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The mathematical film: Dynamic geometry in the post-War era

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From the 1950s, the didactic potential of teaching aids was a main topic of interest within the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM). In particular, mathematical films that illustrated geometric properties in a dynamic way or explored new topics in geometry education were considered promising didactic tools. In this paper, we reconstruct the former role that mathematical films played in the teaching and learning of geometry. Therefore, we discuss the ambitions and analyze the output of two successful realizers of mathematical films in the 1950s, namely Jean-Louis Nicolet (Switzerland) and Trevor Fletcher (UK), and the role Caleb Gattegno played in valorizing their output.

Keywords: CIEAEM, Jean-Louis Nicolet, mathematical film, teaching aids, Trevor Fletcher.

Introduction

In April 1950, Caleb Gattegno, an Egyptian-born mathematician and psychologist, brought together an international group of experts in mathematics, psychology, and education, including some experienced mathematics teachers, in Debden (UK). The aim was to rethink mathematics education from a broad psycho-pedagogical perspective. The meeting in Debden, followed by two similar meetings in 1951, one in Keerbergen (Belgium) and one in Herzberg (Switzerland), led to the official founding of the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM)/International Commission for the Study and Improvement of Mathematics Teaching in La Rochette par Melun (France) in April 1952. Although the CIEAEM was (and remains to this day) an informal organization of mathematics educators and teachers, in the 1950s and 1960s it played an important role in the innovation of mathematics education in Europe.

In the 1950s, two lines of thinking dominated the debates within the CIEAEM: (1) the relationship between mathematical and mental structures, and (2) the role of materials (or aids) for teaching mathematics, respectively leading to the books *L'Enseignement des Mathématiques* [The Teaching of Mathematics] (Piaget et al., 1955) and *Le Matériel Pour l'Enseignement des Mathématiques* [Materials for the Teaching of Mathematics] (Gattegno et al., 1958). The first line gave a strong impetus to the “modern mathematics” movement in Europe, say basing mathematics education on set theory and mathematical structures. The second line of thinking seems contrary to the first, but actually was not. In the view of CIEAEM, learning materials were not concrete or “real” materials, but rather “(pre-)structured materials”, stylized and carefully designed objects, models of or shortcuts to mathematical structures that constituted the actual objective of teaching. Examples include the Cuisenaire rods, cardboard models, light projections, Meccano constructions, geoboards and electrical circuits; followed in the 1960s and early 1970s by Georges Papy's Venn and arrow diagrams, and minicomputer or Zoltán Dienes' logic blocks and multi-base arithmetic blocks.

The mathematical film, a (relatively) new genre in the post-War era, should be seen in the search for teaching aids to improve mathematics teaching in the 1950s. Mathematics teachers and engineers from several countries, including the UK, US, France, and Switzerland, produced or were involved in producing mathematical films. Gattegno et al. (1958) provide a list of 33 recommended films of

that period on pages 207–208 and in four chapters of this book, the topic is discussed in more detail: Nicolet’s *Intuition mathématique et dessins animés* [Mathematical intuition and animated drawings] pages 63–80, Fletcher’s *Les problèmes du film mathématique* [The problems of the mathematical film] pages 81–99, Motard’s *Les techniques du dessin animé mathématique* [The techniques of animated mathematics drawing] pages 101–103, and Gattegno’s *L’enseignement par le film mathématique* [The education through the mathematical film] pages 105–117.

In this paper, we revisit the mathematical film of the 1950s. What were the views and principles of filmmakers of that era? What were their objectives, and how did they hope to achieve these? Finally, what was Gattegno’s role? To answer these questions, we focus on the films of two pacesetters of the mathematical film: the Swiss Jean-Louis Nicolet (1903–1968) and the Brit Trevor Fletcher (1922–2018). First, we investigate written sources of these protagonists to explicate their views, which are then illustrated by examining some films. Second, we sketch how contemporaries, amongst which Gattegno, perceived, used and promoted these mathematical films. Finally, where possible, we discuss related more recent research and literature.

