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On the role of psychologists in the international New Math movement

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The birth of the New Math movement in Europe is closely linked to the work of Bourbaki in pure mathematics, but it was the Swiss psychologist Jean Piaget who provided the psychological justification by assuming a link between the structures of modern mathematics and the structures of children's cognitive development. In the US, Jerome Bruner played an analogous role by providing Edward G. Begle's School Mathematics Study Group a psychological justification for extending its action to the primary level. However, not all psychologists of the 1950s and 1960s had unconditional faith in the New Math reform. Already at Royaumont, William D. Wall, a leading British psychologist and educationalist, reproached the reformers for basing their actions on unverified opinion rather than careful psycho-pedagogical research. Methodological criticism was also voiced by the Belgian school psychologist Fernand Hotyat, but the reformers claimed they had no alternative.

Keywords: Fernand Hotyat, Jean Piaget, Jerome Bruner, New Math, William Douglas Wall.

Introduction

Nowadays, there is considerable scientific interest internationally in the New Math (modern mathematics) movement of the 1960s and early 1970s (see, e.g., Menghini, 2020). However, publications looking at New Math from an international (comparative) perspective are still rare. A pioneer in that respect is Bob Moon who investigated the New Math as a global phenomenon, adding analyses of the movement in five Western European nations as case studies (Moon, 1986). More recent examples are Kilpatrick (2012), Vanpaemel and De Bock (2019), and De Bock (2023).

Views and approaches to mathematics education typically have a connection with more general psychological theories of knowing, learning, and teaching. New Math, say basing mathematics education on set theory and structures, is associated with Jean Piaget's theory of cognitive development. However, it is less known that Piaget also personally played a major role in the birth of the (European) modern mathematics movement (see, e.g., Félix, 1986). Probably more than any other movement in mathematics education, New Math was driven by (academic) mathematicians, but also a number of psychologists took an active role in its development and implementation by either supporting or criticizing the mathematicians' approach. To date, this aspect of the New Math movement has been underexposed and international comparative research on it is lacking. This study is a first step toward filling this gap in the literature. Our research question is: what arguments did then-important psychologists internationally provide to justify or denounce New Math? To that end, we outline the role of four key psychologists related, in some way or another, to the New Math movement and analyze their points of view as reflected in their writings: Jean Piaget (Switzerland), Jerome Bruner (US), William Douglas Wall (UK), and Fernand Hotyat (Belgium).

Jean Piaget at the origin of the European modern mathematics movement

In April 1950, Caleb Gattegno, an Egyptian-born mathematician and psychologist, brought together an international group of experts in mathematics, psychology, and education in Debden (UK). Gattegno also had invited Bärbel Inhelder, Piaget's closest collaborator, to give an introductory talk on the psychological foundations of the mathematical curricula for secondary schools, but she did not

show up (Félix, 1986). Although the number of participants in Debden was limited, the range of competences had to allow for “a thorough reconsideration of the whole problem of the child and mathematics” (Gattegno, 1947, p. 220). This meeting, followed by two similar meetings in 1951, one in Keerbergen (Belgium) and one in Herzberg (Switzerland), led to the official founding of the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM)/International Commission for the Study and Improvement of Mathematics Teaching in La Rochette par Melun (France) in April 1952. In La Rochette, several big names in the fields of mathematics and psychology participated, including Jean Dieudonné, Gustave Choquet, André Lichnerowicz, Jean Piaget, and Ferdinand Gonseth. The meeting was not an accidental encounter between top mathematicians and top psychologists/epistemologists of that time; on the contrary, the meeting was carefully prepared by Gattegno who, as a holder of doctorates in mathematics and in psychology, was familiar with the recent developments in these fields, in this case particularly the work of the Bourbakists and Piaget—as for the latter, he had collaborated on the translation into English of *La Génèse du Nombre chez l'Enfant* [The Child's Conception of Number] (Piaget, 1952). Gattegno also had determined the theme of the meeting in La Rochette (“mathematical and mental structures”), and already at the Debden meeting, he would have announced: “I will have the Bourbakists, I will have Piaget, I will have Gonseth¹” (Félix, 1986, p. 26).

The debate in La Rochette was initiated by Dieudonné who outlined Bourbaki's points of view, paying particular attention to the origin and essence of structures in modern mathematical science (Félix, 1986). He argued that structures are by no means artificial constructs that appear out of nowhere; they are “explicitations” of ideas that were already present in the work of great mathematicians of the past, implicitly and under different guises, but which were not yet recognized as such. Their role in mathematical research was clarified by Lichnerowicz: “A structure is a tool that we search for in the arsenal we have at our disposal. It is not at this stage that it is created” (quoted from Félix, 2005, p. 82). Choquet and Lichnerowicz also testified about how they actually used structures in their research (Félix, 1986, 2005).

