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Representational accuracy in pre-service teachers' analyses of a classroom vignette on a students' difficulty with fractions

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Supportive teacher reactions to students' mistakes require an accurate analysis of the student's thinking. The use of representations of mathematical objects is often a key criterion for such an analysis and allows to detect students' difficulties at the origin of mistakes. However, empirical research on whether pre-service teachers are able to accurately carry out such analyses of classroom situations is still scarce. Consequently, this study examines whether 56 pre-service teachers, who had previously been introduced to a mathematics education background related to representation registers, actually used this analysis criterion framework when confronted with a vignette showing a classroom situation in which a student shows difficulties with fraction calculation. The analysis yielded that only in exceptional cases, pre-service teachers accurately used criteria related to representations – which calls for reinforced support through vignette-based learning opportunities.

Keywords: Representational accuracy, vignettes, noticing, analysis of classroom situations.

Introduction

Representations play a key role in mathematical practice and hence also for its learning and teaching. As mathematical objects are abstract in nature and can only be accessed through the use of representations (Ainsworth, 2006; Duval, 2006): any interaction in which mathematical thoughts are articulated needs the use of representations which stand for the mathematical objects involved in the respective mathematical thoughts. Consequently, looking at how representations are being used is a very powerful analysis tool in teachers' analysis of classroom situations and student's mathematical thinking (Dreher & Kuntze, 2015; Friesen & Kuntze, 2021). In our approach to teachers' analysis of classroom situations, we build on research into noticing (Sherin et al., 2011) and emphasise the aspect of knowledge-based reasoning – in this case, criterion knowledge related to ways of representing mathematical objects plays a key role in the analysis. Even if a hierarchical competence model has shown to describe teachers' analysis of how representations are dealt with (e.g. Friesen & Kuntze, 2021), relatively little is known about whether teachers are able to accurately analyse situations, in which different ways of representing are mixed and have to be separated accurately in the analysis of students' thinking – we call this analysis quality aspect *representational accuracy*. The vignette-based study presented here thus focuses on this research need. In the following, we will present (1) the theoretical background of this study, which leads (2) to its research questions, inform about (3) design and methods, present (4) results and (5) discuss these in a concluding section.

Theoretical background

Building up mathematical knowledge without any knowledge on representations of mathematical objects is not possible (Duval, 2006). Mathematical objects, which are abstract in nature can mostly be represented in different ways; using such different ways of representing as well as converting a

representation into another is key for mathematical competence (Ainsworth, 2006; Duval, 2006). Different ways of representing (according to specific representation rules) are called *registers* of representation (Duval, 2006). So-called *conversions* from one representation register into the other are also based on knowledge about translation rules between the registers. If, for instance, a fraction is represented in the representation register of circle diagrams, and if it is to be translated into the register of symbolic fraction notation, a learner has to know that the whole circle represents a whole, that the number of equal parts the circle is cut into is the number which has to be taken as denominator in the symbolic fraction notation register, etc. Using multiple representations for fractions has been emphasised as helpful for building up flexibly usable mathematical knowledge (e.g. Lamon, 2001). This is also the case for any other content areas, as different representations tend to emphasise different aspects of mathematical objects (Ainsworth, 2006; Duval, 2006). For this reason, being able to change representations is often also a key to problem solving, as the representational change may bring hidden properties of a mathematical object to the foreground (Lesh et al., 1987). However, changing between representation registers has shown to be demanding to learners (Ainsworth, 2006), which highlights the importance of knowledge about representation ‘rules’ in registers and conversion ‘rules’ of translations between registers.

The teacher’s analysis plays a key role when accompanying the students’ learning with this respect: Criteria related to representations are key for describing a student’s difficulties, in case representation registers or certain types of conversions between representations are not mastered, and for providing help which is accurately adapted to the learner’s needs (e.g., explaining connections between representations in different registers). For the notion of teachers’ *analysis*, we build on theoretical and empirical approaches to teacher *noticing* (Fernández & Choy, 2020; Sherin et al., 2011), with a specific focus on teachers’ *knowledge-based reasoning* (Sherin et al., 2011). In our model (Kuntze & Friesen, 2018), teachers’ analysis is described by circular process between observations in the classroom situation and their interpretation against the teachers’ professional knowledge and views. As far as criteria related to dealing with representations is concerned, prior research has focused in particular on whether teachers are able to analyse whether different representation registers were connected or disconnected in teacher reactions (e.g. Dreher & Kuntze, 2015; Friesen & Kuntze, 2021). This has shown to be a hierarchical competence which can be assessed with a vignette-based test, and it can be fostered, e.g. in pre-service teacher education (Friesen et al., 2015). However, building up analysis skills has also shown to be difficult for some pre-service teachers (Friesen et al., 2015), so still, building up this competence can be considered as a complex endeavour. This highlights the need to identify potential difficulties of particularly pre-service teachers: whether they are able to accurately identify registers of representation is likely to make a difference for analysing teacher reactions and corresponding decision-making. In particular, relatively little is known about whether teachers are able to accurately analyse situations, in which different representation registers are mixed and have to be separated accurately in the analysis of student thinking – we call this quality aspect *representational accuracy*. In order to master requirements associated with representational accuracy, professional knowledge (e.g. Kuntze, 2012; Shulman, 1986) is needed, both mathematical content knowledge (CK) and pedagogical content knowledge (PCK) on dealing with representations (as introduced above, e.g. Duval, 2006). Related awareness (Kuntze & Friesen, 2018) helps to

