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# Integrating history of mathematics on the creative process

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*This article intends to discuss the potential of teaching tasks that integrate History of Mathematics for the development of the creative process. Using Sawyer's conceptual model of the creative process (2013), and Boden's perspective of P-creativity (2004), it is concluded that the tasks integrating History of Mathematics that favour the development of the creative process, should be interesting and challenging, involve relevant mathematics, create opportunities to apply and extend knowledge, allow the application of different strategies, with wide possibilities of relations, ideas and solutions.*

*Keywords: Mathematics education, creativity, numbers, history instruction.*

## Introduction

One of the assumptions of the contemporary school curriculum in Portugal and Brazil is the development of critical thinking and creativity in students. Simultaneously, considering the role that the History of Mathematics (HM) may have in promoting the development of transdisciplinary skills and in the apprehension of contents, this article seeks to reflect on the characteristics of tasks to be applied in the mathematics classroom for the development of the creative process.

Under the framework of Fauvel and Maneen (2002), who consider the pedagogical potential of integrating the HM in teaching and learning, we resorted to the conceptual models of Sawyer (2013) on the creative process and P-creativity perspective of Boden (2004) to analyse the pedagogical potential of tasks that integrate the HM for the development of students' creative process.

The analysis showed that HM tasks to develop the creative process should incorporate a multiplicity of characteristics, instigates students' interest, involves relevant mathematics that contributed to students' knowledge, allowing the application of different strategies, with countless possibilities of relationships, ideas and solutions.

## The pedagogical potential of the history of mathematics

Considering the crucial importance of tasks, the question arises about why to use history in teaching and learning mathematics. Mathematics has always been studied and applied in formal and non-formal contexts. Currently, the curriculum is influenced by many factors, from the professional experience of teachers, the expectations of parents and employers and the political and social context. Fauvel and Maneen (2002) refer that it is consensual among teachers all over the world to recognize that knowing history makes a difference and that using the HM as a resource is positive and benefits teaching and learning.

The use of history of mathematics in the teaching and learning of mathematics requires didactical reflection. A crucial area to explore and analyse is the relation between how students achieve an

understanding in mathematics and the historical construction of mathematical thinking. (Fauvel & Maneen, 2002, p. xvii)

The HM plays an important role in mathematics education, revealing a set of curious and content-rich historical details that may awaken students' interest in mathematics. It may also contribute to define a context for the introduction of mathematical concepts, encouraging students to think, and developed creative thinking. The analysis of different points of view existing in historical contexts can be used to develop students' argumentation and their critical and creative spirit (Lakoma, 2002).

It is indisputable that history brings a cultural dimension to knowledge; in this sense Tzanakis and Arcavi (2002) identify five major areas of mathematics teaching that can be supported, developed and improved through the integration of HM in the educational process: i) learning of mathematics; ii) the development of a sense on the nature and activity of mathematics; iii) the increase of teacher's pedagogical knowledge and the enrichment of the pedagogical repertoire; iv) an affective predisposition towards mathematics and v) the recognition of mathematics as a human and cultural enterprise. However, some risks of using history in mathematics education are identified, the most important being the danger of directly transposing the historical event to the present, without a didactic transposition, which allows framing the problem in different times (Menghini, 2002).

Creativity, in official documents from Portugal and Brazil, appears as a competence to be developed. In the case of Portugal, it is one of the key competence areas to be developed in the curriculum and in the document *Profile of Students Leaving Compulsory Schooling* (Martins et al., 2017a). In Brazil, in the *National Common Curricular Base* (ME Brazil, 2018) it appears as one of the general skills of basic education and is present in several parts of the document, covering the various stages of teaching.

Creativity, in the 21st century, has been a topic of interest for several researchers and European Union Institutions. These established broader guidelines and discussions, covering all Member States, and researchers created new models for understanding the process by which creativity occurs, intending that people from different areas can solve problems in a creative way.

## **Creativity and creative process**

A decision illustrating the importance of creativity in our society was the initiative of the General Assembly of the United Nations and the European Parliament regarding the European Year of Creativity (Decision n° 1350/2008/CE, 2009); which establishes April 21 as the World Day of Creativity and Innovation. The creativity is not limited to what is innovative for the individual but integrates the essence of human activity and the way we all act collectively. This drives innovation and develops skills that generate well-being in the individual, and consequently in society, and education is the foundation for promoting this creativity.