During the discussion, Dieudonné emphasized that the Bourbakists were not dealing with questions of a philosophical or metaphysical level, only common logic was used (Félix, 1986). Associations of mathematical structures with extra-mathematical constructs were not suggested by Bourbaki, but they were established by Piaget, who explicitly related Bourbaki's structures to the mental operations through which a child interacts with the world (Piaget, 1955)². More specifically, Piaget identified the fundamental structures and stages of early mathematical thinking, as revealed by (his) psychological research, with the mother structures in the work of Bourbaki:

Now, it is of the highest interest to ascertain that, if we retrace to its roots the psychological development of the arithmetic and geometric operations of the child, and in particular the logical operations which constitute its necessary preconditions, we find, at every stage, a fundamental tendency to organize wholes or systems, outside of which the elements have no meaning or even existence, and then a partitioning of these general systems according to three kinds of properties

¹ All translations were made by the authors.

² Piaget (1955) is the summary of his presentation in La Rochette (1952), as mentioned in a footnote to that book chapter.

which precisely correspond to those of algebraic structures, order structures, and topological structures. (Piaget, 1955, pp. 14–15)

Piaget's identification of Bourbaki's mother structures with the basic structures of thinking, implying a harmony between the structures of "contemporary" mathematics and the way in which a child constructs mathematical knowledge, had a straightforward pedagogical implication: The learning of mathematics takes place through the mother structures of Bourbaki, the structures with which 20th-century mathematicians had founded and built their science. Correspondingly, Piaget (1955) asserted that "if the building of mathematics is based on 'structures', which moreover correspond to the structures of intelligence, then it is on the gradual organization of these operational structures that the didactics of mathematics must be based" (p. 32). In other words: A model for the science of mathematics was promoted as a model for mathematics education. Bourbaki's reconstruction of mathematics from a limited number of basic structures, linking different branches of this science and underlining its fundamental unity, "supported" by Piaget's theory of cognitive development, would continue to stimulate debate within the CIEAEM community in subsequent years (De Bock, 2023, chapter 3).

The view that the structures or developmental stages of intelligence correspond to the structures of modern mathematics, as well as the resulting implication for mathematics education, will be defended by Piaget for life. When invited to deliver a plenary talk at the second International Congress on Mathematical Education (ICME2) in Exeter in 1972, he stated:

[Cognitive development] involves a spontaneous and gradual construction of elementary logico-mathematical structures and that these "natural" ("natural" in the way one speaks of the "natural" numbers) structures are much closer to those being used in "modern" mathematics than to those being used in traditional mathematics. (Piaget, 1973, p. 79)

Certain characteristics of mathematical structures and operations are homologous with the characteristics of certain mental structures, and consequently, according to Piaget, the central problem of mathematics education "is that of the reciprocal adjustment between the spontaneous operational structures proper to the intelligence and the program or methods relating to the particular branches of mathematics being taught" (Piaget, 1977, p. 702).

In a critical review of the modern mathematics movement, Bernard Charlot, French pedagogue and researcher in educational sciences, states: "modern mathematics appeared as the daughter of Bourbaki and Piaget" (Charlot, 1985, p. 28), and she inherited from them—explicitly or more diffusely—structure, formalism and active pedagogy.

Jerome Bruner and the American New Math movement

The period shortly after 1950 saw a large number of projects aimed at improving the mathematics curriculum in American schools, commonly referred to as New Math (see, e.g., Hayden, 1981; Roberts, 2023; Walmsley, 2003). The launch of Sputnik in October 1957 created the political climate for a significant strengthening of efforts, leading to the creation of the School Mathematics Study Group (SMSG), headed by Edward G. Begle. Thanks to massive funding from the National Science Foundation (NSF), SMSG soon became the largest New Math project in the US, enlisting a nationwide network of authors to write curricular materials for each grade of secondary school.

