



**HAL**  
open science

# The impact of content knowledge and pedagogical content knowledge on the development of diagnostic thinking

Jan Philipp Volkmer, Andreas Eichler, Elisabeth Rathgeb-Schnierer

## ► To cite this version:

Jan Philipp Volkmer, Andreas Eichler, Elisabeth Rathgeb-Schnierer. The impact of content knowledge and pedagogical content knowledge on the development of diagnostic thinking. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04421649

**HAL Id: hal-04421649**

**<https://hal.science/hal-04421649v1>**

Submitted on 27 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# The impact of content knowledge and pedagogical content knowledge on the development of diagnostic thinking

Jan Philipp Volkmer, Andreas Eichler and Elisabeth Rathgeb-Schnierer

University of Kassel, Germany; [jan.volkmer@mathematik.uni-kassel.de](mailto:jan.volkmer@mathematik.uni-kassel.de)

*A major issue in the field within teachers' professional development research is diagnostic competence of teachers. Past research has shown that teachers' diagnostic competence can be fostered already in the university part of teacher education. In this research, the influence of prospective teachers' professional knowledge on the development of diagnostic competence has been little researched. Therefore, the presented study examines the impact of two specific areas of professional knowledge (content knowledge and pedagogical content knowledge) on the development of diagnostic competence. The results indicate that both knowledge areas have an impact on the development of at least one part of the diagnostic competence.*

*Keywords: Diagnosis, knowledge base for teaching, professional development, competence.*

## Introduction

Assessment is one of the central tasks of teachers and therefore a much-considered field of research (e.g. Wirth et al., 2022). Situations in which assessment takes place can be called diagnostic situations (Loibl et al., 2020). The ability of teachers to deal with diagnostic situations is often called diagnostic competence (Binder et al., 2018). Diagnostic competence can already be developed in the university education of prospective teachers (e.g. Eichler et al., 2023). A framework which allows to describe diagnostic competence in different diagnostic situations can be found in Loibl et al. (2020) (Fig. 1). The core of this framework is formed by the so-called diagnostic thinking. In the framework, diagnostic thinking is influenced on the one hand by the characteristics of the diagnostic situation, but also by the characteristics of the diagnosing person. Knowledge is one of the person characteristics of prospective teachers that receives much attention in empirical educational research because it is assumed that knowledge is an elementary part of teachers' professional competence (e.g. Kunter et al., 2011). Two central areas of knowledge according to Kunter et al. (2011) are content knowledge (CK) and pedagogical content knowledge (PCK).

The influence of knowledge on diagnostics has also already been considered in various diagnostic situations (e.g. Rieu et al., 2020; Schreiter et al., 2021). We contribute to this research referring to specific diagnostic situations: One relevant diagnostic situation that has received little attention so far is students' solutions to open-ended arithmetic tasks. It is unknown whether research findings on the influence of knowledge on the development of diagnostic competence can be applied to diagnostic situations that have not yet been researched (Loibl et al., 2020). For this reason, the research presented here investigates the extent to which CK and PCK are related to the development of diagnostic thinking when analyzing student solutions to open-ended arithmetic tasks.

## Diagnostic thinking and the impact of knowledge

Diagnostic thinking is understood in the literature as a central part of diagnostic competence (e.g. Loibl et al., 2020). This paper follows this approach and uses the framework of Loibl et al. (2020) as a basic conceptualization of diagnostic competence.

