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# Moves for eliciting and promoting reasoning and proving

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*An important aspect of mathematics teacher education is to prepare pre-service teachers (PSTs) for teaching mathematics with an emphasis on reasoning and proving (RP). Furthermore, a practice-oriented approach to teacher education is suggested in teacher education research. Following, lessons for teacher education were designed through an intervention, using role-plays as a practice-oriented approach to learning to teach RP. This study aims to investigate the opportunities for learning to teach RP in the designed lessons through an inductive analysis of moves for supporting RP in PSTs' plans and enactments. The analysis reveals a high proportion of moves for eliciting and promoting students' RP. Further, the proportion of moves for eliciting and promoting RP increases from the plans to the enactments. The study contributes to knowledge about moves for eliciting and promoting RP and role-plays as an approach to learning to teach RP.*

*Keywords: Validity, teacher education, preservice teachers, content analysis.*

## Introduction

One of the current approaches to teacher education is the practice-oriented approach, trying to narrow the gap between teacher knowledge and enactment by centring teacher education around core practices (McDonald et al., 2013). Work with core practices in teacher education can be implemented through representations, approximations, and decomposition of practice (Grossman, Compton, et al., 2009). Approaches combining these pedagogies for work with core practices, like the cycle of investigation and enactment (Lampert et al., 2013), reveal promising results for the practice-oriented approach.

Reasoning and proving is a central part of mathematical thinking (Stylianides & Stylianides, 2017). Current curriculums in Norway and internationally emphasise RP as a core element of mathematics and call for RP across all topics in mathematics instruction. However, proof is difficult to learn and teach (Stylianides & Stylianides, 2017), and preparing PSTs for teaching mathematics that emphasises RP is necessary. Previous intervention-based research has shown promising results of a practice-oriented approach to RP in teacher education and professional development (e.g., Buchbinder & McCrone, 2020; Kazemi et al., 2021). While these studies are based on enactments in real classrooms, it is also interesting to study the opportunities for learning to teach RP that can take place in teacher education courses.

I report on an intervention for teacher education aiming to provide PSTs opportunities for learning to teach RP through a role-play approach in teacher education. A decomposition of core practices into moves can help unpack PSTs' opportunities for learning to elicit and promote students' RP and make it less complex to study. The research question is: *What moves for eliciting and promoting RP do PSTs plan and enact in role-plays of mathematical conversations?* The research question is answered through an inductive analysis of PSTs' plans and enactments of role-plays. The results are informed by literature on core practices and reasoning and proving.

## **Theoretical framework**

Core practices are not clearly defined (McDonald et al., 2013), but I follow the list of criteria of core practices from Grossman, Hammerness, et al. (2009) as a working definition. Core practices frequently occur in teaching, are research-based, preserve the complexity of teaching, can be enacted by novices across different curricula, allow novices to learn about students and teaching, and have the potential to improve students' learning. Leading classroom conversations that aim at eliciting and promoting RP is a core practice in mathematics, which can be implemented through representations, approximations, and decompositions of practice (Grossman, Compton, et al., 2009). Representations of practice concern making the practice visible to PSTs, including making students' thinking visible, for example, through videos of instruction or records of students' written work. Approximations of practices concern the opportunities for PSTs to enact aspects of the practice. Decompositions of practice concern making complex practice more manageable through unpacking it into smaller skills or routines.

Core practices can be decomposed into a set of moves (Boerst et al., 2011). There are several different ways to decompose a practice, and an extensive list of moves for a practice does not exist. Different studies have decomposed practices for eliciting and promoting RP in different ways. For example, Ellis et al. (2019) found a decomposition of promoting mathematical reasoning as moves in four categories: eliciting, responding to, facilitating, and extending student reasoning. The moves in these categories are further divided into two groups: moves with low and high potential for reasoning. Mata-Pereira and da Ponte (2017) have another list of moves and divide moves for promoting generalization and justification into guiding and challenging moves. In the following, I will define RP, which informs my analyses of moves for eliciting and promoting RP.

I follow Jeannotte and Kieran's (2017) definition of mathematical reasoning as discursive processes. Still, I use the term RP to put a more explicit emphasis on the processes for proving. Jeannotte and Kieran (2017) define mathematical reasoning as an activity aiming to develop and validate mathematical claims. The processes that aim at developing mathematical claims are identifying a pattern, comparing, classifying, generalizing, and conjecturing. Mathematical claims can be validated through justification, proving, and formal proving. Proving and formal proving are mathematically valid arguments following a deductive structure (Reid & Knipping, 2010, p. 84). Mathematical claims that concern conjectures of a single case, finitely many cases, or infinitely many cases can be validated through the process of proving. Further, exemplifying is a process of reasoning in which one infers data that supports processes of developing claims and validating (Jeannotte & Kieran, 2017). Exemplifying can support the other processes for mathematical reasoning.