Nicolet’s animated geometry

Starting from the early 1940s, Jean-Louis Nicolet, a teacher of mathematics at the *École Nouvelle* in Lausanne, Switzerland, was a pioneer in the creation of mathematical films. Nicolet was also very prolific: He realized no less than 22 mathematical films for teaching geometry, both for elementary and more advanced levels. The films by Nicolet, entitled “animated geometry”, were short (2 to 5 minutes), silent hand-animated films presenting simple geometrical situations, but stimulating imagination and provoking reflection (Gattegno, 2007). When someone asked why he made films, Nicolet answered: “Parce qu’on voit la vérité” [Because one sees the truth¹] (Gattegno & Fletcher, 1968, p. 17). According to Nicolet, in the teaching of geometry, intuition should precede logic and proof. In Nicolet (1954–1955), he concisely formulated his position as follows: “Logic proves, but does not convince, intuition convinces, but doesn’t prove” (p. 24). In Gattegno et al. (1958), Nicolet repeats a similar statement and complements it by pointing out that intuition and logic are indispensable one to the other. Moreover, there is one domain in mathematics where the collaboration of intuition and logic is strongly seen, namely geometry. According to Nicolet a mathematical film is a means of evoking images that represent variations of a theorem. However, in Nicolet’s view, a mathematical film must not be a paraphrase of a mathematical proof (Gattegno et al., 1958). “It must awaken, subconsciously, the aesthetical sense and guide it towards the acquisition of a certainty leading to the desire for demonstration” (Geoff, 2007, p. 3).

Initially, Nicolet’s mathematical films received little (international) recognition. When Gattegno attended a screening of his films in Lausanne (in 1947 or 1948) (Gattegno & Fletcher, 1968), this changed to some extent. Gattegno became enthusiastic; he regarded Nicolet’s films as powerful pedagogical situations “since geometry could be filmed every time it involved transformation of some class of figures, the essence of geometry was imagery” (Gattegno & Fletcher, 1968, p. 16). Gattegno proposed promoting Nicolet’s films internationally—a role similar to the one he took up for the Cuisenaire rods (see, e.g., De Bock, 2020). For his part, Nicolet became engaged in the activities of

¹ All translations were made by the authors.

the CIEAEM: In Bernet and Jaquet (1998) he is listed as a “founding member²” and, according to the same source, he was still an “active member” in 1957. Not surprisingly, then, the promotion of Nicolet’s films was mainly through the members and publications of the CIEAEM—for example, by Gattegno et al. (1958).

Discussions of Nicolet’s films

Nicolet’s films are available on DVD (Nicolet, 2007)³, which comes with a leaflet containing a short description of the content of every film and suggestions on how to use it in the classroom or additional questions that can be considered with the pupils. We discuss the film with the title *contact point of parallel tangents to circles*. The animation starts by showing a fixed point and a family of circles of a fixed radius through that point (first image in Figure 1). The family of circles is evoked by rotating the circle around the fixed point. Then a straight line appears which is translated until it is tangent to the circle (second image in Figure 1). The circle and the straight line now continue their movement while they stay tangent (third image in Figure 1). Finally, the locus of the common point of the straight line and the circle is drawn while they continue their movement (fourth image in Figure 1).

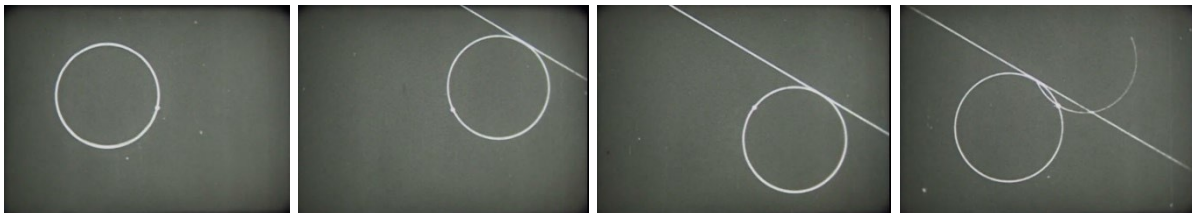


Figure 1: Four screenshots of Nicolet’s film titled *contact point of parallel tangents to circles*