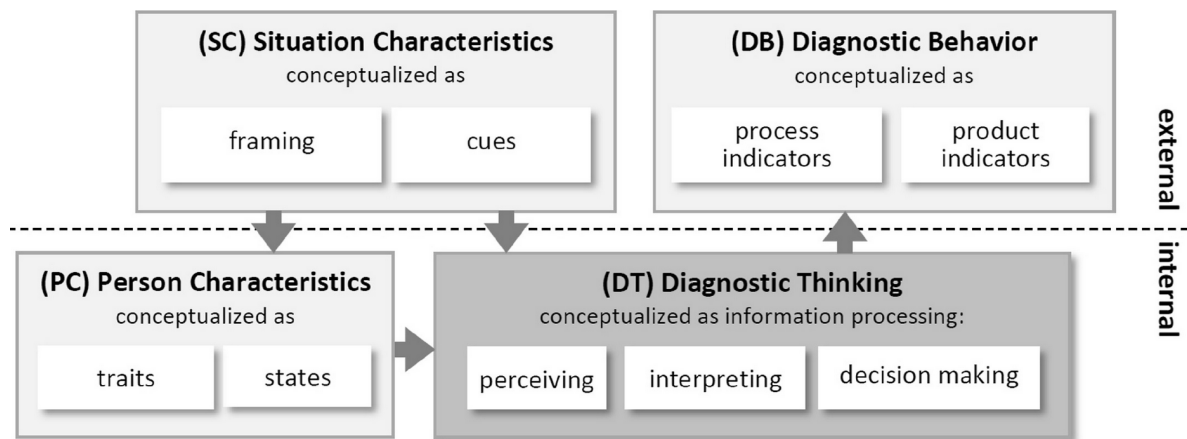


Figure 1: Framework of Loibl et al. (2020, p. 1)

The framework consists of four components, which are divided into external and internal aspects in a diagnostic situation with respect to the diagnosing person. External are the Situation Characteristics (SC) which are given by the diagnostic situation. Depending on the diagnostic situation, the first internal component is the Person Characteristics (PC). These include states such as mindset or stress and traits such as knowledge which is the relevant characteristic for this paper. At the center of the framework is Diagnostic Thinking (DT), which is influenced by SC and PC. The result of the diagnostic thinking process is the externally perceivable diagnostic behavior (DB). The diagnostic thinking is divided into "perceiving", "interpreting" and "decision making". Loibl et al. (2020) state that researchers need to conceptualize this component individually depending on the situation.

As already described above, the person characteristics (PC) include prospective teachers' knowledge. Knowledge as a construct in its own and as an important component of prospective teachers' education is well researched since the work of Shulman (1987). Nowadays, there are different conceptualizations of prospective teachers' knowledge. What they have in common is that the core components include content knowledge (CK) and pedagogical knowledge (PCK) (c.f. Kunter et al., 2011). The framework of Loibl et al., (2020) as well as research stated that knowledge influences the process of diagnostic thinking. For example, Schreiter et al. (2021) found that knowledge about difficulty generating task characteristics, which can be considered as part of PCK, improves prospective teachers' evaluations of these tasks. Rieu et al. (2020) found that judgment accuracy was affected by specific PCK. Judgment accuracy is a possible expression of DB and thus according to Loibl et al. (2020) the inference of diagnostic thinking (DT). Accordingly, professional knowledge is assumed to have a major influence on diagnostic thinking. However, it remains unclear how large the role of knowledge is in different diagnostic situations and whether the knowledge also influences the development of diagnostic thinking. A contribution to this research will be made by the presented study. The study investigates a diagnostic situation in which prospective elementary teachers are asked to analyze students' solutions to open-ended arithmetic tasks. The main research question is:

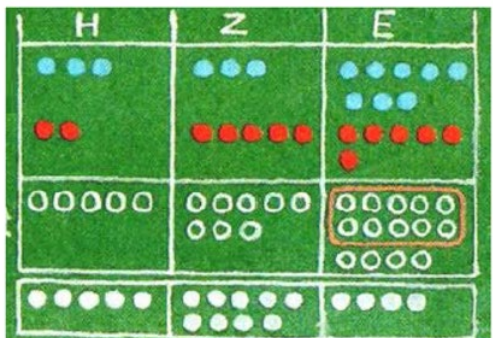
To what extent does CK or PCK influence the development of prospective elementary teacher’s diagnostic thinking regarding perception and interpretation in a situation where students' solutions to open-ended arithmetic tasks are to be analyzed?

## Method

The presented study is in an experimental pre-post-test design. We designed treatments based on the positive effects already researched to promote diagnostic competence. In a university seminar which lasts 14 weeks with 90 minutes session per week the treatments were conducted with 114 prospective elementary teachers. Of these, 89 prospective elementary teachers also completed the knowledge tests and are thus included in the following analyses. These prospective teachers are about halfway through their studies and have already taken courses on mathematics sciences and mathematics didactics. The treatments are based on the aspects of effective promotion of diagnostic competence (Chernikova et al., 2020) and include solving open-ended arithmetic tasks and analyzing students’ solutions to these tasks (for a more detailed description of the treatments see Volkmer et al., 2022).

