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Investigating natural number bias of future primary school teachers: The case of fractions and concept cartoons

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In this paper, we respond to a research study from the last CERME conference and offer an alternative study collecting data via a Concept Cartoon. Our qualitative study investigates how future primary school teachers respond to the Concept Cartoon that is based on a task possibly triggering natural number bias. The task is situated in the domain of fractions and seeks numbers that lie in the middle between given two fractions. We accompany it with a fragmented set of questions that focus on respondents' solutions as well as on their assessment of the solutions and comments of hypothetical pupils presented in the cartoon. According to the results, this instrument of data collection proved its usefulness in revealing natural number bias as well as other knowledge deficiencies related to the topic of fractions or to the learning of mathematics in general.

Keywords: Concept cartoons, fractions, elementary school teachers, natural number bias, initial teacher education.

Introduction

The structural nature of mathematics has its advantages as well as disadvantages (Singer, 2001). The disadvantages include the occasional need “to forget old knowledge for new one”, the seeming impossibility to apply all previously learned ways of reasoning in newly emerged contexts. In primary school, such cases involve matters related to so-called natural number bias, i.e. to the process of expanding the field of interest from natural numbers to rational numbers (fractions, decimal numbers, negative integers; Rezat, 2019). One of the TWG18 contributions at the last CERME conference discussed in detail natural number bias related to the transfer from natural numbers to fractions and presented difficulties that future primary school teachers might be facing regarding this topic from the perspective of pedagogical content knowledge (Reitz-Koncebovski et al., 2022). The authors collected their data during a knowledge written test, via a mathematics problem accompanied by a set of indicative questions.

Following up on our research on Concept Cartoons and their possible role as an assessment tool in professional preparation of primary and secondary school teachers (e.g. Samková, 2019, 2022), we offer an alternative study concerning natural number bias in fractions and future primary school teachers. This alternative collects data via a Concept Cartoon. As indicated in previous research (e.g. Samková, 2018, 2022), such an alternative approach might provide an additional perspective on the issue, since it seems to be able to highlight deficiencies in conceptual knowledge that might have stayed hidden when using standard written tests.

Our study is of a qualitative design and addresses the following research question: “*What kind of reasoning can be revealed when a Concept Cartoon that indirectly aims at natural number bias in fractions is assigned to future primary school teachers after the topic of fractions and rational numbers has been covered in their mathematics content course?*”

Natural number bias

During first years of primary school, pupils work intensively with natural numbers, and so they tend to unreasonably transfer many considerations and procedures learned in the field of natural numbers to their later work with rational numbers. As mentioned above, this tendency is called *natural number bias*. Van Dooren et al. (2015) distinguish four different types of such transfers, depending on whether they concern *number size* (the way numbers are compared), *operations* (their properties and the properties of their results), *representations* (the meanings of different symbolic notations), or *density* (the existence of infinitely many rational numbers between any two numbers as opposed to the linear discrete structure of natural numbers). Among typical displays of the fourth type we include the idea that rational numbers each have a uniquely given predecessor and successor, and that no rational numbers lie between the numbers and their (supposed) successors. From this point of view, there is seemingly no number between $\frac{1}{2}$ and $\frac{1}{3}$ or between $\frac{3}{5}$ and $\frac{4}{5}$, and classroom discussions about this topic can be challenging not only for pupils but also for teachers (Tunç-Pekkan & Kılıç, 2015; Kleve & Solem, 2015). Even if the roots of natural number bias lie in primary school mathematics, its displays can influence reasoning even of educated adults (Van Hoof et al., 2020).

Czech educational context

In the Czech educational environment, where our study takes place, both decimal numbers and fractions are taught in primary school (pupils from 6 to 11 years of age) but only partially. The primary school part of the topics includes just basic understanding of the main concepts: notation and its meaning (place value in decimal numbers, part and whole in fractions), visualization (number line, illustrative diagrams), comparisons, and associated word problems. Operations are taught later, in lower secondary school (pupils from 11 to 15 years of age). In Czech curricular documents for the primary school level (Ministry of Education Youth and Sports, 2021), the topics of decimal numbers and fractions are separated and their order is not specified. This makes the educational context difficult for teachers: the two topics are usually separated in textbooks with some other topics placed between them, and their order may vary. However, most textbooks offer fractions first. Under these circumstances, primary school teachers have to be cautious and, ideally, should be able to treat the topics of fractions and decimal numbers autonomously when needed.