Researchers, in order to understand creativity, noticed that it occurs in stages and several models were elaborated to describe the creative process. Among them, Kindersley (1994), establishes principles for the development of creativity with the use of games and mathematical logic, in a discussion about types of reasoning, creative process (3 stages) and group creativity (3 stages). In problems solutions, the author considers Margaret Boden's types of creativity: the psychological, relatively common and

can be reached by everyone, and the historical, which are new ideas in the history of mankind, this is the rarest. Sawyer (2013) developed an eight-step model for the development of creative thinking and explains that they are not necessarily sequential, in her system there are several aspects of Kindersley's principles (1994). Sawyer's model (2013) was used by Mártires (2017) to analyse tasks in the classroom, in order to assist students in their creative results, the data obtained led to the construction and validation of a Creative Process Assessment Scale.

Based on our experience using tasks related to the HM, in teacher training, we highlight some of the activities analysed for this research: Moura and Brito (2009) carried out a didactic sequence, using HM, in the study of logarithms, tangents and the numbering system; Neves (2007) prepared seven episodes of HM content adapted from the 3rd to the 12th grade; Miguel (2005) prepared a historical-pedagogical study of irrational numbers. These tasks were analysed from the perspectives of the pedagogical potential of HM by Tzanakis and Arcavi (2002) and Sawyer's model (2013), but it is possible to discuss their creative potential according to frameworks proposed by other authors. We chose the task of irrational numbers, proposed by Miguel (2005), because it is characterized by multiple resolutions that allow a cultural, social and historical discussion, and, admits in a single activity to develop all stages of the creative process proposed by Sawyer.

Sawyer's (2013) steps to generate a creative solution can be performed in a non-linear way. They are:

- Ask: create and ask good questions to try to define the problem to solve. Knowing how to ask the right question in order to arrive at a creative solution.
- Learn: acquire knowledge relevant to the problem. Constantly learning, practicing, mastering, until become an expert.
- Gather information: Collect a wide range of information that potentially relates to the problem. Structure the search, see the new, out of the ordinary and the unexpected, be open to spotting opportunities to link new information to the existing problem.
- Play: incubation time of ideas. Taking the time away from the project to allow the mind to synthesize and come up with new and better ideas.
- Generate ideas: Always think of many ideas, because some will be useful. They come after a lot of hard work, that promotes potential solutions.
- Combine ideas: Combining ideas in unexpected ways. Many creative ideas result from the combination of already existing ideas or mental concepts.
- Select the best ideas: choose one idea and test it. Try to make improvements to it. The creative person must know how to select, among the many ideas obtained, which one is most favourable to develop.
- Make: externalize ideas, make it real.

We draw on creative ideas from the perspective of P-creativity (Psychological), that states “The psychological sense concerns ideas [...] that are surprising, or perhaps even fundamentally novel, with respect to the *individual mind* which had the idea” (Boden, 2004, p. 43). But, when “The historical sense applies to ideas that are novel with respect to the *whole of human history*,” it is H-

creative. We agree with Boden (2004) “While H-creativity is the most fascinating notion and is what people usually have in mind when they talk about real creativity, P-creativity is the most important one for our purposes.” (Boden, 2004, p. 43). So, in this investigation, what matters is how ideas are reached and not who thought of them first.

For Sawyer (2013), creativity involves daily cognitive processes and creative products result from long periods of work, associated with specific domains. The proposed task, using a HM framework, allows us to identify the creative process in a context of mathematics education. It is necessary to internalize the languages, symbols and conventions related to that particular domain in order to be able to be creative.

## **History of mathematics for the development of the creative process**

The historical dimension encourages thinking about mathematics on an ongoing basis and reflection on practices in the sense of improvement, instead of believing in mathematics as a succession of invariable truths (Barbin et al, 2002). At the same time, this perspective brings to the teacher a new dimension of knowledge, which he can incorporate into his practices, looking at HM as a tool and not as a goal.

The problem that we will propose is not intended to be a reproduction of a problem from the HM, but a recreation of it, thought with didactic and pedagogical purposes. We propose to explore a task that will contribute to the understanding of irrational numbers. According to Jesus and Oliveira (2018), it is necessary to rethink the teaching of irrational numbers in basic education.