We tested the professional knowledge of the prospective elementary teachers at the post-test. The content knowledge test regarding arithmetic was adapted from Kolter et al. (2018). The test includes items on prime number, divisibility, and the decimal system. Individual items were taken and modified with the aim of a Rasch model. The pedagogical content knowledge test was adapted from Baumert et al. (2009). In the test from Baumert et al. (2009), the PCK items are divided into three areas: "Explaining and Representing, Student Cognition - Typical Errors and Difficulties, Multiple Solution Potential". Baumert's test version targets secondary teachers, so the elementary school-specific adaptation was needed. For this purpose, new items were generated with the aim of developing a Rasch-scaled test, which are based on the areas according to Baumert et al. (2009). Below is an item from the CK test and an item from the PCK test.

**Table 1: Item examples**

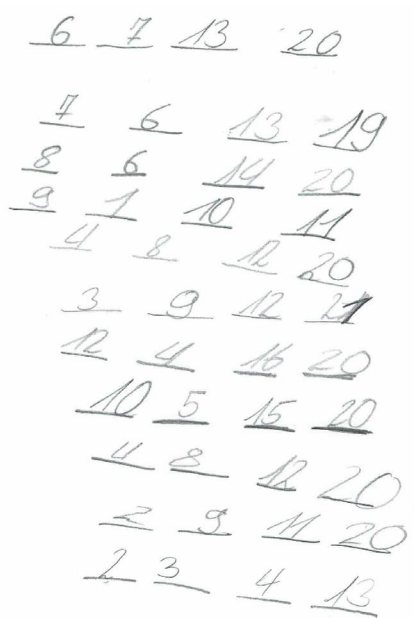
CK Item example			PCK Item example
	True	False	<p>You see the following panel in a colleague's classroom:</p>  <p>What was the presumed purpose of the panel?</p>
The amount of prime numbers is infinite:			
The number 77 is a prime number because it is divisible by 1 and 77.			

The statistical software R with the “exhaustiveRasch” package was used to test the Rasch models. The tool calculates all possible Rasch models with a maximum-likelihood method including quality

criteria for entered items under predefined rules. The main criteria are the MSQ fits, which should be between 0.5 and 1.5 or even better between 0.7 and 1.3 (Wu et al., 2016). We identified a Rasch model for both the CK of arithmetic and the PCK of arithmetic with MSQ fits between 0.7 and 1.3. For the scale on PCK, a Rasch model of 7 items is formed. For the scale on CK, a Rasch model of 5 Item is formed. The person parameter distributions are approximately normally distributed and the item difficulties differentiate well between all skill levels in both scales by having at least one item between each skill level.

To measure the prospective teacher’s diagnostic thinking we designed test-items in which the prospective elementary school teachers are asked to analyze written solutions to open-ended arithmetic tasks. The solutions to open-ended tasks have the special feature that they allow multiple perceptions and interpretations. The diagnostic tests were conducted before the semester and after the semester. In each test the prospective teachers had to analyze two solutions to two different open-ended tasks.

**Table 2: Example of an open-ended arithmetic task and a students’ solution**

Task example	Solution example of an elementary school student (3 <sup>rd</sup> grade)
<p>How can you get to the number 20 in a sequence? The number sequence includes four numbers and the 4th place is supposed to be 20. You get to the number at a specific place by adding up the two numbers before.</p> <p>For example, if you start with 3 (1st place) and 4 (2nd place) you will get 7 (3rd place) and 11 (4th place). Your aim is to find all number sequences with 20 at the fourth place. (Birnstengel-Höft &amp; Feldhaus, 2006)</p>	 <p>The student has written several sequences of four numbers where the sum of the first two equals the sum of the last two, resulting in 20. The sequences are:</p> <ul style="list-style-type: none"> <li>6, 7, 13, 20</li> <li>4, 6, 13, 19</li> <li>8, 6, 14, 20</li> <li>9, 1, 10, 11</li> <li>4, 8, 12, 20</li> <li>3, 9, 12, 11</li> <li>12, 4, 16, 20</li> <li>10, 5, 15, 20</li> <li>4, 8, 12, 20</li> <li>2, 9, 11, 20</li> <li>2, 3, 4, 13</li> </ul>