To bridge the gap between primary school mathematics and mathematics learnt by future teachers during their secondary education, mathematics content courses in professional preparation programs usually discuss all primary-school-related mathematical content in terms of the SRCK model of Dreher et al. (2018). In case of the Czech educational environment and the topic of fractions, the corresponding matters are included in content courses, and later on, also in didactical courses. The content courses cover various meanings and forms of fractions, various ways of visualising fractions, various ways of transforming a fraction into an equivalent fraction, comparisons, operations with fractions, ratios, percentages, mixed numbers, various ways of transforming a fraction into a decimal number and vice versa, the concept of rational and irrational numbers, rounding and approximations, various types of word problems etc. As for equivalent fractions, the name itself might not be taught, but the two ways to obtain a same-size fraction (reducing to lower terms, raising to higher terms) are introduced at lectures and practiced at seminars, usually under shortened names *reducing* and *raising*.

Concept cartoon

Educational and assessment tools in initial teacher education might take different forms, including various types of educational vignettes – real or hypothetical recordings of classroom situations (Skilling & Stylianides, 2020). To develop or assess future teachers' professional knowledge, vignettes are either discussed in groups or presented with indicative questions to respond to individually. In our more than ten years long research, we have systematically studied vignettes called Concept Cartoons – individual cartoon pictures with children discussing a content related situation. We have accompanied these pictures by various sets of indicative questions (Samková, 2019) and such a constellation repeatedly proved to be a valuable assessment tool in teacher education. For instance, in Samková (2018, 2022), Concept Cartoons allowed to reveal aspects of weak conceptual knowledge that was not revealed by standard written tests. The three key features playing the role in such revelations seemed to be: (i) the combination of a mathematical task to be solved and hypothetical children's solutions to be assessed; (ii) the task to be solved being unusual and/or not directly aiming at the assessed mathematical content; (iii) one of the bubbles referring to a diagram or scheme that is not shown in the cartoon.

