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Evaluating the influence of a multiaxial static preload on the dynamic properties of elastomers

A.-S. Lectez¹, E. Verron¹, B. Huneau¹

¹ *École Centrale de Nantes, GeM-UMR CNRS 6183,
1 rue de la Noë, BP 92101, 44321 Nantes cedex 3, France*

ABSTRACT: A general experimental procedure is proposed to characterize the dynamic properties of elastomers under large static multiaxial preloads through tension-torsion tests. The experimental set-up is presented, before paying attention to the post-treatment. Because of the heterogeneity of the strain induced by torsion, calculating stresses is too complex, and it is proposed to use some global equivalent quantities. Storage and loss moduli in both tension and shear are then deduced from these quantities, extending classical definitions of dynamic moduli around an undeformed state to a deformed one. Then, the influence of frequency and amplitude on loss and storage moduli is studied, as well as the influence of the preload. The preload is represented either by measures of the strain, through invariants of the Hencky strain tensor, either by its static moduli in tension and shear. Finally, the validity of the proposed procedure is discussed.

1 INTRODUCTION

Elastomers are widely used in industry for their anti-vibration properties. One of such application concerns engine mounts, which are subjected to complex multiaxial loading conditions. Indeed, engine mounts are often submitted to small oscillations, caused by either perturbations from road or excitations of the engine, superimposed on a large multiaxial static preload due to engine weight. The objective here is to characterize elastomers under such conditions, focusing on frequencies between 0.1 Hz and 30 Hz. In the case of small uniaxial tension or shear oscillations without any preload, DMA analysis is classically used to measure the dynamic properties of elastomers (see for example Menard (2008), and Höfer and Lion (2009)). Other authors have included a static preload, considering uniaxial tension and/or compression for both preload and oscillations (see for example Meinecke and Maksin (1980), Dutta and Tripathy (1990), Cho and Youn (2006), Rendek and Lion (2010), Thorin et al. (2012), and Wollscheid and Lion (2013)) but very few consider multiaxiality and preloading during their measurements (Azoug et al. (2013), Suphadon et al. (2010), and Suphadon (2010)). These authors have performed biaxial experiments on cross-shaped specimens, simple shear oscillations superimposed on a pure shear preload and free pure torsion oscillations superimposed on a uniaxial tension preload respectively. However, considering tension-torsion tests

on a cylindrical specimen is particularly adapted for multiaxial characterization. Indeed, it offers the possibility to combine several levels of uniaxial extension (tension or compression) with several levels of shear through torsion. Thus, multiple amplitude levels as well as multiple modes of deformation (that is to say multiaxiality) can be prescribed on a unique sample. In this context, this paper follows three objectives:

- To present these original tests, consisting in applying a tension-torsion preload with several amounts of extension and angles, followed by small oscillations, either in uniaxial tension/compression or in pure torsion while the preload is maintained;
- To develop an adapted procedure of post-treatment;
- To observe the influence of the amplitude and direction of the preload on the dynamic properties of the material, as well as the influence of both frequency and amplitude of oscillations.

First, the method of tension-torsion characterization is presented through the description of the experimental procedure and the chosen analysis of raw data. Then, raw and post-treated results will be exposed, before discussing the influence of preload, frequency and oscillations on the dynamic properties of the materials.

Attention will be paid on the validity of the approach and on its limitations.

2 METHOD

2.1 Experimental procedure

An Instron ElectroPuls E10000 Linear-Torsion machine has been used for the tests. The maximal displacement reachable with this machine is 60 mm, while several turns can be imposed. Capacities of the load and torque cells are respectively 1 kN and 25 N.m. Quasi-cylindrical specimens have been specially designed for this application. The design was driven by several rules as the stability of deformation, and the amplitude and modes of deformation to reach while respecting machine specifications. Figure 1 shows the specimen placed in the machine. The ra-