The prospective teachers’ analyses are the basis to investigate their diagnostic thinking. Perceiving and interpreting are the processes of diagnostic thinking (Loibl et al., 2020) that are relevant for the present study. Therefore, the prospective teachers’ diagnostic thinking was measured as mentioning of manifest characteristics in students’ solution as perceiving and the prospective teachers’ generation of hypotheses as interpreting. One example of perceiving a manifest characteristic is: “The child calculates all tasks correctly”. One example of a generated hypothesis is: “The child calculates safely in the number range up to 20” (Eichler et al., 2023). To analyze prospective teachers’ diagnostic thinking, we counted the amount of identified manifest characteristics and the number of generated hypotheses that the teachers’ used in their responses.

## Results

Examination of the development of diagnostic thinking between pre-test and post-test shows significant improvements. These were obtained in both the identification of manifest characteristics and the generation of hypotheses representing teachers' diagnostic thinking. To examine the extent to which CK and PCK influence significant improvement in diagnostic thinking, partial correlations are calculated. Partial correlation in the present research means that the correlation between CK or PCK and the diagnostic thinking score in the post-test is corrected for the correlation of the diagnostic thinking score in the pre-test.

**Table 3: Partial correlation results**

	Identification of manifest characteristics	Generation of hypotheses
CK	.17	.31**
PCK	.08	.26**

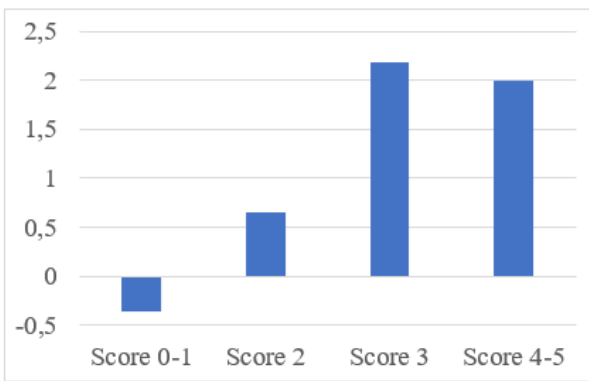
$p < .05 = *$ ;  $p < .01 = **$

Partial correlations between knowledge and hypothesis generation are significant at the  $p < .01$  level with a medium effect  $r = .31$  for the CK and a small effect  $r = .26$  for the PCK (Cohen, 1988).. Partial correlations between knowledge and the identification of manifest characteristics are not significant at all. We can see that the development of diagnostic thinking regarding the identification of manifest features is not influenced by either the PCK or the CK of prospective elementary school teachers. This means that the prospective elementary school teachers in the sample undergo stronger development with respect to their generated hypotheses if they also have higher CK or PCK. To give a more detailed insight into the correlations, the case of the highest partial correlation and the case of the lowest partial correlation are presented below. In the diagrams below, groups are formed depending on the score in the knowledge tests. The differences of the identified manifest features or the generated hypotheses are then averaged for these groups.

For the case of the highest partial correlation found between CK and the number of hypotheses generated, the following diagram is obtained.

**Table 4: Development of generation of hypotheses in relation to the CK score**

CK Score	N	mean
0 - 1	25	-0.36
2	23	0.65
3	27	2.19
4 - 5	15	2

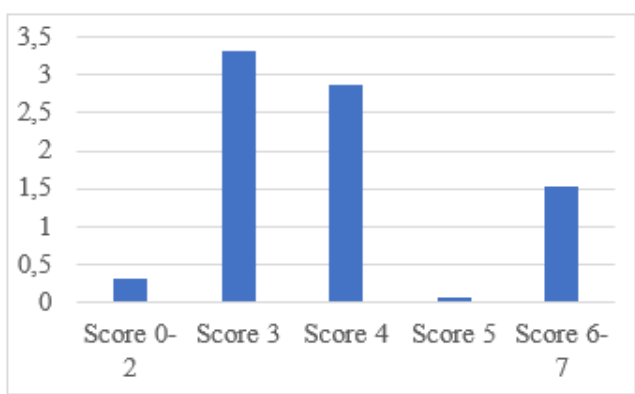


Four groups were formed according to the respective CK test scores. In group 1 are all prospective teachers with 0 - 1 score in group 2 are all prospective teachers with 2 score. The height of the bars is defined by the mean value of the difference of the generated hypotheses in pretest and posttest. We can see that the group with the lowest CK test score shows a slightly negative development (mean = -0.36) while the group with a CK test score of 2 shows a positive development by a mean of 0.65. The two groups with 3 or 4 - 5 points scored in the CK test develop with a mean of 2.19 respectively a mean of 2. This shows the tendency that with a higher CK a stronger development of the diagnostic thinking between pre- and post-test related to the generation of hypotheses takes place.