Soon the elementary level also came into the reformers' sights, but Begle realized that problems associated with designing a curriculum and writing related materials for elementary schools are more psychological rather than mathematical (Phillips, 2015). Begle recognized the need to incorporate psychological understandings about child development and concept-formation into the teaching of arithmetic, but the question was how to do it. From February 12–13, 1959, SMSG succeeded in bringing together a number of psychologists at a conference on elementary school mathematics, but they did not seem to be able to reach a consensus on the nature of children's learning (School Mathematics Study Group, 1959). Marshall H. Stone, mathematician at the University of Chicago and president of the International Commission on Mathematical Instruction (ICMI), concluded:

While psychologists can provide much more information about the unfolding of the child's mind than they could twenty-five or fifty years ago, it seems clear that they are only on the threshold of understanding of the problems of mental development and concept formation. (Stone, 1959, p. 178)

Although psychologists played an important role in this initial conference and they were occasionally consulted, SMSG would follow the lead of its mathematicians (Phillips, 2015).

SMSG could in part justify this decision based on the views of Jerome Bruner, a leading Harvard psychologist. From September 9–18, 1959, the NSF sponsored an important conference on education in the sciences convened by the Education Committee of the National Academy of Sciences in Woods Hole, Massachusetts, and directed by Bruner. At Woods Hole, influential curriculum designers, scientists and mathematicians of the period, including Begle and Stone, met with leading psychologists and pedagogues to discuss the basis for further development of reform projects in mathematics and natural sciences. It is likely that Woods Hole was also the first contact between reform movements in the US and the European reform movement. Indeed, the conference was attended by Inhelder who discussed Piaget's theories on pedagogy and the justification for teaching the structure of a subject in its own logical or axiomatic order rather than its order of historical development (Bruner, 1960).

As a psychologist, Bruner was strongly influenced by "structuralism" and the research of Piaget. In *The Process of Education* (Bruner, 1960), the proceedings of Woods Hole conference, Bruner defined structure vaguely as the relatedness of things and considered the teaching and learning of structure, rather than mastery of facts and techniques, to be at the heart of the classical problem of transfer. He claimed that knowledge could be transferred to other domains if the emphasis of pedagogy is on teaching the structure of knowledge rather than its content. In particular, properly teaching mathematical structures could accelerate intellectual development and facilitate learning in other areas. Bruner viewed Piaget's research as evidence that modern mathematics can be taught at an earlier age than previously attempted because it is consistent with children's cognitive development. In contrast to Piaget and other stage theorists, who assumed developmental stages of relatively fixed length, Bruner was convinced that any subject can be taught effectively in some intellectually honest form to any child at any developmental stage, as long as instruction is organized appropriately (Bruner, 1960). It provided Begle and his team with a psychological justification for going ahead with the New Math reform at the elementary level, without worrying too much about the psychology of learning (Phillips, 2015).

Critiques from a methodological point of view: William Douglas Wall and Fernand Hotyat

From November 23 to December 4, 1959, the Organisation for European Economic Co-operation (OEEC, currently the Organisation for Economic Co-operation and Development or OECD) organized a Seminar on “New Thinking in School Mathematics” at Royaumont, near Paris. The Seminar proved to be a turning point in the European modern mathematics movement. It is generally acknowledged that Royaumont was monopolized by (pure) mathematicians, including Dieudonné and Choquet, members of or strongly influenced by Bourbaki. These mathematicians were primarily focused on the preparation for university studies in mathematics and science of secondary school students in the so-called scientific streams of upper secondary education. However, the official goals of the Seminar, as listed in the proceedings, suggest that the original conception was broader, including “the needed research in mathematical education” (OEEC, 1961, p. 12). Research by Gert Schubring in the OECD archives in Paris revealed a preparatory document to the Seminar entitled “Psychological and Educational Researches into the Teaching of Arithmetic and Mathematics”, authored by William Douglas Wall (1913–2004) (Figure 1) and John B. Biggs (Schubring, 2014), confirming that empirical research in mathematics education was originally on the Seminar’s agenda. Only Wall participated in the Seminar, as the only psychologist and in the role of guest speaker. Wall, a specialist in educational psychology, was the director of the National Foundation for Educational Research in England and Wales (NFER) and chairman of the International Project for the Evaluation of Educational Attainment. Biggs, a mathematics educator, was an Assistant Research Officer at NFER. Their joint paper presented itself as a genuine and innovative research report on the state of empirical research on mathematics education (Schubring, 2014).