Remark that there are no letters or symbols in Nicolet’s films and neither is there any text, which makes the films usable regardless the language and the taste of the teacher for specific notation. “Not a word has been read, nothing has been heard; there is nothing but the vision of the moving and living figures” as Nicolet phrases it in Gattegno et al. (1958, p. 77), while Gattegno (2007) calls the absence of text, numbers and symbols “an additional silence” (p. 43). In Gattegno et al. (1958), Fletcher refers to his own and Nicolet’s silent films as truly internationally usable. As soon as symbols or text are inserted, a mathematical film loses its value for some groups of people and it becomes merely regionally usable. According to Fletcher (1954–1955) words are not needed in a mathematical film, the drawings should speak for themselves. Lucien Motard writes in Gattegno et al. (1958) that the black background is not only good for projecting, it has also some familiarity for pupils who are used to the blackboard and the white lines of the drawings are eyecatchers in a darkened room.

In Fletcher (1954–1955) examples of several mathematical films are found: films on differential equations made by Fairthorne and Salt in 1936–1937 in the UK, films on methods present in school books made in France by Cantegrel, Jacquemard and Motard, and Nicolet’s films. Fletcher admires Nicolet’s films because these show mathematical ideas that can be changed freely by the imagination.

² Gattegno used the term “founding member” also as an honorary title; it did not necessarily imply presence at the founding meeting in La Rochette in 1952.

³ See also the YouTube ATM channel for two of Nicolet’s films: <https://www.youtube.com/watch?v=4D3ttrC2Wdk> and <https://www.youtube.com/watch?v=gum9kvxR9K8>

In Gattegno et al. (1958), Fletcher uses a film by Nicolet as an excellent example of how the logical structure of the film is flawless; without using a single letter, the construction of an ellipse is shown.

After an exposition on mathematical intuition, Nicolet discusses two of his films in detail in Gattegno et al. (1958). According to Nicolet, many pupils need to reserve so much of their attention to a correct reading of a theorem that there is not much left over to evoke the matching images, let alone possible variations of those images. A way to harmonize the attention, the evocation of infinite analogous figures and the imagination is the animated drawing. After having seen the animation “The intuition has played its role: the logic may enter the scene” (Gattegno et al., 1958, p. 77).

Also Gattegno discusses two of Nicolet’s films in Gattegno et al. (1958), including a detailed description of the mathematical content. Moreover, he explains how he uses the films in class: sometimes a film is shown several times in a row without any explanation or without asking questions but at the end, sometimes he starts by asking questions from the beginning of the film or sometimes he starts with a preliminary class and shows the film only later. Additionally, Gattegno describes how one should pass to the stage of replacing the images by numbers (for example the radius of the circle) and to the construction with the ruler and the compass or to a generalization of the property shown in the film. Gattegno notes that the pupils in the classes where the films were used were ordinary pupils and there was always a large audience of observing teachers. At the end he remarks that not all authors of mathematical films proceed similarly and that not all mathematics teachers expect the same. Some films illustrate, some revise a large section of a program, some are mathematical documentaries on all aspects of a particular problem. The latter may even lead to the discovery of new theorems and Gattegno refers to Fletcher for this type of mathematical films.

A brief discussion of the mathematical content of all 22 Nicolet films, including drawings and suggestions on how to use the films in class, was presented by Leujes and Vredenduin (1959–1960). The authors refer to the manual that comes with the films and from that we can see the similarities with the leaflet that still accompanies the DVD (Nicolet, 2007). Leujes and Vredenduin (1959–1960) stress the absence of audio, text and symbols in the films, offering the individual mathematics teachers freedom in how to use them. The authors started by noting that it was difficult to obtain the films in the Netherlands. Moreover, they mention the price for the films, a large investment for a school or potentially a group of schools⁴, but according to Leujes and Vredenduin (1959–1960) worth the money.

From Monteiro (2018), we learn that Nicolet’s films were also known and used in Portugal during the 1960s, more specific in the mathematics teacher training school “Pedro Nunes” in Lisbon. We refer Portuguese speaking readers to Monteiro (2018) and to the references therein for more detailed discussions and information.