In rethinking this teaching, we perceive the development of the ideas of approximation and infinity as fundamental for the student to understand the concepts and operations related to irrationality. These ideas can be (re)treated and (re)discussed even before the formal introduction of irrational numbers content and, we believe, (re)treated and (re)discussed at other times and contexts after such introduction. (Jesus & Oliveira, 2018, p. 339)

Throughout history, there are several reports about the development of the idea of irrationality, from the “shock” of the Pythagoreans with numbers that could not be measured to Dedekind's notion of cuts (Jesus & Oliveira, 2018). Different perspectives enrich approaches to irrational numbers that can be promoted. So, the (re)creation of situations close to those that may have happened historically seems to us a possible and didactically interesting approach (Jesus & Oliveira, 2018).

Our proposal for the task, based on Miguel's (2005) didactic activity, is contextualized with the story of a murder. In the 5th century BC, legends tell us that the philosopher Hippasus of Metapontum, a member of the Pythagorean School, discovered that some numbers could not be expressed as a ratio between two different numbers and therefore were irrational. This discovery contradicts the idea that rational numbers could express any number in the universe, as held by the Pythagorean School, and as a result, Hippasus was killed by the Pythagoreans. In another version, he was lost and died at sea as a punishment from the gods for having divulged the discovery. What is the significance of the discovery of Hippasus? And what are its implications? Shall we investigate?

Several sanskrit texts collectively called the *Sulbasutra*, written by the Vedic Hindus probably between 800 and 500 BC, constitute a geometry textbook detailing the geometry necessary for

designing and constructing of *Agni* (Joseph, 2011). The task of creating a new square that has the area of the sum of two other squares, can be found in several *Sulbasutras* in Indian mathematics, see Joseph 2011 (Crest of the Peacock 3rd ed. pp. 330-331).

The objective of the task is to verify if, given any two squares, it is always possible to construct a single square whose area is equal to the sum of the areas of two given squares.

**Task:** For a pair of squares with side lengths 4 and 3 units, construct, a single square whose area is equal to the area of the given squares.

- Repeat the procedure for two squares with a side measurement of 12 and other with 5 units in length. What about squares 3 and 2 units long? And yet 4 and 2 units long?

- Do you think it is possible to build a single square whose area is equal to the sum of the areas of any two squares? Justify your answer.

Suggestion: for the students, this is an exploratory task; the teacher is the mediator of knowledge.

Analysing the task and its resolution using Sawyer's (2013) framework, the following steps of the creative process are identified:

- Gather information: in order to answer the various questions of the activity, the student needs to look for different information in his investigation, such as: the context of irrational numbers in the Pythagorean school, the characters involved, the history of immeasurable irrational numbers and the necessary mathematical knowledge. Creativity for solutions and answers to questions may arise from information from the past/present context and/or from other information apparently unrelated to the problem situation.
- Learn: after acquiring information, acquiring knowledge related to the problem: mastering the historical context and mathematical content.
- Ask: in possession of knowledge, ask questions in search of solutions, such as: which technology can I use? Can geometric design, geometry and/or algebra help? How?
- Play: at this stage, emphasize to students the importance of releasing their imagination, flowing their thoughts, maturing their ideas.
- Generate ideas: matured ideas make it possible to think of several other ideas and their possible solutions. In the Hipassus problem we present three solutions, none of which is shown in the *Sulbasutras*.
- Combine ideas: creating combinations of several ideas in unforeseen ways, such as using dynamic geometry software and ruler and compass techniques in geometric design.

1. Exclusively algebraic resolution proposal:

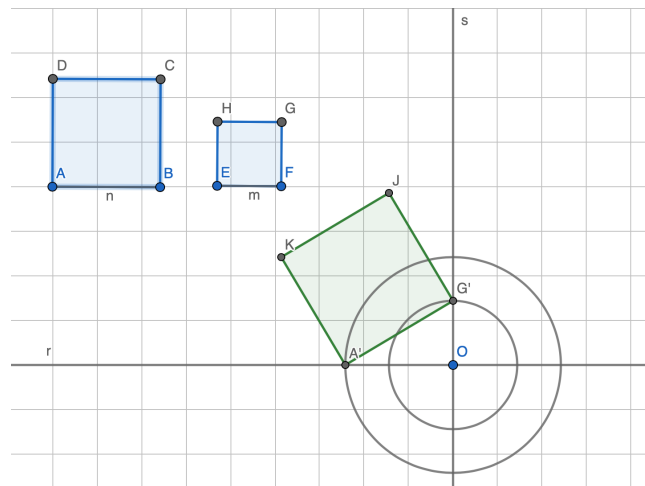
$n$  is the length of the side of square 1, then  $A_{square1} = n \times n = n^2$

$m$  is the length of the side of square 2, then  $A_{square2} = m \times m = m^2$

So, the sum of the areas is given by  $n^2 + m^2 = (\sqrt{n^2 + m^2})^2$ . So, this is the area of a square with side  $\sqrt{n^2 + m^2}$ .