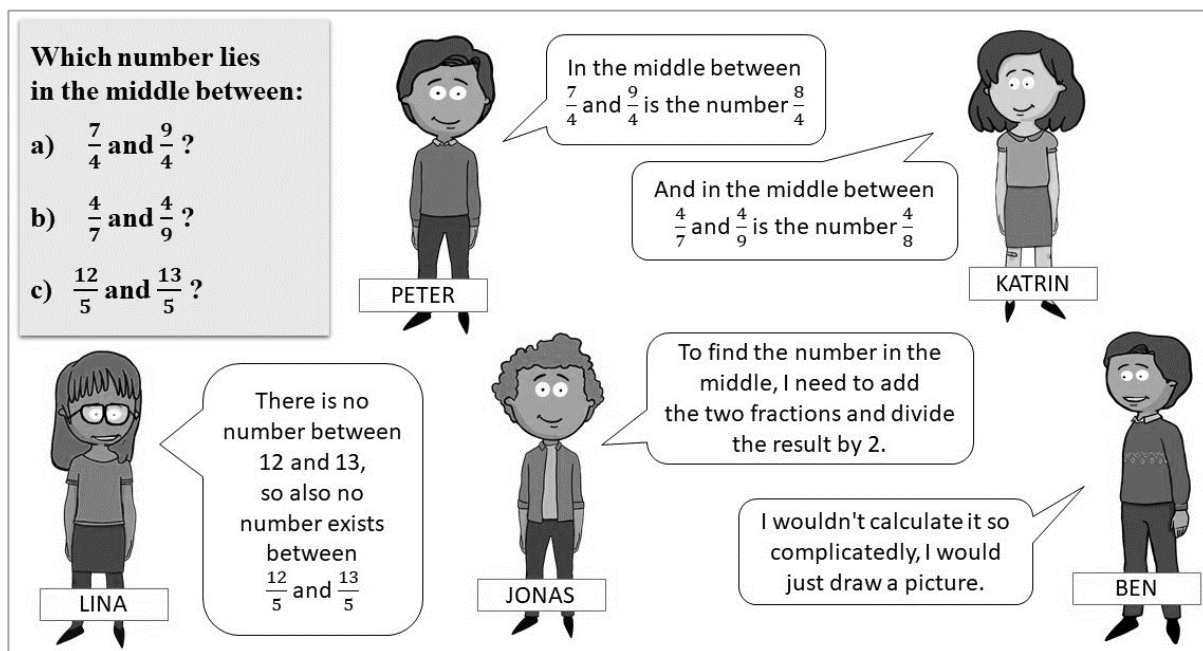


Figure 1: The concept cartoon for our study; graphical elements: DIVER (coReflect@maths, 2022)

Employing these key features, we created a Concept Cartoon (Figure 1) for the density type of natural number bias in fractions. To divert the aim slightly from the desired direction, we based the cartoon on a task that does not seek any numbers between two given fractions but the number in the middle. The task consists of three sub-tasks with three different pairs of fractions that might trigger natural number bias in three different ways (Van Hoof et al., 2020): (a) the same denominators, the numerators differ by 2 (i.e. there is exactly one natural number between the numerators), (b) the same numerators, the denominators differ by 2 (i.e. there is exactly one natural number between the denominators), (c) the same denominators, the numerators differ by 1 (i.e. there is no natural number between the numerators). One of the children (Lina) presents an opinion driven clearly by natural number bias, other two opinions (Peter, Katrin) might or might not be driven similarly (when building

on the fact that 8 is in the middle between 7 and 9). The last two children do not offer particular results but general procedures: Jonas describes how to get the middle number using the arithmetic mean, and Ben refers to drawing a picture but the picture itself is not shown.

Design of the study

Participants of the research study were 47 future primary school teachers – full time students of the second year of the 5-year teacher training program at the University of South Bohemia in České Budějovice. As a part of their studies, they attend three mathematics content courses. The first one covers geometrical topics, the second one is on logic, natural numbers, decimal numbers, and integers. In the time of the research study, the participants were in the middle of the third course where they had partaken in all lectures and seminars on the topic of fractions, rational numbers and irrational numbers. These lectures and seminars covered all primary and secondary school content relevant for teaching the topic of fractions at the primary school level. To support the autonomous approach to the topic of fractions, all calculation and word problems with fractions were encouraged to be solved using only fractions. A task on fractions in-between was discussed from the perspective of density of fractions during one of the seminars: *Find at least 4 fractions between a) $\frac{1}{3}$ and $\frac{1}{4}$; b) $\frac{2}{5}$ and $\frac{3}{7}$. Can you find 400 of them?* Another similar task was assigned as homework. None of the assigned tasks focused on looking for a fraction lying in the middle between two given fractions.

Data collection took place immediately after the participants had finished their official review test on the topic of fractions. For the purpose of our study, we assigned them two Concept Cartoons with indicative questions and asked them to answer the questions as individual written homework. The first Concept Cartoon focused on natural number bias in decimal numbers; the second one was the cartoon from Figure 1. When composing the indicative questions, we combined two different focuses: (1) *Which children in the picture are right? Which are wrong? Why?* (2) *How would you solve the task in the box?* Such a fragmented aim of questions had proved useful in our previous research (e.g. Samková, 2022), as it was able to pinpoint the instances when solution procedures were learnt correctly but without deeper conceptual understanding.

During data analysis stage, we processed responses to the Concept Cartoon from Figure 1. We analysed them qualitatively using open coding and constant comparison (Miles et al., 2014), and primarily focused on various traces of existence/nonexistence of natural number bias. Additionally, during the coding process, we also began to consider relevant excerpts that related to the ability to handle fractions autonomously (i.e. without employing decimal numbers). The comparison process went across participants, across bubbles, and across indicative questions. For the purpose of anonymity, the participants were randomly labelled by code names K1 to K47.