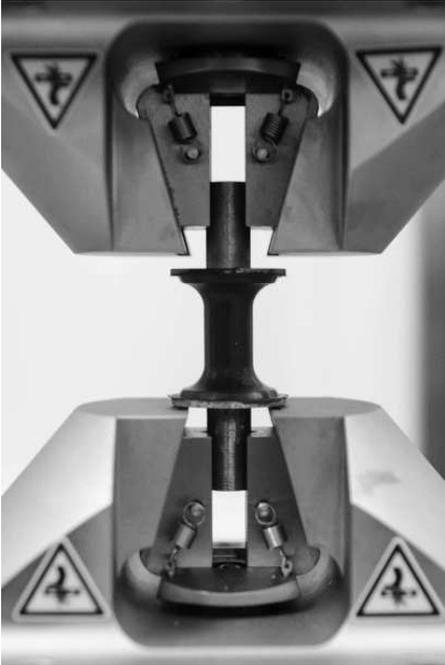


Figure 1: View of the specimen in the machine tests

dius of the perfectly cylindrical part of the specimen is 7 mm, while the total length of the rubber part is 34 mm. The elastomeric material of the specimen is closed from the one of an engine mount: it contains natural rubber (NR) and is carbon black filled (30 phr). In order to determine the relationship between strain, and displacements and angles practically prescribed by the machine, stereo digital image correlation (SDIC) has been used: large quasi-static combined tension-torsion loading conditions have been prescribed on a specimen on which a white speckle pattern has been applied, filmed by two synchronized cameras. From these measurements are deduced the relationship between λ , the extension in the central zone of the specimen and D , the global displacement imposed by the machine on the one hand, and between τ , the angle per unit of length and α the global

angle imposed by the machine on the other hand:

$$\lambda = 0.039D + 1 \quad (1)$$

$$\tau = 0.033\alpha \quad (2)$$

with D in mm, α in rad and τ in $\text{rad}\cdot\text{mm}^{-1}$. Before each test, specimens have been accommodated in order to reduce the Mullins effect, with 5 loading/unloading cycles at the highest loading conditions, which are combined uniaxial tension until $\lambda = 2.5$ and torsion until $\tau = 0.09 \text{ rad}\cdot\text{mm}^{-1}$ ($D = 38.5 \text{ mm}$ and $\alpha = 158^\circ$). A rate of extension $\dot{\lambda} = 10^{-2} \text{ s}^{-1}$ is imposed during accommodation. Then, each tests is composed of two steps:

- A large quasi-static preload of combined torsion and uniaxial tension or compression. In order to impose quasi-static conditions during the first step, the uniaxial displacement rate is fixed at $\dot{\epsilon} = 10^{-3} \text{ s}^{-1}$.
- uniaxial tension-compression or pure torsion small oscillations with a frequency $0.1 \text{ Hz} \leq f \leq 30 \text{ Hz}$.

A map of loading conditions during the first step is shown in Figure 2: each final point of loading is represented by its coordinates in terms of extension λ and angle per unit of length τ . These points have been chosen in order to ensure different levels of amplitude and multiaxiality of deformation. All loading are proportional. For uniaxial tension-compression oscil-

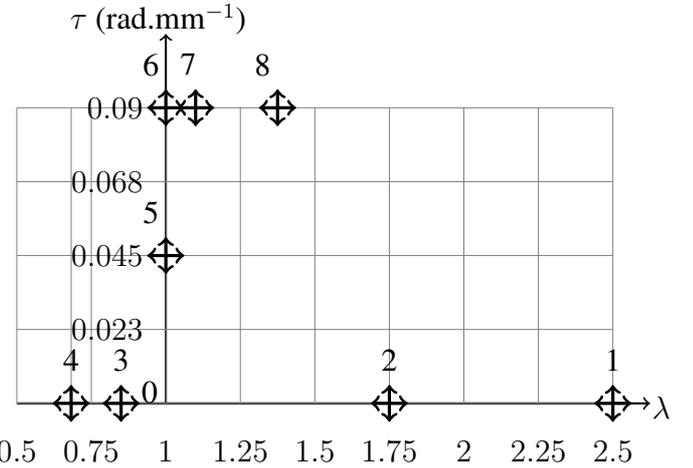


Figure 2: Mapping of the loading points in terms of extension and angle per unit of length during the preload

lations, 9 levels of amplitude have been chosen in terms of strain with respect to the unloaded configuration: $0.01\% \leq \epsilon_0 \leq 7\%$. For pure torsion oscillations, 10 levels of amplitude have been considered: $0.01\% \leq \epsilon_0 \leq 10\%$.