RP develops students' deep learning in mathematics, and working with proof already in the elementary grades provides students with a way of thinking about mathematics that decreases the gap between elementary and secondary mathematics (Stylianides & Stylianides, 2017). In an intervention aiming to support PSTs in implementing proof-related tasks in secondary school, Buchbinder and McCrone (2020) found that the PSTs successfully planned for proof-related tasks. However, the enactments differed from the plans, and one of the PSTs' challenges was to enact precise language for proofs. Through working with core practices in cycles of planning, enactment, and reflection,

Kazemi et al. (2021) identified a shift in teachers' sensemaking about how tasks and language routines can be enacted to support students' mathematical argumentation in primary school. Although work on core practices can support PSTs' learning to elicit and promote RP, Boerst et al. (2011) suggest that PSTs' practices depend on the mathematical content because the complexity of the mathematical content makes teaching more difficult. Thus, PSTs' practices for RP might differ from those in other subjects.

## **Methods**

In an intervention in teacher education, a series of four lessons were developed to provide PSTs with the opportunity to learn to enact core practices for RP through role-play. Role-play is an approximation to practice that is quite far from real practice (Grossman, Compton, et al., 2009). Although there is a risk of oversimplifying practice when working with approximations to practice, the advantage of approximations is that one can isolate and rehearse certain aspects of the practice without the full complexity of teaching. In the intervention reported on here, PSTs rehearsed leading classroom conversations aiming at eliciting and promoting RP without, for example, the complexity of classroom management.

The four lessons were revised and refined through three cycles of implementation. Inspired by the cycles of investigation and enactment (Lampert et al., 2013), the lessons consisted of four components. First, the PSTs were presented with a representation of practice: a case from a classroom involving a task and a record of students' work. Second, the PSTs were prompted to plan a mathematical conversation aiming to elicit and promote the students' RP. The PSTs planned in groups of 2-4 persons. Third, the plans were enacted in groups or the whole class. In the enactments in the whole class, one or several PSTs from one planning group played the teacher, PSTs from the other groups played students, and some PSTs observed the enactments. Finally, the PSTs were prompted to reflect on the plans and enactments based on the observations made by the observing PSTs. The intervention was conducted in a first-year mathematics and methods teacher education course, and the PSTs had participated in one lesson about theory of RP before the intervention.

The data material for this study consists of the transcriptions of when PSTs planned and enacted the role-plays in six of the lessons. In every lesson, the planning and enactment were audio and video recorded and were further transcribed and anonymised. First, I reduced the transcriptions of the group work in the planning into a sequence of the PSTs' planned questions, the predicted student responses, and, if they explicitly stated it, the purpose of their planned questions. Next, the enacted role-plays were reduced to the questions asked by the PST who played the teacher and the answers from those who played students.

The data analysis focused on the moves for eliciting and promoting RP that PSTs plan and enact in role-plays. The questions and (predicted) answers in the PSTs' plans and role-plays were analysed inductively using content analysis (Cohen et al., 2018). For every question the PSTs planned and enacted, a description of the questions and the processes for RP they supported was made. Based on the descriptions, the questions were labelled as moves for eliciting and promoting RP, and the moves were grouped into four categories based on the processes for RP they support. In two of the categories, some of the moves only supported RP processes if they were in a sequence with other moves. These

were labelled as support moves. Not all questions supported processes for RP and were thus grouped in a separate category. The data material was coded again, using the moves and categories of moves found in the previous analysis phase. The last coding resulted in a list of moves and the number of occurrences for each move.

## Results

In total, the PSTs planned 115 moves and enacted 274 moves throughout the six lessons. Since the number of moves in plans and enactments was different, I present the proportions of moves relative

**Table 1: Moves for eliciting and promoting students' reasoning and proving. The percentage of moves in a category is given in the title of the category. The percentage of moves within each category is relative to the total number of moves in planning and enactment in the category.**