2. Geometric resolution proposed (1):

i) Consider the squares  $[ABCD]$  and  $[EFGH]$ , where  $AB$  has measure  $n$  and the segment  $EF$  has measure  $m$ , and  $n$  is greater than  $m$ ; ii) Start by drawing two lines  $r$  and  $s$ , perpendicular to each other; iii) Mark point  $O$ , intersection of  $r$  and  $s$ ; iv) Trace the segment  $OA'$ , of measure equal to  $n$ , over  $r$ ; v) Draw the segment  $OG'$ , of measure equal to  $m$ , over  $s$ ; vi) Draw the segment  $A'G'$ , which is the measure of the side of the requested square (applying the Pythagoras Theorem). Figure 1 illustrates the construction carried out.



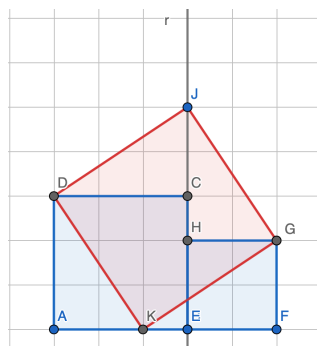
**Figure 1: First proposed geometric resolution**

3. Geometric resolution proposed (2):

This resolution happens by decomposing figures and translating the parts. It is illustrated in Figure 2.

i) Consider the squares  $[ABCD]$  and  $[EFGH]$ , where the length of  $AB$  is  $n$  and the segment  $EF$  has length  $m$ ,  $n$  is greater than  $m$ ; ii) Start by representing the squares juxtaposed by the sides  $[BC]$  and  $[EH]$ ; iii) Draw the line  $r$  that passes through points  $H$  and  $C$ ; iv) Construct the square  $[DJGK]$ ,  $J$  belongs to the line  $r$ ,  $D$  and  $G$  are vertices of the previous squares.;

Figure 2 illustrates the construction: the area of the square  $[DJGK]$  is equal to the sum of the two previous squares.



**Figure 2: Second proposed geometric resolution**

- Select the best ideas: after thinking about several ideas, it is necessary to know how to choose which fits best in solving the problem, in order to justify the answers.

- Make: concretize the choice in a clear and precise mathematical language, in the correct use of its symbols and representations, the same in the written language for the answers to the questions.

## Final considerations

The selected task, based on a problem from the HM, aims to develop creativity, following Sawyer's (2013) creative process model. Since the focus is on creativity, anachronism in possible solutions is irrelevant, such as the algebraic solution presented. Algebra (9th century) did not cohabit with irrational numbers in the historical time of Hipassus (5th century BC). In that period, it was not possible to think of a solution of this nature, but the task allows discussing the mathematical “tools” of each period. Neither of the problem solution is new (H-creativity), but the resolution proposals for the task, highlight the application dimension of Boden's P-creativity, since they appeal to the student's individual creative process, that found a possible solution to the problem.

The initial question of the task “What is the meaning of the discovery of Hippasus?” it is subject to several answers, according to personal, school or social experience, synergistically with knowledge about the HM and mathematics. The student can respond from the importance of irrational numbers to the Pythagorean school, to the present day, to not having any importance since it is a legend and the creation of irrational numbers by Hippasus is questionable. The questions allow to place the discussion and start to form ideas to solve the problem of the area of squares. The multiple resolutions and answers, algebraic and geometric, whether by digital or manual technologies, if presented to the other students in the classroom, allow to encourage discussions that broaden the horizon on mathematics in its diverse aspects: cultural, social and historical.

We consider that within the classroom, not all HM activities have potential for the development of the creative process, such as, for example, problems with direct solutions, immediate application of formulas and verification of mastery of techniques and contents. We consider the tasks developed in the HM could be in the form of a problem, an exploratory question, a project or a survey, and incorporate a multiplicity of characteristics: be interesting and challenging, instigating students' interest, involve relevant mathematics that contributed to students' knowledge, create opportunities to apply and extend knowledge or allow for different strategies, make it possible to track learning and student progress and with ample possibilities of relationships of ideas and solutions as being relevant to creativity. However, the activity does not happen by itself; it is necessary for the teacher to elaborate a possible didactic strategy and adopt a posture of knowledge mediator, according to each class. We believe that the tasks that make up the HM have the potential to enable students to make creative discoveries.

