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## ► To cite this version:

Dinah Reuter. Professional competencies of prospective teachers in the area of patterns and structures - a pilot study. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. <hal-04421405>

**HAL Id: hal-04421405**

**<https://hal.science/hal-04421405v1>**

Submitted on 27 Jan 2024

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# Professional competencies of prospective teachers in the area of patterns and structures - a pilot study

Dinah Reuter

University of Education, Freiburg, Germany; [dinah.reuter@ph-freiburg.de](mailto:dinah.reuter@ph-freiburg.de)

*In various countries, a special role is attributed to patterns and structures in mathematics education. Furthermore, various studies show the high importance of domain-specific professional knowledge for teaching quality. Therefore, it seems reasonable and important to take a closer look at teachers' professional knowledge related to patterns. In order to do this, a questionnaire was developed and piloted to investigate prospective elementary teachers' professional competencies in the area of patterns and structures. The analyses show a wide variety of responses and a vast amount of heterogeneity in all areas. The results suggest that it is important to focus even more on professional knowledge in the area of patterns and structures as well as pattern and structure recognition competences during teacher education and professional development.*

*Keywords: Patterns and structures, pattern recognition, teacher knowledge, mathematics teachers' professional knowledge, elementary school teachers.*

## Introduction

Mathematics can be seen as the science of patterns and structures (e.g., Sawyer, 1955; Devlin, 1998). This (fundamental) view of mathematics has found its way into the respective standards in various countries. In some countries it is defined as an independent area, with a superordinate function with regard to other content areas, e.g., arithmetic or geometry (Germany: Ständige Konferenz der Kultusminister, 2022). In other countries it has an integrated function within the listed content areas, such as algebra (e.g., USA: National Council of Teachers of Mathematics, 2000).

In the view of mathematics as the science of patterns and structures, working with patterns and structures can be classified as a fundamental mathematical activity. Thus, fostering children's ability to recognise patterns and structures is of high importance (e.g., Käpnick, 1998; Reuter & Bruns, 2022). For a clarification of the terms *pattern* and *structure* and a related distinction between the two terms, no consistent definitions can be found in the literature. On the one hand, this is due to the fact that these terms can also be found in everyday language without a clear delimitation of content. On the other hand, it can be seen with regard to mathematics that the concept of structure in particular is understood in different ways in the various subfields. According to Lüken (2012b, p. 22, translated by author), however, "consistent basic features in the understanding of the terms" can be identified. Thus, Mulligan and Mitchelmore (2009) describe *pattern* as the occurrence of a regularity between objects that stand in a numerical, spatial, or logical relationship to one another, and *structures* as "the way a mathematical pattern is organised" (p. 34).

Recognising a mathematical pattern is challenging to children as they must detach from individual objects: For example, they have to interpret numbers in a more abstract way to identify a general relationship or structure (Akinwunmi, 2013). Or, as Battista and Clements (1996) put it, "[w]e assume that an individual does not 'read off' a structure from objects, but instead, creates a structure of his or her mental actions concerning the objects" (p. 282). Accordingly, pattern recognition is an

individual interpretive performance that is based on the ability to recognise patterns and structures (e.g., Lüken, 2012b).

Recent research indicates a large correlation between pattern and structure recognition and mathematical development as well as performance (e.g., Lüken, 2012a; Mulligan & Mitchelmore, 2009; Rittle-Johnson et al., 2019; Schmerold et al., 2017). This concerns associations between pattern and structure recognition and cognitive flexibility (Bock et al., 2015, 2018; Schmerold et al., 2017), working memory (Bock et al., 2018; Schmerold et al., 2017), and general mathematical ability, which in this context has typically been operationalized via arithmetic ability (Lüken, 2012a; Schmerold et al., 2017). Moreover, several studies demonstrate an impact of pattern and structure recognition on mathematical performance (e.g., Rittle-Johnson et al., 2019; Schmerold et al., 2017) and indicate that promoting pattern and structure recognition leads to improved arithmetic performance (Lüken & Kampmann, 2018; Hendricks et al., 2006).

