In the debate, students were able to step out of their present state and into historical times, being forced to justify arguments which may not be valid by today's standards. Moreover, while doing so, students are also challenged to engage in important sense-making, which is required for connecting the historical content/sources with the modern perspectives they have developed while studying disciplinary mathematics. Given the potential of the debate activity at the center of the research we shared in this paper, we are currently preparing a manuscript of the online periodical, *Convergence*. In that contribution, we share the source materials and the design of the two-day debate activity, as well as the details of implementation of the activity and discussion for the potential for enhancing participation, sense-making, and access to practices of the discipline.

In our previous work (Clark et al., 2022), the transgressive action of *participation in the practice of mathematicians* culminated in outcomes of *access to practitioner practice*; however, we were only aware of the practices mentioned by students who engaged with PSPs such as proving, conjecturing, and discovering. Analysis of students' engagement in the debate activity revealed a new dimension of mathematical practice which may not have been readily accessible by students completing PSPs. Specifically, the mathematical practice of questioning mathematical claims and having a critical eye towards others' claims is afforded by participating in the debate task because it places the responsibility of upholding standards of rigor (and therefore causes some students to examine how standards of rigor differ between modern mathematics and historical mathematics) and holding others accountable for their mathematical assertions. We conjecture that because PSPs are scaffolded with a description of the historical context, excerpts from historical sources, guided questions (tasks), and author commentary, the more open-ended debate activity affords opportunities of access to other mathematical practices generated in an organic fashion.

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