actively use corresponding professional knowledge in the analysis (e.g. Dreher & Kuntze, 2015). The knowledge components mostly have to interact when analysing classroom situations. We will discuss an example in the design and methods section below, showing the corresponding requirements of representational accuracy in the teachers' analysis.

Research interest

As representational accuracy in the analysis of student thinking in classroom situations can play a key role for teacher's ways of reacting adaptively to the learner's needs, this study explores in a first approach whether and to what extent teachers are able to accurately analyse a correspondingly purposefully designed vignette. This vignette shows a classroom situation in which a student ("Enzo") has difficulties with a calculation of mixed fractions. In order to assure the teachers had access to PCK related to the use of representations (as introduced above, Duval, 2006) we chose a sample of pre-service teachers who had previously been offered corresponding learning opportunities in a university course. In order to find out about these pre-service teachers' analyses and the representational accuracy in the analyses in particular, we focus on the following research questions:

- (1) Do the pre-service teachers accurately distinguish registers used by the vignette student in their analysis of the classroom situation vignette?
- (2) Which role does potential representational accuracy play within the pre-service teachers' analysis, i.e. which other potentially meaningful characteristics does their analysis show, such as, for instance, (2a) the mathematical description of the student's difficulty? Do the teachers suggest (2b) the use of an additional graphical representation register in order to support the students' understanding in this case of fraction calculation?

Design, sample and methods

For answering the research questions given above, 56 pre-service teachers (48 female, 8 male, most of them at the beginning of their master's degree studies) were asked to analyse a purposefully designed vignette in cartoon format. Before working on this vignette, the pre-service teachers had received learning opportunities related to the theory elements of representations, representation registers and conversions of representation between registers. Moreover, they had been given opportunities to use this knowledge in the analysis of classroom situations as well as of task and textbook material which were part of these situations.

The vignette started with a short introductory description of the classroom situation context: It is mentioned that the content of the lesson is about solving problems of adding/subtracting mixed fractions with different denominators, followed by a short text: "Xavi is a pre-service primary school teacher doing his internship in Lara's classroom (5th grade). During the first part of the internship, Xavi has to observe Lara's teaching. [...]. Today the maths lesson is focused on solving problems involving adding or subtracting mixed fractions with different denominators. Lara proposes the following problem:" – the follow-up cartoon images of the vignette are shown in Figure 1. After viewing the cartoon slides, the pre-service teachers were asked questions on the classroom situation: "(1) Describe the understanding of Enzo's group. Justify your answer based on extracts from Enzo's

answers and the mathematical elements in it. (2) Starting from the answer by Enzo's group: If you were the teacher, what would you do next? [...]"



Figure 1: Cartoon vignette on Enzo's answer (translated key cartoon slides)

The classroom situation was presented in vignette form. Vignettes are considered as representations of practice (Buchbinder & Kuntze, 2018), which can be purposefully designed so as to represent also relevant theory and learning goals (Skilling & Stylianides, 2020) and elicit the teachers' analysis. As a base for the analysis of the pre-service teachers' answers, the vignette (Figure 1) was analysed from the representation perspective. Of course, other interpretations of the vignette are possible. In the vignette, two main representation registers can be detected: A language-based situation context register of "pizzas in the fridge", and a symbolic register of fraction notation. In both of them, it would have been possible to carry out the full solution process of the given task. As a possibly helpful additional representation register, a further graphical "pizza register", e.g. with circles, could have been introduced, e.g. in a thinkable teacher reaction.

In Enzo's answer, the first two registers are somehow mixed or amalgamated. For instance, the operation "to subtract" belongs not to the register of *pizzas in the fridge* (which can rather be "eaten up" or "taken away"), but to the symbolic calculation register of mixed fractions and fractions written in their symbolic number notation. Enzo probably has integrated knowledge linking situation context register and operations with natural numbers – probably it is very easy for Enzo to change between these registers in the case of natural numbers (possibly for this reason he also mixes the register-specific languages: "We subtracted from the pizza that was in the fridge...").