To summarise, fostering children's recognition of patterns and structures plays a key role in the development of children's mathematical ability and is therefore given special importance in mathematics education. In order for teachers to be able to promote children's understanding of patterns and structures, they need a good understanding of patterns and structures and their importance for mathematics themselves. This leads us to the professional competencies of teachers.

### **Professional competencies of teachers**

For many years, scholars have grappled with the issue of teacher professionalisation. In this context, many of the studies of the past 30 years tie in with Shulman (1987), who conceptually described teacher professional knowledge by distinguishing between different facets. On this basis, various other competency models have been developed that include various other (e.g., affective) components in addition to cognitive aspects (e.g., Blömeke et al., 2010; Prediger, 2019a).

For example, Prediger (2019a) has developed a conceptual framework for content-specific teacher expertise, which consists of five constructs (Jobs, Practices, Pedagogical Tools, Orientations and Categories). There are close relationships between these constructs, based on the idea that the requirements for content-specific teacher expertise can be described via the analysis of specific teaching content. In this framework the cognitive aspects represent only one of five constructs. It is the construct *Categories* which encompass specific levels of knowledge and include both domain-specific areas of competencies, content knowledge (CK) and pedagogical content knowledge (PCK) and domain-independent areas of competencies like pedagogical and psychological knowledge as well as organisational knowledge (Shulman, 1987). Following Bromme (1992), the starting point of this approach is the concrete instructional requirements (Prediger, 2019b), which serve to identify the required professional competencies. Thus, an explicit combination of "a situated perspective on teachers' classroom practices with a cognitive perspective on underlying knowledge categories and orientations" takes place (Prediger et al., 2022, p. 6). The direct reference to the respective teaching content is accompanied by the fact that the professional competencies are directly understood in a content-specific way and thus have to be considered separately for each specific content area. According to Prediger et al. (2022), the model offers the possibility to be used in a *prescriptive mode*

by analysing for a specific domain which pedagogical tools, categories, and orientations teachers should have.

## Research questions

Various studies show the high importance of domain-specific professional knowledge for the quality of teaching (Ball et al., 2008; Blömeke et al., 2010). Research results for various areas of mathematics teaching are available, providing valuable information about where training and professional development programs should take place in order to improve teachers' domain-specific professional knowledge (e.g., Prediger, 2019a; Prediger et al., 2022; Schulz, 2014).

In view of the high importance attributed to patterns and structures, it seems reasonable and necessary to also take a closer look at teachers' professional knowledge specifically for this area. In Germany, the idea of *productive practice* is one way of trying to address this importance. "Productive practice (...) integrates the practice of skills with the exploration and explanation of patterns" (Wittmann, 2021, p. 274). Therefore, in Germany, mathematics textbooks for elementary school are permeated with various tasks on patterns and structures following the idea of productive practice, such as arithmetic sets (see an example of such tasks in Figure 1). The tasks in these sets have an operative relationship to one another (structure), which in this example can be traced back to a regularly change (pattern) of the second summand: In the left packet the first summand remains constant (structure) while the second summand is reduced by one (structure) in each row (pattern), hence the sum is reduced by one as well (structure).

$$\begin{array}{rcl} 7 + 4 = \_ & & 4 + 4 = \_ \\ 7 + 3 = \_ & & 3 + 4 = \_ \\ 7 + 2 = \_ & & 2 + 4 = \_ \\ \_ + \_ = \_ & & \_ + \_ = \_ \end{array}$$

**Figure 1: Example of arithmetic sets**

Students are not only asked to calculate these tasks, but are encouraged to continue them, describe their discoveries, and explain the relationships. However, there is no evidence on the extent to which teachers appropriately guide the discovery and description of such relationships, thereby fostering children's recognition of patterns and structures.