Findings

The four following code categories appeared as relevant at the end of the analytic process: Whole numbers (codes *explicit bias, assignment, operation*), Density awareness (codes *explicit density, indicated density, not sure*), Decimal numbers (codes *2.5 numerator, periodic numbers*), and Accent on own procedures (codes *correct alternative, incorrect reasoning, excessive disapproval*). Below, we describe the code categories in detail and provide illustrative data excerpts related to them.

Whole numbers

Some of the respondents let themselves be fooled by the Lina bubble, and responded exemplary in accordance with natural number bias. They either explicitly stated that there is no number between the given fractions, or particularized that Lina would be right if considering whole numbers:

- K39 Lina She is right. No number can lie between numbers $\frac{12}{5}$ and $\frac{13}{5}$, because they are in order.
- K29 Lina Lina, I think, is right, I see it the same way. In case the assignment is in whole numbers.
- K34 Lina Linda would be right if we were to look for numbers only in whole numbers.

Other respondents employed natural number reasoning in their response to the Jonas bubble:

- K25 Jonas He is not right. The result of the fractions is not divisible by two.

Density awareness

On the other hand, some of the respondents presented more or less explicitly their awareness of the density of rational numbers:

- K24 Lina She is not right. There are infinitely many numbers between 12 and 13.
- K10 Lina She is not right, we can always find a number in the middle. We just have to raise both numbers by 10, 100 etc.

However, sometimes we were not able to distinguish whether the awareness concerned really the concept of the density or just a learned procedure that related to the solved problem:

- K21 Lina She is not right. The important thing is that we solve fractions – raise by 2, get $\frac{24}{10}$ and $\frac{26}{10} \rightarrow$ between is $\frac{25}{10}$.
- K18 Lina If we raise the fractions, we obtain 9 more fractions in-between.

Decimal numbers

Many respondents based their solutions or explanations on decimal numbers. Some of them proposed a decimal number as the numerator, others used the fraction with the decimal numerator just as an intermediate result. All of them looked very similar, so we present just two samples:

- K07 Lina Between 12 and 13 can be numbers: 12.5; 12.6; 12.7... \rightarrow she is not right. When I divide $12:5 = 2.4$
 $13:5 = 2.6 \Rightarrow$ between numbers is the number 2.5
- K06 Lina Lina, there is not a whole number between 12 and 13, but there are decimal numbers \Rightarrow 12.1; 12.2; 12.3...
So, there are numbers between $\frac{12}{5}$ and $\frac{13}{5}$, for example $\frac{12.1}{5}$; $\frac{12.2}{5}$...
In the middle among them is $\frac{12.5}{5}$ which is $\frac{125}{50} = \frac{25}{10} = \frac{5}{2}$.

One of the respondents employed rather unusual, creative (but not quite correct) reasoning based on the decimal form of the two fractions in the (b) assignment:

- K06 Katrin Katrin, think! $\frac{4}{7}$ is roughly 0.5714 and $\frac{4}{9}$ is $0.\overline{44}$.
If one number is periodic and the other is complicated like this [*an arrow towards 0.5714 above*], then the number between them will also be complicated, and not $\frac{4}{8} = \frac{1}{2} = 0.5$.

Accent on own procedures

Majority of the respondents excessively referred to a solution procedure they preferred to the procedure offered or indicated in the cartoon; it concerned particular procedures (e.g. a procedure using the common denominator, the arithmetic mean procedure) or general approaches (e.g. raising the fractions, drawing a picture). Such a favouritism might be in order when the bubble is incorrect:

- K22 Katrin Katrin did not realize that she had to work with the same denominators, not numerators. It is necessary to determine the common denominator of the given fractions, i.e. to raise the fractions appropriately. After that, we just look for the number lying in the middle between the numbers in the numerators.
- K08 Katrin Katrin is not right. Instead of the common denominator, she was looking for the middle number in the denominator.