2.2 Post-treatment

During tests, displacements and angles are prescribed, and load and torque are measured. As the objective

is to characterize the material, some intrinsic mechanical quantities have to be calculated from these raw data. Practically, we want to extend the definition of the storage and loss moduli, classically defined for small oscillations around an undeformed state, with uniaxial tension/compression tests (quantities are noted E' and E'' respectively), or shear tests (G' and G'' respectively). Yet, their calculation is based on stresses and strains. From Eqs. (1)-(2) and the hypothesis of a perfect cylinder, strains quantities are easily determined. While it is possible to calculate stresses at the outer radius of the cylinder in the case of quasi-static pre-extension followed by torsion with the method of Penn and Kearsley (1976) (see Lectez et al. (2014)), the calculation is too complex and requires too many hypothesis for multiaxial and non quasi-static conditions, which is the case for some of oscillations here. Thus, calculation of stresses is different according to the loading conditions: the case of uniaxial tension-compression during the preload as well as the oscillations is treated differently as the other cases.

2.2.1 Uniaxial case: tension-compression during preload and oscillations

In this case, the Cauchy stress tensor is easy to calculate since deformation is homogeneous. It reduces to a unique component in the direction of extension: during the preload, $\sigma = N/s$, where N is the measured load and s is the deformed surface, during oscillations, this relationship is linearized to obtain the linearized stress $\Delta\sigma$, and finally, the infinitesimal strain with respect to the preloaded configuration in the direction of extension during the oscillations is: $\epsilon = \Delta u/l$, where Δu is the small displacement imposed during oscillations and l is the length of the specimen after preload.

2.2.2 Multiaxial case: tension-torsion during the preload and/or the oscillations

In this case, there is no simple way to calculate stresses. Thus, we choose to introduce average global quantities which are close to the definitions of stresses, but which are not rigorously stresses. It is chosen to study the global behavior in tension on the one hand, and the global behavior in torsion on the other hand, that is to say it is chosen to simplify the problem and to consider both contributions as decoupled. For the equivalent stress in the tension direction, the same definition is chosen as the stress in pure uniaxial case: $\sigma = N/s$ for the preload, $\Delta\sigma$ for the oscillations. The infinitesimal strain in the tension direction remains $\epsilon = \Delta u/l$.

Concerning the global quantity in the torsion direction, it is chosen to consider the torque reported to the area moment of inertia, as an analogy with torsion of a cylindrical bar: $T = M/I_g$, where ΔT is the equivalent quantity, expressed in $\text{N}\cdot\text{mm}^{-3}$, M is the torque

during the preload and I_g is the area moment of inertia after the preload. This relation is linearized to find ΔT , which corresponds to oscillations. The shear dynamic moduli are deduced from curves of ΔT with respect to $\Delta\tau$, the linearized angle per unit of length. Table 1 shows the mechanical quantities calculated for each type of test.

	Oscillations	Uniaxial tension	Pure tension
Preload			
Uniaxial tension		$\Delta\sigma$ vs. ϵ	
Uniaxial compression			ΔT vs. $\Delta\tau$
tension-torsion		$\Delta\sigma$ vs. ϵ ,	
pure torsion		ΔT vs. $\Delta\tau$	

Table 1: Mechanical quantities calculated for each test

2.2.3 Calculation of dynamic moduli

The dynamic moduli are defined in the framework of linear viscoelasticity (see for example Ward (1983)). An harmonic uniaxial infinitesimal strain is imposed:

$$\epsilon(t) = \epsilon_0 \sin(\omega t), \quad (3)$$

where ϵ_0 is the strain amplitude, ω the angular frequency and t the time. For small oscillations, the stress can be approximated by:

$$\sigma(t) = \sigma_0 \sin(\omega t + \phi), \quad (4)$$

where σ_0 is the stress amplitude and ϕ is the phase. The stress can be rewritten as follows:

$$\sigma(t) = \epsilon_0 E' \sin(\omega t) + \epsilon_0 E'' \cos(\omega t), \quad (5)$$

with $E' = \cos \phi \sigma_0 / \epsilon_0$ and $E'' = \sin \phi \sigma_0 / \epsilon_0$. E' appears in the part of the stress which is in phase with the strain, and is called storage modulus. E'' appears in the part of the stress which is out-of-phase with the strain, and is called loss modulus since it is directly related to the energy dissipated during one cycle ΔW :

$$\Delta W = \int \sigma d\epsilon = \pi E'' \epsilon_0^2. \quad (6)$$

In our case, E' and E'' , the dynamic moduli in tension, are deduced from the curves $\Delta\sigma$ versus ϵ , while G' and G'' , the shear dynamic moduli, are calculated from curves ΔT versus $\Delta\tau$.