In the context of this pilot study, following Prediger's framework (2019a), conceptual knowledge is exemplarily considered as a subdomain of the construct categories by focusing on conceptual understanding in the domain of patterns and structures. Conceptual understanding includes not only conceptual knowledge but also the teachers' ability to recognise patterns and structures. This leads to the following research question: Which conceptual understanding of patterns and structures can be found among prospective elementary school teachers?

## Method

In order to investigate this research question, an explorative design was chosen, which enables the targeted collection of data "to shed light on different aspects of an issue" (here: Professional competencies of prospective teachers on patterns and structure) to be able to describe them "subsequently in a differentiated way" (Döring & Bortz, 2016, p. 192; translated by author).

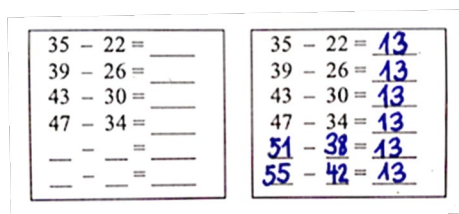
In a first step, a guided interview (Döring & Bortz, 2016, pp. 372–374) was developed to assess teachers’ professional competencies related to patterns and structures. Following Prediger’s framework (2019a) in developing the questions, an analysis of the specific area of patterns and structures with regard to the different levels of knowledge (e.g., CK and PCK, including classroom-related tasks) was carried out. Through subsequent interviews, initial insights into teachers’ professional competencies related to patterns and structures were obtained.

In contrast to an interview, questionnaires offer the possibility of obtaining answers from many people in a short period of time. In empirical educational research, questionnaires are a frequently used survey instrument to record knowledge and opinions (Reinders, 2015). Therefore, a pilot questionnaire was developed by condensing the questions from the guided interview.

In the following, I will focus on two items of the pilot questionnaire that address teachers’ conceptual understanding of patterns and structures.

### Overview of the task selection

To assess teachers’ understanding of patterns and structures, teachers had to describe their understanding of both terms as an answer to the three open questions: “What is a *pattern*?”, “What is a *structure*?” and “How do *pattern* and *structure* differ?” These tasks were chosen for this study as they might give a good overview with regards to teachers’ conceptual understanding of patterns and structures.



**Figure 2: Classroom-related task on teachers’ understanding of patterns and structures**

In addition, to provide further insight into teachers’ understandings of the concepts of patterns and structures, and how they apply this understanding on classroom tasks, we included the task shown in Figure 2. The prospective teachers were asked to provide a correct student solution for the task. In these types of tasks students are usually asked to calculate results, fill in blank spaces to continue the pattern, describe their discoveries, and explain the relationships. Therefore, teachers need to understand the task themselves and be able to identify and explain the structure and the pattern in order to accompany the discussion. The answers of this task provide information about the ability of the prospective teachers to recognise patterns and structures.

### Sample

The questionnaire was piloted with a casual sample of N=39 prospective teachers. These were prospective elementary school teachers who were at the end of their studies and participated in a mathematics education course that was part of their master studies at the PH Freiburg. In previous courses so far, the view of mathematics as the science of patterns and structures has been repeatedly addressed from both a content-specific and a pedagogical content perspective.

## Data analysis

The open-ended questions from the questionnaire are evaluated with the help of Qualitative Content Analysis (Mayring, 2022). This is an established method for the evaluation of qualitative data, which is characterised by a systematic and rule-guided procedure. The basis for this method is the development of a category system that contains both deductive and inductive categories. Furthermore, this form of evaluation also offers the possibility of a supplementary quantitative analysis for further interpretation of the results. The basis for the creation of the deductive categories were terms that are often used in the literature in the context of patterns and structures (e.g., *repetition*, *regularity*, *continuation*, *arrangement*, *organisation*) as well as possible links and relationships between these terms.