<b>Moves for developing mathematical claims (21,6%)</b>	P	E	<b>Moves for validating (36,8%)</b>	P	E
<ul style="list-style-type: none"> <li>• Revoice conjecture</li> <li>• Ask students to find a better or more effective strategy</li> <li>• Ask students to generalise</li> <li>• Ask students to compare</li> <li>• Ask students to look for a pattern</li> </ul>	13%	18%	<ul style="list-style-type: none"> <li>• Ask students to find all combinations</li> <li>• Ask students to prove a single-case conjecture</li> <li>• Ask students to prove a single case conjecture using a visual representation</li> <li>• Ask students to prove a general case conjecture</li> <li>• Support students in refuting a claim</li> </ul>	3%	0%
<i>Support moves:</i>					
<ul style="list-style-type: none"> <li>• Support students in developing a claim</li> <li>• Ask students to elaborate on a claim</li> </ul>	17%	25%	<ul style="list-style-type: none"> <li>• Support students in refuting a claim</li> </ul>	13%	5%
	4%	3%	<i>Support moves:</i>		
			<ul style="list-style-type: none"> <li>• Revoice and make clearer</li> <li>• Ask students to explain a step in a procedure</li> </ul>	0%	42%
				9%	35%
<b>Moves for exemplifying (2,3%)</b>	P	E	<b>Non-RP moves (39,3%)</b>	P	E
<ul style="list-style-type: none"> <li>• Ask students to use a conjecture to solve a problem</li> <li>• Give more examples that fit the conjecture</li> </ul>	14%	50%	<ul style="list-style-type: none"> <li>• Revoice student answer</li> <li>• Ask students to give a model or context</li> <li>• Ask students if there are other strategies</li> <li>• Ask students to explain their choice of strategy</li> <li>• Ask students to explain their procedure</li> <li>• Ask for students' answers</li> <li>• Confirm a strategy</li> </ul>	0%	21%
	86%	50%		17%	5%
				2%	1%
				23%	14%
				27%	34%
				31%	21%
				0%	5%

to the total number of moves in the plans (P) and enactments (E), as seen in Table 1. In the following, examples of the moves are given for each category, along with the main trends for relevant lessons.

### **Moves for developing a mathematical claim**

The moves for developing mathematical claims were present in all lessons. Particularly, I will go into detail about the moves for developing mathematical claims in the two lessons where the PSTs were prompted to plan and enact role-plays aiming to develop a mathematical claim as the main or subsidiary goal. In one lesson, the goal of the role-play was to develop and prove a conjecture about how the dividend and the divisor can be changed without the quotient changing in a whole-class discussion. Approximately 18% of the planned moves in this lesson were moves for developing a mathematical claim; *ask students to compare* (“We have now seen some examples, and I wonder if anyone can tell me what is the same and what is different in these examples”) and *revoice conjecture* (“So you have found that the answers are the same and that the numbers in the arithmetic problems are doubled, is that right?”). In the enactment of this role-play, the proportion of moves for developing a mathematical claim was increased to 42%. In particular, the PSTs enacted the moves *support students in developing a claim* (“We will hear from Kristian and Odin. Do you agree that these numbers are doubled?”) and *ask student to compare* were enacted moves that were not planned.

In another lesson, the main goal of the role-play was to develop a conjecture about how you can easily compare fractions that are a unit fraction smaller or larger than a whole in a conversation between a teacher and one student. The PSTs planned and enacted approximately the same proportion of moves for developing a mathematical claim, 31% and 35%, respectively. However, there are some differences in the moves they planned and enacted. About half of the questions the PSTs planned were the move *ask students to look for a pattern*, for example, “What did the fractions have in common?”. However, in the enacted moves, there was a higher proportion of the support moves *ask students to elaborate on their claims* and *support students in developing a claim*. For example, the PSTs asked students to elaborate on their claims by asking them to make part of their mathematical claims more concise: “What do you mean by “the fraction with the largest denominator is the largest”?”

### **Moves for validating**

Moves for validating are presented through two lessons where the PSTs were prompted to plan and enact mathematical conversations aiming at proving single-case and general conjectures. In both lessons, the proportion of moves for validating increased from plans to enactments. In the first lesson, the single-case conjectures were different strategies for solving the division problem  $328:8$ . The PSTs’ plans for conversations leading towards proofs for the strategies contained only a few moves for validating (15%). However, the enacted role-plays contained a larger proportion of moves for validating, where the support move *ask students to explain a step in a procedure* was one of the most frequent moves. In the following excerpt, the PST playing the teacher (Simen) uses this move and *revoice and make clearer*, thus making the deductive structure of a proof visible.

- Simen: Okay, so you jumped from each number. 41. Did you count the number of eights you added?  
Elias: Yes, maybe. I don’t know.

- Simen: It looks like it, at least. But what did you find here? What does the number in between mean?
- Elias: It is one game, two games, three games, four games, five games [Simen: Ah, okay], six games, seven games.
- Simen: Yes. Okay, it is the number of games. But what if we take three games? What do 24 dollars on three games mean?

In the other lesson, the goal was to prove the general conjecture that if you divide one by a fraction, the answer is the inverted fraction. The PSTs built upon a student solution for  $1:2/3$  in which the student first divided one by the unit fraction before he divided the answer by the numerator. The plans contained only one move for validating: *ask students to prove a single case conjecture*. However, the enacted role-plays contained several moves for validating (48%). In the following excerpt, the PST playing teacher (Nadia) first *asks the student to explain a step in the procedure*. Next, she asks the student to generalise and *give proof of the general conjecture*. Finally, she *revoices and makes the proof clearer*.