Gattegno recreated seven of Nicolet’s films in 1979 (Tahta, 1981). The computer animated series explores various families of circles in the plane. According to Tahta (1981), the subtle timing of the

⁴ Motard writes in Gattegno et al. (1958) that mathematical films are expensive, due to the large amount of work that is needed to produce these. He expresses his hope that mathematics teachers, convinced of the pedagogical value of the films, form a solid international organization to produce and distribute mathematical films.

original films is preserved in the remakes. For a more detailed description of the differences between (the content of) the old and the new series of films, we refer to Tahta (1981).

Nicolet's films as a contemporary source of inspiration

Gattegno (2007) already suggested in 1951 that Nicolet's films make useful tools for research. Recently, in the field of research on mathematics education, some researchers seem to have (re)discovered Nicolet's films. In Markle (2022), Nicolet's films on the construction of the conic sections are used in an experiment on how students in a mathematics classroom use their hands as a landing site during spatial reasoning and visualization. For each of the conic sections (circles, ellipses, parabolas and hyperbolas), the test subjects (undergraduate pre-service teacher candidates) had to visualize what the investigator was reading from a script, their response (through gesture, speech or drawing) had to be submitted using photos or short videos and then they watched a corresponding animation of Nicolet.

Sinclair (2016) showed Gattegno's remake of a Nicolet film to both pre-service and in-service teachers. She hereby proceeds as Gattegno recommended: first the film was watched, without giving any instructions. Then a short discussion follows, after which the film is watched again. This is repeated and in between test subjects need to use hands/arms or drawings to show what they saw.

The silent video⁵ in the experiment described in Kristinsdóttir (2021), made with GeoGebra, is based on Nicolet's films. The test subjects (pupils) could watch the video as many times as they wanted, after which they had to record a voice-over to the video, explaining what is shown in the video. See Kristinsdóttir (2021) and the references therein for more examples and details.

Noël and Midavaine (2011) discuss two of Nicolet's films in detail and situate the topics in the current school program. They complement this with questions one can raise after seeing them as well as with objectives one could aim to achieve by showing the films to pupils. Noël and Midavaine (2011) conclude that computer animations without any doubt have a better visual quality than do Nicolet's and Fletcher's films. But when it comes to the pedagogical quality, the films of Nicolet are still models that can serve as inspiration for making contemporary versions using a computer program.

Fletcher's films aimed at creating "new" mathematics

Trevor Fletcher, a lecturer at Sir John Cass College in London, UK, was an early member of ATAM and was quickly co-opted in the Association's steering committee. When *Mathematics Teaching* launched in November 1955, he became the first editor. He was elected President of the Association in 1960 and instigated the first book by members of the Association, *Some Lessons in Mathematics* (Fletcher, 1964). Eventually he became Staff Inspector for Mathematics i.e., the chief of all Her Majesty's Inspectors (ATM, 2004). Fletcher had an extensive international network, including the CIEAEM of which he was an "active member" in the late 1950s (Bernet & Jaquet, 1998).

Fletcher's mathematical films were generally longer, more sophisticated and intended for older students than Nicolet's. A main theme was the illustration of properties of geometric curves, such as

⁵ <https://www.geogebra.org/m/BfRqGSKq>

epicycloids and hypocycloids, topics that were not part of secondary mathematics programs in the UK at the time. Fletcher showed himself quite ambitious about the potentials of the new medium:

It is not only a matter of producing films that illustrate the mathematics as it is taught today. By making films, we will create new mathematics, and if the films are of a sufficient quality, they will change the mathematics that will be taught in the future. (Fletcher, 1954–1955, p. 29)

Apparently, his films could make an indelible impression. Leo Rogers, an undergraduate student of Fletcher in the late 1950s, vividly remembered a screening of the cardioid⁶ nearly fifty years before (Rogers, 2008). For him, it was a first encounter with “living mathematics” which he described as:

A way of working where the visual experience brought the algebra off the page and into life in the most unexpected ways. Repeating a showing enabled us to see new and unexpected things each time and to realize how powerful the film medium was, not only for displaying geometrical objects and loci, but also for following the development of an idea – a kind of visual “What would happen if?” (Rogers, 2008, p. 43)

Fletcher (1954–1955) writes about how much more demanding making a mathematical film is compared to making one on history or on geography for example. The reason for this is that the mathematical diagrams require a special type of animated drawings. As a consequence, according to Fletcher, mathematical films are rarely used in teaching mathematics, despite the fact that the silent mathematical films are universal since they can be understood regardless of language. Because of this little experience with mathematical films in teaching, Fletcher (1954–1955) esteems it needed that one starts experimenting with the existing material and that feedback on the adequate duration of the film, speed of the movements, ... is gathered. This feedback could also lead to setting up conventions in the world of mathematical films, similar to conventions in for example notation in textbooks. Fletcher ends by writing that one should not underestimate the film, it is the most important visual aid that is introduced in the teaching of mathematics since the blackboard.

Discussions of Fletcher’s films



Figure 2: Five screenshots of Fletcher’s film titled *the four point conics*

There are three films by Fletcher available on the YouTube ATM channel, *the Simpson line*⁷, the previously mentioned *the cardioid* and *the four point conics*⁸. Due to the much higher complexity of Fletcher’s films compared to Nicolet’s, we limit ourselves to give a flavor of Fletcher’s films in *the four point conics*. The film is introduced by drawing the spectators attention to the fact that, although two different points determine a unique straight line and three non-collinear points determine a unique

⁶ See, e.g., <https://www.youtube.com/watch?v=OX12aOh9brw>

⁷ <https://www.youtube.com/watch?v=uKCCEk8tsQY>

⁸ <https://www.youtube.com/watch?v=Jef2Ax4EcL4>

circle, four non-collinear different points determine a family of conics (first two images in Figure 2). Following some properties of conic sections are presented (three last images in Figure 2).

Fletcher's contribution to Gattegno et al. (1958) focusses mainly on technical issues and this both on the techniques used (drawings versus models, white on black versus black on white, ...) and on the choices about content (add symbols and text or not, what to show, ...). Fletcher gives in this respect remarks on his own films *the Simpson line* and *the cardioid*. On the former he writes that certain wrong techniques were used, something he only recognizes now he knows more about this. It would also have been better if it would have contained less ideas. The final scene, which is very complicated, contains so many questions of which Fletcher was unaware when he started making this film and which he would never have discovered if he had not made this film. His second film *the cardioid* is the result of these lessons.

A modest reference to Fletcher's films by his contemporaries is the one in Monteiro (2018), where it is said that Fletcher's films were not be used in Portuguese secondary schools because his films aim at higher-level students. We did not find any discussion of Fletcher's films by other authors, nor any recent use of these films.

Conclusion

Due to the different scopes and target audiences of Nicolet's and Fletcher's films, combined with a different number of sources on their films, we have treated these two protagonists in an asymmetric way. However, from our exposition it is clear that Nicolet and Fletcher had very similar views on mathematical films, on how these should be set up and on how these could be used. Since Gattegno's interests were situated mainly in primary and secondary school, he was only involved in promoting Nicolet's films. Despite Gattegno's efforts, Nicolet's films were never able to match the international success of the Cuisenaire rods (Gattegno & Fletcher, 1968). One possible reason is that from the late 1950s onward, reformers of mathematics education focused primarily on developing programs for "modern mathematics", which gave little or no attention to classical Euclidean geometry, the inspiration for Nicolet's films.

Although curriculum reforms and new technologies made Nicolet's films outdated, their contribution to the pedagogy of mathematics and research on mathematics education lingers on, be it modestly and only rarely. Due to their higher complexity and older target audience, Fletcher's films were not only less discussed in their times, we also have not found any recent research mentioning these films.

Our research has no direct implications for existing research, however, a conclusion for contemporary mathematics education seems to be the possibility to use animations not merely for illustrating mathematics, but also for activating pupils. Further research and classroom experiments following this line could be of interest.

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