Considering the case of the lowest partial correlation found between PCK and the identification of manifest characteristics, the following diagram is obtained.

**Table 5: Development of the identification of manifest characteristics in relation to the PCK score**

PCK Score	N	mean
0 - 2	19	0.32
3	16	3.31
4	22	2.86
5	19	0.05
6 - 7	13	1.54



Five groups were formed according to the respective PCK test scores. This time in group 1 are all prospective teachers with 0 – 2 score in the PCK test. In the second group are all prospective teachers with a PCK test score of 3 while in group 3 are all prospective teachers with a PCK test score of 4 and so on. The height of the bars is defined by the mean value of the difference of the identified manifest characteristics in pretest and posttest. We can see that the group with the lowest development from the pretest to the posttest is the group that has a score of 5 in the PCK test. The highest development (mean = 3.31) is observable in the group that has a score of 3 in the PCK test. Therefore, no tendency can be found for the relationship between the development of diagnostic thinking regarding the identification of manifest features and PCK.

## Discussion

It can be concluded that, in accordance with the theory, both the CK and the PCK have an influence on the development of diagnostic thinking. However, in the present study, this only applies to generation of hypotheses and therefore the interpreting as part of the process of diagnostic thinking. No influence of PCK or CK could be detected on the development of the identification of manifest characteristics and therefore the perceiving as a part of diagnostic thinking. Thus, for the specific situation of analyzing open-ended arithmetic tasks, even a low CK and PCK seems to be sufficient for our specific training to be successful in fostering the identification of manifest characteristics. In contrast, the training we use is particularly successful in fostering generating hypotheses when there

is high CK and PCK. Further, the present study descriptively suggests a tendency for the CK to appear to have a greater influence than the PCK.

A limitation to be discussed in the presented research is found in the selection of variables. The dependent variable was diagnosing student solutions to open-ended arithmetic task. The covariates relate to arithmetic in general. Would there be a greater impact by measuring knowledge related to open-ended arithmetic tasks or would such a measurement confound with the dependent variable? What is actually the best possible fit between covariates and dependent variable? More research on this could be helpful to design further studies.

## References

- Baumert, J., Blum, W., Brunner, M., Dubberke, T., Jordan, A., Klusmann, U., Krauss, S., Kunter, M., Löwen, K., Neubrand, M., & Tsai, Y.-M. (2009). *Professionswissen von Lehrkräften, kognitiv aktivierender Mathematikunterricht und die Entwicklung von mathematischer Kompetenz (COACTIV): Dokumentation der Erhebungsinstrumente* [Teachers' professional knowledge, cognitively activating mathematics teaching and the development of mathematical competence (COACTIV): Documentation of the survey instruments]. Max-Planck-Inst. für Bildungsforschung. [https://doi.org/10.48644/mpib\\_escidoc\\_33630](https://doi.org/10.48644/mpib_escidoc_33630)
- Binder, K., Krauss, S., Hilbert, S., Brunner, M., Anders, Y., & Kunter, M. (2018). Diagnostic skills of mathematics teachers in the COACTIV study. In T. Leuders, K. Philipp, & J. Leuders (Eds.), *Diagnostic competence of mathematics teachers* (pp. 33–53). Springer International Publishing. [https://doi.org/10.1007/978-3-319-66327-2\\_2](https://doi.org/10.1007/978-3-319-66327-2_2)
- Birnstengel-Höft, U., & Feldhaus, A. (2006). Gute Aufgaben in der Lehrerausbildung und -weiterbildung [Good tasks in teacher training and further education]. In S. Ruwisch & A. Peter-Koop (Eds.), *Gute Aufgaben im Mathematikunterricht der Grundschule* [Good tasks in primary school maths lessons] (pp. 196–210). Mildenberger.
- Chernikova, O., Heitzmann, N., Fink, M. C., Timothy, V., Seidel, T., & Fischer, F. (2020). Facilitating diagnostic competences in higher education—a meta-analysis in medical and teacher education. *Educational Psychology Review*, 32(1), 157–196. <https://doi.org/10.1007/s10648-019-09492-2>
- Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2. ed.). Erlbaum.
- Eichler, A., Rathgeb-Schnierer, E., & Volkmer, J. P. (2023). Das Beurteilen von Lernprodukten als Facette diagnostischer Kompetenz fördern [Promoting the assessment of learning products as a facet of diagnostic competence]. *Journal für Mathematik-Didaktik*, 44, 29–58. <https://doi.org/10.1007/s13138-022-00216-8>
- Kolter, J., Blum, W., Bender, P., Biehler, R., Haase, J., Hochmuth, R., & Schukajlow, S. (2018). Zum Erwerb, zur Messung und zur Förderung studentischen (Fach-)Wissens in der Vorlesung „Arithmetik für die Grundschule“ – Ergebnisse aus dem KLIMAGS-Projekt [On the acquisition, measurement and promotion of student (specialised) knowledge in the lecture "Arithmetic for primary schools" - results from the KLIMAGS project]. In R. Möller & R. Vogel (Eds.), *Konzepte*