Excerpts from Wall’s discussion paper at Royaumont are included in the proceedings of the Seminar (OEEC, 1961). In this paper, Wall outlined some steps that educational research should look forward to, but first he admitted that:

Research into the learning and teaching of arithmetic and mathematics has, on the whole, been short-term, scattered and piecemeal. For the most part, too, it has erred by examining only one or two aspects of the problem at the time—whereas education essentially concerns a complex of interrelated variables whose dynamic interaction is qualitatively and quantitatively different from its parts taken separately or additively. (OEEC, 1961, p. 101)

But, he continued, results of psycho-pedagogical research are often ignored by education administrators, teacher training institutions and the schools themselves. “They prefer the cosy comfort of unverified opinion and rule of thumb to the dangers of objectively verifiable hypotheses” (OEEC, 1961, p. 102). He further called for a coordinated, long-term and multidisciplinary research effort and explained how such research might be conducted. Wall was critical of the suggestions for change made during the Seminar:

The suggestions for change made in this Seminar have a justification in reason and logic perhaps but we have no means of knowing whether they will be as successful or more than the old ways, unless into our reform we build from the outset means of objective study and evaluation of results. (OEEC, 1961, p. 103)

But, as Schubring (2014) has shown, this road was not taken.

A similar call for objective study and verification was voiced by Fernand Hotyat (1896–1975) (Figure 2). Hotyat was a prominent Belgian school psychologist and the director of the *Institut Supérieur de Pédagogie du Hainaut*. Hotyat had special interest in mathematics education: He had published about “Psychological Difficulties of Beginners in Mathematical Reasoning” in *Mathematica & Paedagogia*, the journal of the Belgian Society of Mathematics Teachers (Hotyat, 1955–1956), and in May 1960, he was elected as a board member of that Society.

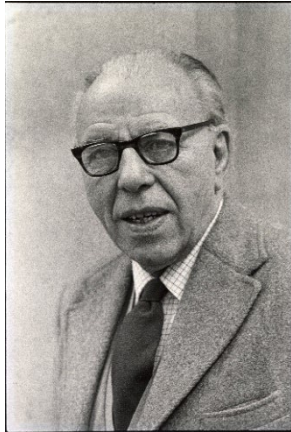


Figure 1: William Douglas Wall
(n.d., UC London © Sally Greenhill)



Figure 2: Fernand Hotyat
(first half of the 1970s)

Hotyat critically commented on the “audacious” experiments carried out by Georges Papy, the uncompromising Belgian New Math reformer (see, e.g., Hotyat, 1960, 1961). In particular, he pointed to the lack of control and of objective measurement, necessary to build a truly experimental science. The personal experiences of the mathematicians led to a form of “experiential” (or experience-based) psychology, but did not contribute to “experimental psychology”. Their proposals remained mere hypotheses as long as they were not supported by objective verification.

Mathematics teachers have proposed many initiatives intended to improve the performance of their teaching. Their practical experience is irreplaceable because it results from the combination of a thorough knowledge of the matter and lessons suggested by their daily contact with students. But their projects are still at the stage of hypotheses as long as they have not undergone the test of objective verification. (Hotyat, 1961, p. 244)

To our knowledge, however, at that time there was little or no direct cooperation between the communities of mathematicians and school psychologists in Belgium, possibly because that latter community could not offer usable alternatives. Jean Nachtergaele, a mathematics teacher at the *Collège Saint-Michel* in Brussels and vice president of the Belgian Society of Mathematics Teachers, formulated the reason for this lack of co-operation as follows:

Efforts of psychologists related to the learning of mathematics have been very conscientious and often very detailed. But their conclusions necessitate significant reservations: The experiences commence from the mathematical knowledge acquired by a child of a certain age, within a particular pedagogy. So their observations give an account of the results of that pedagogy, about that specific body of knowledge.

But to know what a child of 12 can assimilate, for example, about sets and relations, it should have received a pedagogically valid education on these subjects. The psychologist can then take suitable tests, prepared in collaboration with mathematicians, to measure the results obtained.

But until now, studies that meet those requirements are missing. So, for the time being we must be satisfied with empirical evidence, gleaned from the teaching itself, and at the same time hope for a closer collaboration between psychologists and mathematicians. (Nachtergaele, 1964, p. 84)

Conclusion

Psychologists did not occupy a passive role in the New Math reform of the 1960s. Through the study of four cases, we examined the different roles psychologists took. Piaget was partly at the basis of the reform and provided European reformers with a justification for their actions. A similar role was taken by Bruner in the United States. But criticism also came from psychologists. Already at Royaumont, Wall pointed out the lack of objective study and evaluation. Similar criticism was voiced by Belgian school psychologist Hotyat, who referred to the experiments of Papy, but it did not stop the reformers of mathematics education from continuing along the intended path...

This paper represents a first step in the study of the various roles psychologists assumed in the international New Math movement. Their roles, supportive or dismissive, can situate existing and future research on the New Math in a more comprehensive perspective.

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