2.2.4 Characterizing the influence of the preload

Firstly, dynamic moduli are plotted as functions of frequency and amplitude of oscillations, in order to study their respective influences. Then the influence

of the preload on the dynamic moduli is investigated. Usually, authors chose to plot dynamic moduli with respect to the extension. This method is of course not applicable if torsion is considered during preload. Thus, it is proposed here to represent the preload by two very interesting quantities, K_2 and K_3 which are invariants of the Hencky strain tensor \mathbf{H} (see Criscione et al. (2000)):

$$K_2 = \sqrt{\text{dev}\mathbf{H} : \text{dev}\mathbf{H}} \quad (7)$$

$$K_3 = \frac{3\sqrt{6}}{K_2^3} \det(\text{dev}\mathbf{H}), \quad (8)$$

where dev is the deviatoric operator. These invariants have a special interest through their physical meaning: K_2 is the amplitude of distortion and K_3 is a measure of multiaxiality (it is worth -1 for uniaxial compression, 0 for pure shear and 1 for uniaxial tension). Thus, they are well adapted to represent the preload. An other try is to characterize the preload with its static modulus rather than its strain state. The tension static modulus is calculated as the tangent of the (λ, σ) curve, while the torsion static modulus is the tangent of the (T, τ) curve. Of course, static moduli depend on the level of strain.

3 RESULTS

8 preloads, 5 frequencies and 10 amplitudes of oscillations are studied, it is hence not possible to show all the results here. A few curves are shown as examples, and the main tendencies observed on the whole data are summed up.

3.1 Influence of frequency and amplitude of oscillations

Three main observations are made from all tests:

- Storage modulus decreases when the amplitude increases and increases when the frequency increases;
- No clear trend appears concerning the evolution of the loss modulus with respect to the frequency;
- For most of preload levels, the loss modulus first increases with amplitude, then decreases. The amplitude point at which the evolution changes depends on preload.

Figure 3 illustrates the first item, showing the evolution of E' with respect to the frequency and the amplitude of oscillations, for a pure torsion preload $\tau = 0.09 \text{ rad.mm}^{-1}$ (point 6 in Figure 2).

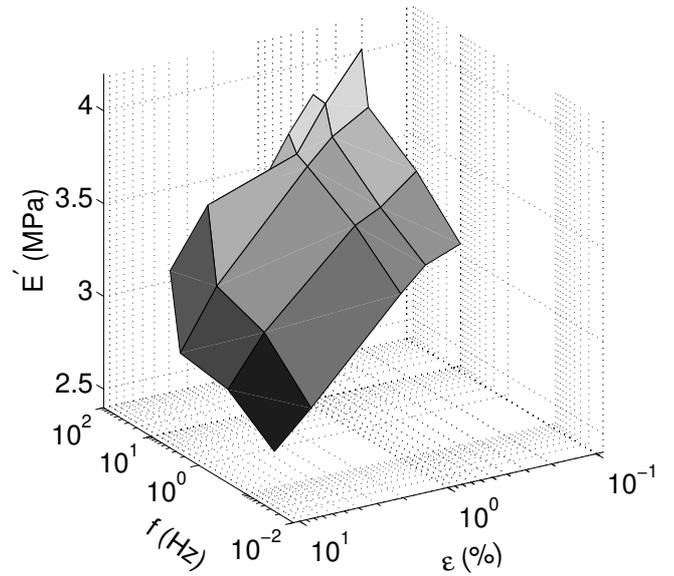


Figure 3: Evolution of the tension storage modulus with respect to frequency and amplitude of oscillations for uniaxial tension-compression oscillations after a pure torsion preload $\tau = 0.09 \text{ rad.mm}^{-1}$

3.2 Influence of preload

In order to study the influence of preload on dynamic moduli, E' and E'' are plotted against K_2 , which represents the amplitude of the preload, and K_3 , the measure of multiaxiality during preload. Figure 4 shows the evolution of E' with respect to K_2 and K_3 for uniaxial tension-compression oscillations with an amplitude $\epsilon = 0.4\%$, a frequency $f = 0.1 \text{ Hz}$. Each point on graphs represents a preload of Figure 2. No very clear