The given task uses the expressions $2\frac{3}{8}$ and $1\frac{1}{4}$ within the text-based situation context register. It would require a conversion of representation from the students to imagine a context-based representation of how 2 whole pizzas and 3 pizza pieces (of originally 8 equally big pieces cut from one pizza) could be stored in the fridge (in line with the "pizza in the fridge" register). Without this conversion, the objects " $2\frac{3}{8}$ " and " $1\frac{1}{4}$ " within the "pizza in the fridge" register have the status of "representational aliens", as they belong to the register of symbolic fraction/mixed fraction notation.

In the pizza context register, from which subtractions of whole numbers probably can be easily converted back and forth by Enzo, the natural number subtraction is an operation at hand. The corresponding calculation rules are closely connected across the registers for him. So it is plausible that he uses the natural number operation and calculation laws for all components of the mixed fractions – even if he might not necessarily have a general problem with fractions in all thinkable representation registers he might have learned of.

In conclusion and acknowledging that this is not be the only possible interpretation of the vignette, the analysis shows that distinguishing the two registers appearing in the vignette allows more specific insight and deeper follow-up analysis than if the registers were just amalgamated without distinction.

The pre-service teachers' answers were collected in the form of written texts. This data was analysed in a top-down interpretive coding procedure according to pre-defined and theory-based categories (Mayring, 2015). For an easier reading, the categories will be presented together with the code frequencies in the results section. Accounting for possible individual "versions" of representational accuracy, the analysis of the empirical data was done according to the justifications given in the respective answer. For research question (1), it was coded whether there was evidence of a distinction between registers related to the mathematical objects which appeared in the vignette student's answer, as required for representational accuracy. For research question (2), it was additionally coded whether the pre-service teachers gave a mathematical analysis of the mistake in Enzo's answer, and whether they suggested using an additional graphical representation register.

Results

Figure 2 presents overview results of the coding related to representational accuracy (research question (1)). The evidence indicates that pre-service teachers' answers with evidence of representational accuracy are rather exceptions. In a majority of the pre-service teachers' answers, there is evidence of an amalgamation of registers (code "3" in Figure 2). Here is a sample answer:

"Enzo's group subtracts from the left-over Pizza, i.e. from $2\frac{3}{8}$, the $1\frac{1}{4}$ eaten-up part by Emma's parents. The solution of the group is $1\frac{2}{4}$. It is correct that the group subtract the whole "pizzas"

from each other, so $2-1$ equals a whole. Afterwards they subtract the denominators and numerators from each other and forget about taking into account the calculation rules for fractions. Enzo's group would have had to expand the fractions to have a common denominator (common multiple), so as to be able to subtract the fractions from each other."

In this answer, Enzo's amalgamation of registers in the vignette is taken over by the pre-service teacher and varied (e.g. " $2-1$ equals a whole", where Enzo uses the pizza register, and "the $1\frac{1}{4}$ eaten-up part by Emma's parents" where Enzo uses numbers outside of the pizza context in his calculation). Hence, the registers are not distinguished and corresponding representations are just mixed, as if the "pizzas" were directly part of the symbolic notation register, in which also the calculation rules with the common denominator are rooted, which are emphasized by this pre-service teacher.

In contrast to answers with evidence for the amalgamation of representation registers, the following examples from other pre-service teachers contain evidence that the registers are distinguished, the first example concerning the subtraction as mathematical object which is being considered:

"First of all, it is good that Enzo's group has understood that it turns about 'subtracting' here. The parents eat pizza, which means that pizza "goes away" and that less pizza is in the fridge. Consequently, the group has used the right calculation sign and reached the understanding of a 'subtraction task'".

This answer was interpreted to underline that a kind of translation had to be made from the pizza context to the calculation context, even if representation registers were not mentioned explicitly. The following example answer focuses also on the mixed fractions/fractions as mathematical object(s):

"○ They recognise that it turns about the subtraction here (context situation of taking away/detracting).

○ It appears however, as if Enzo's group cannot imagine anything under the notion of fraction (basic understanding of fraction is missing), or at least as if they do not know, what the $\frac{3}{8}$ mean in this context, as he is at first speaking of pizza ("two pizzas minus one pizza, remains one pizza"), but with the fractions, he is unable to denominate them in [pizza] pieces, for example. Here he clearly changes to the symbolical level without taking into account the [pizza] situation context ("and then $3 - 1 = 2$, and $8 - 4 = 4$ "). Equally [the fact] that he is unable to say afterwards, that one pizza and 2 small quarter pieces of a pizza are left (or similar) allows the implication that he is not able to translate his result back completely to the context situation."