*und Studien zur Hochschuldidaktik und Lehrerbildung Mathematik. Innovative Konzepte für die Grundschullehrerausbildung im Fach Mathematik* [Concepts and studies on university didactics and teacher training in mathematics. Innovative concepts for primary school teacher training in mathematics] (pp. 95–121). Springer Fachmedien Wiesbaden. [https://doi.org/10.1007/978-3-658-10265-4\\_4](https://doi.org/10.1007/978-3-658-10265-4_4)

- Kunter, M., Baumert, J., Blum, W., Klusmann, U., Krauss, S., & Neubrand, M. (Eds.). (2011). *Professionelle Kompetenz von Lehrkräften: Ergebnisse des Forschungsprogramms COACTIV* [Professional competence of teachers: Results of the COACTIV research programme]. Waxmann.
- Loibl, K., Leuders, T., & Dörfler, T. (2020). A framework for explaining teachers' diagnostic judgements by cognitive modeling (DiaCoM). *Teaching and Teacher Education, 91*, 1–10. <https://doi.org/10.1016/j.tate.2020.103059>
- Rieu, A., Loibl, K., Leuders, T., & Herppich, S. (2020). Diagnostisches Urteilen als informationsverarbeitender Prozess – Wie nutzen Lehrkräfte ihr Wissen bei der Identifizierung und Gewichtung von Anforderungen in Aufgaben [Diagnostic judgement as an information-processing process - How teachers use their knowledge to identify and weight requirements in tasks]? *Unterrichtswissenschaft, 48*(4), 503–529. <https://doi.org/10.1007/s42010-020-00071-x>
- Schreiter, S., Vogel, M., Rehm, M., & Dörfler, T. (2021). Teachers' diagnostic judgment regarding the difficulty of fraction tasks: A reconstruction of perceived and processed task characteristics. *Research in Subject-Matter Teaching and Learning 4*(1), 127–146. <https://doi.org/10.23770/rt1846>
- Shulman, L. (1987). Knowledge and teaching: Foundations of the New Reform. *Harvard Educational Review, 57*(1), 1–23. <https://doi.org/10.17763/haer.57.1.j463w79r56455411>
- Volkmer, J. P., Eichler, A., & Rathgeb-Schnierer, E. (2022). Contrasting and comparing solutions to foster diagnostic competence. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)* (pp. 1–8). Free University of Bozen-Bolzano and ERME.
- Wirth, S., Friesen, M., Philipp, K., & Streit, C. (2022). Pre-service teachers' use of information when diagnosing first graders' number sense in text-image vignettes. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)* (pp. 1–8). Free University of Bozen-Bolzano and ERME.
- Wu, M., Tam, H. P., & Jen, T.-H. (2016). *Educational measurement for applied researchers*. Springer Singapore. <https://doi.org/10.1007/978-981-10-3302-5>