## Results on prospective teachers' understanding of the terms *pattern* and *structure*

The analyses of the data confirm a vague and undifferentiated understanding of the two terms *pattern* and *structure*. For the explanation of both terms, the prospective teachers often use words like “regularity”, “repetition”, and “possibility of continuation”. Thus, responses can be found in which the term structure is not distinguished from the indications of the term pattern as in these examples: “A pattern is something that repeats itself” as well as “A structure repeats itself.” In some other answers, on the other hand, terms such as “arrangement” or “manner of construction” are used for the term structure to distinguish it from the term pattern, such as: “A structure is an arrangement in a pattern”. In addition, there are answers in which the terms pattern and structure are related to each other, such as: “A structure stands above a pattern” or “A structure is a smaller unit than a pattern”.

With this in mind, the descriptions for the words could be assigned to four different categories: The category ***misleading understanding*** was assigned to descriptions which, due to the wording, show insufficient definition and are ambivalent, as for example in the description for a pattern “In a pattern, symbols repeat more or less regularly”. Here the regularity underlying a pattern is weakened by the formulation “less regularly”. Most of the responses were categorised into the category ***undifferentiated understanding***, that included the answers in which aspects describing the terms pattern and structure were mixed, e.g., for the term *structure*: “The arrangement occurs regularly over and over again”. The category ***differentiated understanding*** was used for responses providing a suitable description, e.g., for the term *structure*: “The structure is the construction and interrelationships of the individual parts of a whole”. In addition, some descriptions could be assigned to an ***everyday understanding***, e.g., for the term *pattern*: “A beautiful picture that has something of art”.

Regarding the usage of the terms in different mathematics areas, geometric patterns are more often referred to in connection with the term *pattern*, while arithmetic relationships are also related to the term *structure*. Some answers are limited to statements such as “patterns consist of shapes” or “a pattern is the big picture”, so that no clarification of content could be obtained.

Prospective teachers' answers to the classroom-related task (Figure 2) provide insight into what structures are recognised and which are taken as relevant and how the findings are presented. The responses could be classified into four main categories with further differentiating subcategories. The categories differ in terms of the amount of processing and the associated insight into the teachers' pattern and structure recognition. They range from answers in which only calculations were made but not continued to answers in which calculations were made, continued, described and explained.

Particularly striking here were responses showing that no relationships were recognised, as in the example “The result is always 13, but no further correlation can be seen between the individual tasks” and answers that did not provide a valid structural description, e.g., “The result is always 13. There are three pairs. The tens of the first number increase by 1 ten per pair”.

## Discussion

The results are consistent with those from the interview study and show a large heterogeneity in terms of conceptual understanding and structural recognition competence. To explain the terms *pattern* and *structure*, the prospective teachers use many different words. In doing so, they often use the same words to describe both terms. However, it is striking in the descriptions that there is exactly one word - the German word “Aufbau” - translated as “organisation” - that was not used for both explanations. It was used exclusively to explain the term *structure*. All other words are used equally to describe both terms, including the word “regularity”, which according to Mulligan and Mitchelmore (2009) can be classified as a reference point for the term *pattern*. Results show that a large proportion does not have consistent knowledge about the terms pattern and structure. Instead, in many cases the two terms are explained by referring to the same words or are even used synonymously, and only in a few cases consistent and suitable descriptions are given. Furthermore, the prospective teachers show high differences regarding their own pattern and structure recognition competence.

In light of the underlying content framework (Prediger, 2019a), these initial findings seem to be problematic. Conceptual knowledge, as part of the construct *Categories*, is a fundamental aspect for successful teaching. It seems to be important that teachers are able to distinguish between pattern and structure aspects, e.g., in classroom-related tasks, as this is fundamental with regard to description and explanation processes. The results indicate that prospective teachers, however, due to their diffuse understanding of concepts, do not have sufficient knowledge to support and thus promote children’s understanding of patterns and structures in this respect. However, this is exactly what is relevant for children’s further mathematical learning. This poses a great challenge for teacher education, as it is necessary to enlighten how it is possible to support teachers in expanding their understanding of patterns and structures.

Thus, with regard to teacher professionalisation, it seems important to continue the study on a larger scale to find out to what extent the results from the piloting will be confirmed. In this case, it might be important to focus even more on professional knowledge in the area of patterns and structures in initial and (in-service) teacher education.

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