However, sometimes the preferred procedure included incorrect reasoning or incorrect terminology:

- K41 Katrin Katrin is not right. 8 is not a common denominator of numbers 7 and 9.
- K36 Katrin She is right for numbers 7 and 9. But if they are in a fraction, she has to transform them to the same base. ... Number $\frac{4}{8} = 0.5$ is 7 thousandths smaller than the number actually in the middle.

The favouritism of the own solution appeared also in reactions to the correct bubbles:

- K07 Jonas We would first have to convert the numbers to a common denominator, then his reasoning would apply (I can't add together numbers that have a different denominator).
- K19 Jonas Jonas is not right. This is how the mean is calculated. We have to raise the fractions.
- K37 Jonas No. If we add, for example, fractions a) $\rightarrow \frac{7}{4} + \frac{9}{4} = \frac{16}{4} = 4$, and $4 : 2 = 2$, so that it did not help us with solving the problem.
- K08 Ben Ben is not right. Comparing several pictures will not give me the middle number.
- K18 Ben We cannot tell from the picture what fractions lie between.
- K38 Ben Pictures cannot help us in this case. They could help when comparing 2 fractions, but the task does not ask us to do that.

Discussion and conclusion

As the answer to the research question “*What kind of reasoning can be revealed when a Concept Cartoon that indirectly aims at natural number bias in fractions is assigned to future primary school teachers after the topic of fractions and rational numbers has been covered in their mathematics content course?*” we may state that the reasoning was diverse. From the perspective of natural number bias, the responses went from explicit displays of this phenomenon (K39/Lina), through various implicit indications of it, to explicit displays of the awareness of the density of rational numbers (K24/Lina). The implicit indications of natural number bias varied from attempts to reassign the task to whole numbers (e.g. K34/Lina), through an inability to see that $8/4$ equals 2, or maybe that 2 can be considered a correct result in a task assigned in fractions (K37/Jonas), to incidental statements that the operation of addition or the operation of division is not possible for some fractions (K07/Jonas, K25/Jonas). From the perspective of the typology of natural number bias (Van Dooren et al., 2015), the data collection tool aimed at the density type and majority of the incorrect transfers found in data

belong to this type. However, the two last mentioned excerpts refer to the operations type. The displays of natural number bias appeared in data even though the respondents had finished all the respective lectures and seminars, and were just after the review test. Such a finding is in accordance with the ascertained persistence of the phenomenon (e.g. Van Hoof et al., 2020).

From the perspective of the topic of fractions and future primary school teachers, we found many responses that went outside the topic of fractions and sought the explanations either solely within the set of decimal numbers (K07/Lina), or at least with the help of them (K06/Lina). Unfortunately, such explanations are in conflict with the autonomous approach to the topic of fractions, so that the prospective teachers will not be able to apply them in their future pedagogical practice

In accordance with our previous research (e.g. Samková, 2022), the fragmented aim of the accompanying questions had again proved useful in revealing knowledge deficiencies rooted in procedures learnt correctly but without deeper conceptual understanding. The third set of illustrative excerpts from the *Accent on own procedures* category provide the needed evidence, especially the responses to the Jonas bubble presenting an alternative way of solving and to the Ben bubble referring to an unknown drawing. While disagreeing with these bubbles, some of the respondents displayed natural number bias even if their own solutions of the task were correct, others clearly showed that their knowledge is more procedural than conceptual: that their procedures are fixed to particular contexts and they are not able to generalize or alternate them (e.g. K19/Jonas, K38/Ben).

Similar as in previous CERME empirical contributions concerning natural number bias and future or in-service teachers (Tunç-Pekkan & Kılıç, 2015; Kleve & Solem, 2015), the tasks triggering natural number bias were challenging for future teachers, not only to solve but also to assess hypothetical pupils' solutions and comments. We started this paper with a reference to a study (Reitz-Koncebovski et al., 2022) that approached the topic of natural number bias with a similar mathematical task but different accompanying questions: their participants did not assess pre-prepared hypothetical pupils' solutions, instead they proposed their own ideas how pupils' solutions might look like. Due to this distinction, the results of the two studies are complementary and, if used with the same respondents, have the potential to provide us with a richer picture of respondents' professional knowledge.

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