- Nadia: So, you divided the answer from 1 divided by  $1/3$  by two. Why?
- Lone: Because  $2/3$  is double as much as  $1/3$ .
- Nadia: Okay, but what if we use another fraction? What happens then? Does the same happen then?
- Anders: Yes.
- Nadia: Why?
- Lone: Because it is numerator times as big as, sort of, you have to divide by the numerator to find the answer.
- Nadia: So, you divide the answer from 1 divided by... Can you explain again?

### Moves for exemplifying

The moves for exemplifying were not present in all lessons. *Give more examples that fit the conjecture* was planned and enacted in some of the role-plays that aimed at a proof. For example, in one of the lessons, the goal was to give a proof of the general conjecture about the distributive property in division. Here, the PSTs planned to give the students another problem that can be solved using the distributive property (“We can provide a new arithmetic problem, for example,  $48:8=40:8+8:8$ ”) before they planned to ask the students what happens when they change the numbers. *Ask students to use conjecture to solve a problem* was planned and enacted in the role-play that aimed at finding a conjecture for easily comparing fractions (“Can you use what you found now to find the solution without drawing?”).

### Moves that are not directly connected to RP

Almost 40% of the planned and enacted moves did not directly support RP. In almost all the lessons, the proportion of moves in this category decreased from planning to enactment. One move that was enacted in almost all the role-plays but never planned was *revoice student answer*. The PSTs revoiced in the enactments of several role-plays. In one of the lessons, the role-plays aimed at a proof for the single case conjecture that you can multiply the fractions  $2/5$  and  $3/4$  by multiplying the numerators and the denominators. The role-play was based on several student solutions and their drawings of an area. The PSTs planned to *ask students to explain their solution* (“Selma and Klara, what did you think about this task? Try to explain”) and to *ask students to give a model or context* (“Ivan and

Marius, you made a drawing that shows how you can get a fraction. Can you show us?”). The model they would get would later be used to prove the conjecture.

## Discussion

The analysis shows that PSTs plan and enact many moves for eliciting and promoting RP in the role-plays. Thus, planning and enacting role-plays of mathematical discussions in teacher education can provide PSTs opportunities for learning to elicit and promote RP. The PSTs planned and enacted moves for developing claims, validating, and exemplifying. Further, the proportion of moves for developing claims and validating increases from plans to enactments in most lessons, and the proportion of moves not directly connected to RP decreases from plans to enactments. These results are in contrast to Buchbinder and McCrone’s (2020) study, which found that PSTs’ plans contained more RP than the enactments. Research has shown that it is difficult for PSTs to teach RP (Stylianides & Stylianides, 2017). However, in this role-play approach to learning to teach RP, PSTs enact moves that elicit and promote RP. Although the results in this study are promising, the quality of the moves with regard to the potential for supporting RP has not been investigated, as it is outside the scope of this paper.

In the literature on RP, not much of it deals with exemplifying. However, Jeannotte and Kieran (2017) state that exemplifying processes can support both processes for validation and developing claims. The moves for exemplifying that the PSTs used in this study aim at supporting validation, and they are most prominent in the role-plays that aim at proving a general conjecture. Exemplifying can seem irrelevant for RP at first glance because giving more examples results in an empirical argument, not proof of a general conjecture. However, by giving more examples, students can start to generalise and notice the structure of the mathematical objects, giving them opportunities to produce proofs (Jeannotte & Kieran, 2017). Similarly, the moves not directly connected to RP can support RP later in the mathematical discussion. For example, by asking students to explain their procedure and make a model representing their thinking, one can later use these procedures and models as a stepping stone for constructing a proof. In the present study, the moves not directly related to RP are put into a separate category. In Ellis et al.’s (2019) study, similar moves can be found in the categories of moves with low potential for reasoning.

The proportion of moves for developing conjectures and validating increased from planning to enactments in almost all the role-plays, where the support moves increased the most. A straightforward question about validation or a generalisation rarely leads to a proof or a conjecture (Mata-Pereira & da Ponte, 2017), and support moves for these categories of moves are needed. Similarly to Mata-Pereira and da Ponte’s (2017) moves for generalisation and justification, the categories of moves found in the present study are divided by the processes for RP they support. Despite the promising number of moves for validating in the enactments, it is evident from the results that a tiny proportion of the moves asked the student to give proof of a general conjecture. Since PSTs’ practices are highly dependent on the mathematical content (Boerst et al., 2011), and proving general conjectures is one of the main challenges with reasoning and proving (Stylianides & Stylianides, 2017), these results are not very surprising. However, the designed lessons did not



sufficiently support PSTs in planning and enacting role-plays leading to proofs of general conjectures. Future interventions in teacher education should aim to solve this problem.

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