## References

- Barbin, E., Bagni, G. T., Grugnetti, L., Kronfellner, M., Lakoma, E., & Menghini, M. (2002). Integrating history: Research perspectives. In J. Fauvel, & J. Van Maanen (Eds.), *History in Mathematics Education* (pp. 63–90). New ICMI Study Series, Vol 6. Springer. [https://doi.org/10.1007/0-306-47220-1\\_3](https://doi.org/10.1007/0-306-47220-1_3)
- Boden, M. A. (2004). *The Creative Mind: Myths and Mechanisms* (2nd Edition). Routledge.



- Decision n.º 1350/2008/CE of the European Parliament and of the council. Official Journal of the European Union, L 348/115 (2008). <https://eur-lex.europa.eu/eli/dec/2008/1350/oj>.
- Fauvel, J., & Maanen, J. (2002). *History in Mathematics Education*. New ICMI Study Series, vol 6. Springer.
- Joseph, G. G. (2011). *The crest of the peacock. Non-European roots of mathematics* (3<sup>rd</sup> Edition). Princeton University Press.
- Jesus, B. C. D., & Oliveira V. C. A. (2018). Sobre números irracionais e possibilidades para o seu ensino [About irrational numbers and possibilities for teaching them]. *Instrumento: Revista de Estudo e Pesquisa em Educação*, 2, 331–342.
- Kindersley, D. (1994). *Desenvolva sua criatividade* [Develop your creativity]. Editorial Verbo.
- Lakoma, E. (2002). History of mathematics in curricula and schoolbooks: a case study of Poland. In J. Fauvel & J. van Maanen (Eds.), *History in Mathematics Education* (pp. 19–28). New ICMI Study Series, Vol 6. Springer.
- Martins, G. O., Gomes, C. A. S., Brocrado, J. M. L., Pedrosos, J. V., Carrillo, J. L. A., Silva, L. M. U., Encarnação, M. M. G. A., Horta, M. J. V. C., Calçada, M. T. C. S., Nery, R. F. V., & Rodrigues, S. M.C. V. (2017). *Perfil dos alunos à saída da escolaridade obrigatória* [Profile of students leaving compulsory education]. DGE.
- Mártires, M. I. C. C. (2017). *O processo criativo: Construção e validação de uma escala de aferição do processo criativo* [The creative process: Construction and validation of a creative process measurement scale]. [Doctoral's thesis, Universidade de Huelva]. Repositório Institucional da Universidade de Huelva. <https://rabida.uhu.es/dspace/bitstream/handle/10272/15222/O%20processo%20criativo.pdf?sequence=2>
- ME Brazil. (2018). Base nacional comum curricular. Ensino médio [Common national curriculum base. Secondary education]. CNE. <http://basenacionalcomum.mec.gov.br/abase/>
- Menghini, M. (2002). On potentialities, limits and risks. In J. Fauvel & J. van Maanen (Eds.), *History in Mathematics Education* (pp. 86–90). New ICMI Study Series, vol 6. Springer.
- Miguel, A. (2005). Números irracionais: A constituição de um estudo histórico-pedagógico [Irrational numbers: The constitution of a historical-pedagogical study]. In A. J. Brito, A. Miguel, D. L. Carvalho, & I. A. Mendes (Eds.), *História da Matemática em Atividades Didáticas* (pp. 89–156). EDUFRRN.
- Moura, E. C. M., & Brito, A. de J. (2019). History of mathematics in didactic sequences in teacher's initial formation. *Educação: teoria e prática*, 29(62), 609–625.
- Neves, E. F. (2007). *Episódios da história da matemática para o ensino: Apresentações e actividades* [Episodes from the history of maths for teaching: Presentations and activities]. Departamento de Matemática da Faculdade de Ciências, Universidade de Lisboa.
- Sawyer, R. K. (2013). *Zig zag - The surprising path to greater creativity*. Jossey-Bass.
- Tzanakis, C., & Arcavi, A. (2002). Integrating history of mathematics in the classroom: an analytic survey. In J. Fauvel & J. van Maanen (Eds.), *History in Mathematics Education* (pp. 201–241). New ICMI Study Series, Vol 6. Springer.