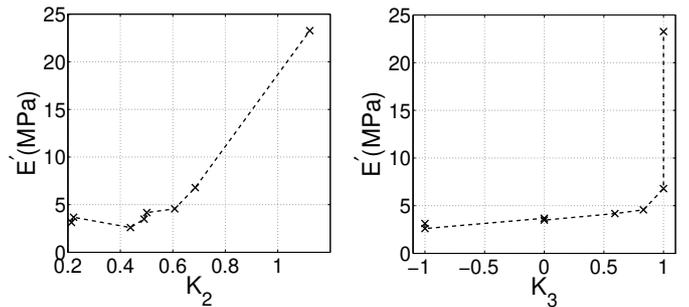


Figure 4: Evolution of the tension storage modulus with respect to the amplitude of distortion and mode of distortion of the preload, for uniaxial tension-compression oscillations of amplitude 0.4 % at 0.1 Hz.

trends appear on these graphs, it is quite difficult to describe the evolution of E' as a function of K_2 and K_3 . Similar observations are made for G' , E'' and G'' .

Figure 5 shows the evolution of E' with respect to E_s , the static modulus for the same conditions of frequency and amplitude as above; a logarithmic scale is adopted. Except for the fourth point, a monotonic evolution appears, the storage modulus increases with the static modulus, in a quite linear manner on the logarithmic plot.

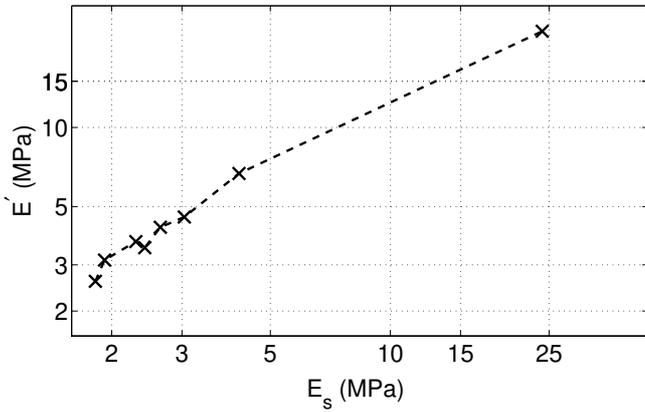


Figure 5: Tension storage modulus with respect to the static modulus in logarithmic scale, for uniaxial tension-compression oscillations at 0.1 Hz, with 0.4% of amplitude

4 DISCUSSION

4.1 Influence of the frequency, amplitude and preload

The evolution observed on the storage moduli with frequency and amplitude when data are not too noisy are coherent with the trends observed by other authors who studied the dynamic moduli after a preload (see for example Dutta and Tripathy (1990) and Cho and Youn (2006)). Few authors studied the evolution of the loss moduli with the frequency and the amplitude under a preload. Rendek and Lion (2010) observed also an increase followed by a decrease of the loss modulus when the amplitude increases, while Wollscheid and Lion (2013) observed a quasi constant evolution. The change of evolution of the loss modulus is comparable to the Payne effect (or Fletcher-Gent), observed during oscillations without preload (see for example Payne (1962a), Payne (1962b), Luo et al. (2010)). Yet, this effect is highly dependent on the elastomer composition, which could explain the difference of observations (Payne and Whittaker (1971)).

Concerning the influence of the preload, characterizing the preload with the strain measures K_2 and K_3 is not pertinent, since no clear trend appears. However, some interesting results are obtained when studying the evolution of E' as a function of E_s : a quasi-linear curve is obtained on the log plot. This quasi-linearity is also found for others frequencies. Of course, curves are not superimposed, since E' depends on the frequency while E_s does not. Hence, a linear regression can be proposed, giving two parameters which depend on the frequency. Figure 6 shows the comparison between the experimental curve of Figure 5 and the linear regression:

$$E' = \alpha E_s^\beta, \text{ where } \alpha = 0.59 \text{ and } \beta = 0.83, \quad (9)$$

β being dimensionless and α in $\text{MPa}^{1-\beta}$. This relationship is very interesting, since it directly relates the static property E_s , which can be known either experimentally or deduced from a hyperelastic model,