Again in this answer, the notion of "register" is not used explicitly, however the distinction appears to have been made consistently and the aspect of "translating" is emphasised. There is thus evidence of representational accuracy and related awareness, even if the corresponding attending and interpretation could have been deepened. The example stands for code "1" in Figure 2 (left and right)).

However, the numerous amalgamations and consequently this lack of representational accuracy does not mean that these pre-service teachers' answers were completely inadequate also with respect of other indicators of their analysis. Figure 3 presents the results related to the second research question (questions (2a) and (2b)). These data show that a majority of the pre-service teachers gave a

mathematical analysis of Enzo’s mistake and close to 80% suggested using an additional graphical register in the reaction to Enzo, in most cases to support the understanding of the fractions.

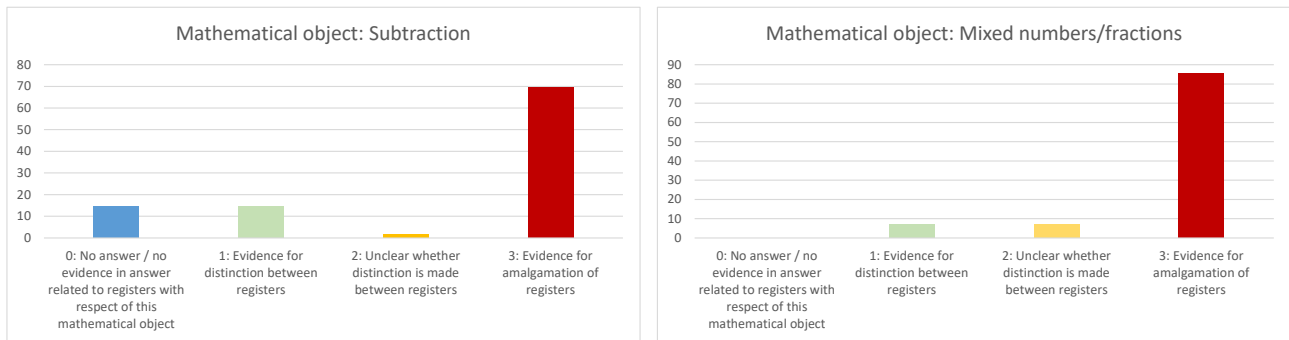


Figure 2: Code frequencies related to distinguishing registers concerning the subtraction (left hand side) and the mixed fractions/fractions (right hand side) as mathematical object(s)

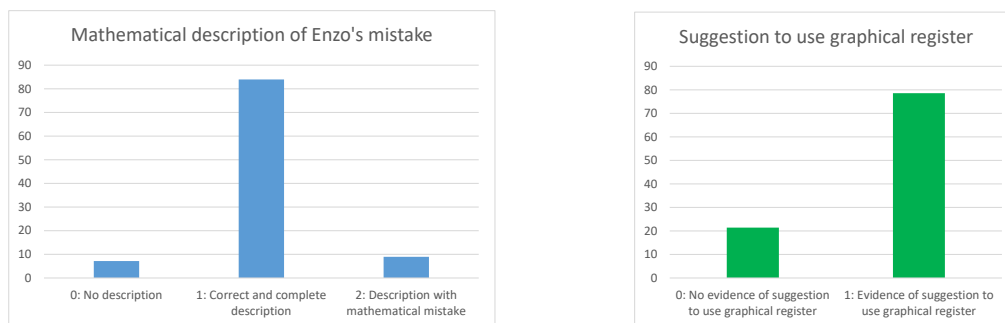


Figure 3: Code frequencies related to the mathematical description of Enzo’s mistake (left hand side) and to suggestions to use a graphical register (right hand side)

Discussion and conclusions

Even if the study has limitations such as its non-representativeness and the restriction to a one-vignette design, the results show that a large majority of the teachers did not show evidence of representational accuracy in their analysis answers. Still, most of them noticed other relevant aspects of the situation. However, for analysing classroom situations according to representations systematically, corresponding practices in knowledge-based analysis are needed (e.g. Dreher & Kuntze, 2015; Friesen et al., 2015). Awareness for possible analysis questions (e.g. “which representation registers appear in the classroom situation?”) might have been missing for the majority of the pre-service teachers, or, alternatively, their own fraction-related CK had such strong links between registers that they might have noticed only symbolic fractions, despite the pizza context. For improving pre-service teachers’ representational accuracy, the findings call for reinforced vignette-based work in teacher education, in order to provide the teachers with a spectrum of challenging possibilities to use their theoretical knowledge for accurate analysis.

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