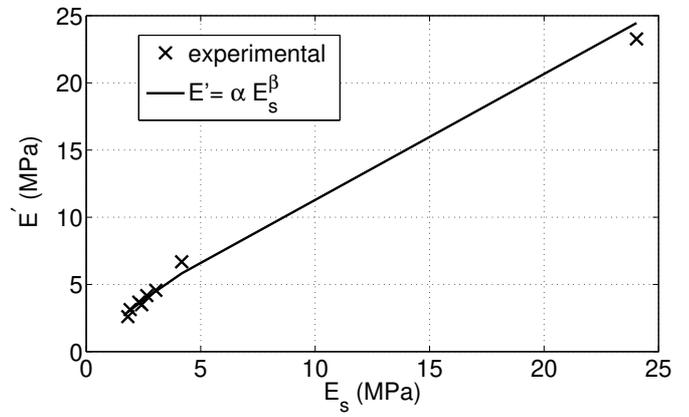


Figure 6: Comparison between the measured and calculated tension storage moduli with respect to the static modulus, for uniaxial tension-compression oscillations at 0.1 Hz, with 0.4% of amplitude

with the storage modulus E' . It has been found that the tension loss modulus E'' follows also this kind of law. However, it is not the case for the shear storage and loss moduli, G' and G'' , on which the results are noisy.

4.2 Validity of the method

4.2.1 Using a large displacement machine for small oscillations

Using this kind of machine, and particularly load and torque cells adapted for large loading conditions can be questionable. Indeed, variations of load and torque during oscillations are often very small in front of the maximum measurable. Data are thus very noisy in some cases, which renders the study of the influence of the frequency and amplitude on dynamic properties harder, even if some trends have been observed. However, using this machine allows us to perform various multiaxial tests with a unique experimental set-up and above all, to study the influence of a large preload on the dynamic properties.

4.2.2 Extending definitions of loss and storage moduli

For some of the amplitudes imposed, non-linearities are observed on the linearized stress-strain curves. Yet, the calculation of loss and storage moduli is based on the linearity hypothesis. Thus, the approximated relationship (5) does not hold anymore, since harmonic of higher orders are no longer negligible. One can speak of several loss and storage moduli, which interpretation becomes complex (see for example Govindjee and Simo (1992)). Despite of this loss of accurate meaning, the loss and storage moduli as calculated in this paper still remain global comparison tools: E' and G' are indicators of the evolution of the stiffness while E'' and G'' are dissipation indicators. This is illustrated by Figure 7: in this case, the linearized stress-strain curve is slightly non-linear, but E' can be plotted and indicates the global change of stiffness. The loss and storage moduli remains use-

ful, but does not permit to completely characterize the dynamic behavior since it does not characterize the non-linearity.

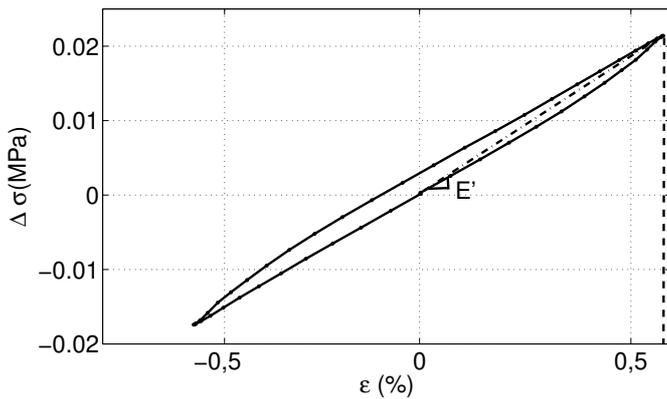


Figure 7: Linearized stress with respect to infinitesimal strain, uniaxial compression preload to $\lambda = 0.67$, $f=20$ Hz

5 CONCLUSION

In this paper, an original experimental procedure dedicated to dynamic properties of elastomers under a large multiaxial quasi-static preload has been presented. A method to post-treat these tests have been presented, based on the calculation of loss and storage moduli, usually calculated from non preloaded experiments. The validity of their use for our experiments has been discussed. Even if non linearity appears for small oscillations, the loss and storage moduli are used as global indicators of stiffness and dissipation. Finally, it has been highlighted that it is more interesting to study the influence of the preload on the dynamic properties comparing the static modulus to the dynamic ones rather than using strains as representation of the preload. It would be interesting to study if a similar power law is obtained for other elastomers.

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