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Inquiry to me: Four mathematics teachers' conceptualizations of inquiry preceding a professional development course

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Despite the centrality of inquiry in educational research, teachers might not have a clear idea of what inquiry entails. In this study, four teachers, ready to enter a professional development course, were asked about their conceptualizations of inquiry in teaching and learning mathematics. The teachers emphasized multiple aspects that are also emphasized in research and in the new Norwegian mathematics curriculum, such as open structures with few prescriptions and students exploring mathematical connections. However, other key aspects were only vaguely conceptualized, e.g., collaborative and argumentative features and the teachers' role in inquiry in mathematics. Based on the findings, I propose that mathematics professional development courses should portray inquiry as a daily approach, balance inquiry examples and experiences, and promote teacher (inter)actions to support the inquiry teacher.

Keywords: Mathematical inquiry, professional development, teacher conceptualization.

Introduction

Inquiry is promoted in educational policies and implemented in mathematics curriculums worldwide (Maaß et al., 2017). However, the teachers, who ultimately are responsible for bringing inquiry to life in the mathematics classrooms, often have limited experience with inquiry both as students and as teachers. Moreover, inquiry might not manifest itself as a part of teachers' practices because they "simply do not understand what inquiry is" (Capps et al., 2012, p. 292). Teachers' conceptions of inquiry in teaching and learning mathematics contribute to their amalgam knowledge of mathematics, students, and teaching – their pedagogical content knowledge (PCK) (Shulman, 1986) – as inquiry represents a powerful learning approach for representing and grappling with mathematical concepts through activities that draw on students' own knowledge and ideas. In the dynamic relationship between conceptions and enactment, teachers' PCK can both inform their teaching and be informed by their teaching. Thus, how teachers conceptualize inquiry might both have implications for, and reflect, how it is realized in their mathematics classrooms.

Conceptualizing inquiry here means the forming of an idea of what inquiry entails. This includes identifying various features of inquiry and recognizing pedagogical decisions and classroom actions that are considered essential to inquiry. Research on mathematics teachers' conceptualizations of inquiry is scarce, but Maaß et al. (2017) reported three main ways teachers viewed inquiry-based learning at the beginning and end of a professional development (PD) program: as recreational activities to motivate students, as various approaches to promote student thinking, or as a way to support mathematical learning. Studies in science imply that teachers' conceptualizations of inquiry are vague and often emphasize exploration more than other qualities (e.g., Romero-Ariza et al., 2020).

This paper is part of a research project following four in-service Norwegian secondary school teachers as they engage in an out-of-school PD course in mathematics that draws on inquiry in multiple ways.

Maaß et al. (2017) found that PD can promote more sophisticated interpretations of inquiry within teachers' existing views. It is therefore relevant to illuminate teachers' pre-PD conceptualizations of inquiry – which is the focus of this paper – to gain knowledge about possible ideas teachers might have and inform course designs that address and build on those existing conceptions. I explore the following research question: *what characterizes four teachers' conceptualizations of inquiry in teaching and learning mathematics before they enter a PD course?* The focus is on their conceptualizations related to three facets of inquiry, seen as a natural triplet that is easily experienceable (can be observed and felt “in-action”), frequently highlighted across research, and that forms a foundation for inquiry in teaching and learning mathematics: the role of *the students*, *the teachers*, and *mathematical inquiry problems*. Further, I reflect on how the findings from this study can inform the design of mathematics PD courses that seek to promote inquiry.

Conceptualizing inquiry in teaching and learning mathematics

Inquiry remains a notion without a universally agreed-upon definition (Laursen & Rasmussen, 2019). However, it is often loosely described as a way for students to work with mathematics like mathematicians do. I adopt this flexible description, and stress that through inquiry-based approaches, students actively engage in processes of exploration, argumentation, communication, and evaluation of mathematical ideas and relationships. These processes are enhanced by collaborative actions where students share their ideas, listen to and try to make sense of other students' ideas, build on, challenge, and justify their own and each other's reasoning, and negotiate and assess mathematical strategies (e.g., Artigue & Blomhøj, 2013; Laursen & Rasmussen, 2019; Naalsund et al., 2022). Inquiry can thus enrich the student voice and responsibility for learning as students “reinvent or create mathematics that is new to them” (Laursen & Rasmussen, 2019, p. 131). The teacher is often portrayed as a guide, orchestrator, or broker between students' ideas and formal mathematics (e.g., Kuster et al., 2018; Laursen & Rasmussen, 2019), seeking to prompt and illuminate inquiry through purposeful interactions that focus on students' explanations and justifications. This includes using students' reasoning as starting points in one-to-one interactions and plenary discussions. Another key teacher role is to structure fruitful inquiry activities (e.g., Laursen & Rasmussen, 2019). Open tasks, or problems, with multiple methods and solutions are often used in such activities to promote exploration, argumentation, communication, and evaluation. The problems used can subtly focus the students towards mathematical ideas and connections while at the same time give encouraging them to build on what they already know (e.g., Bråtlien et al., 2022; Laursen & Rasmussen, 2019).

The study is situated in Norway, where inquiry has been implicitly circulating in mathematics curriculum for a while and is central in the new mathematics curriculum (e.g., Norwegian Directorate for Education and Training, 2020). The Norwegian language does not have a direct equivalent to ‘inquiry’. A commonly used translation of the notion is ‘utforsking’. The new curriculum highlights searching for mathematical patterns and connections, discussing to reach a shared understanding, and students developing persistence, independence, and increased awareness of their own learning, as key to mathematical ‘utforsking’ (e.g., Norwegian Directorate for Education and Training, 2020). It is also emphasized that students, when engaging in mathematical ‘utforsking’, should focus on possible strategies and methods rather than their final solutions. In everyday language, ‘utforsking’ is used for exploration-like processes (whereas inquiry, as emphasized above, involves more than exploration).

Method

Four teachers (Alex, Cameron, Darcy, Eli) volunteered to participate in the research project for its full duration. Three of them worked in upper secondary schools and one in a lower secondary school, and their teaching experience ranged from one to nearly 15 years. They all reported that their mathematics classroom practices were quite teacher-centred. I use data from one individual, semi-structured pre-PD interview with each teacher, conducted in the summer of 2022. The aim of the interviews was to get familiar with the teachers' conceptions of inquiry in teaching and learning mathematics (*Q: What do you think inquiry is in mathematics?*). Follow-up questions were asked to highlight their conceptions related to the teacher and student roles and mathematical inquiry problems, and to address any potential or challenges the teachers themselves had experienced or believed that inquiry could bring. This gave rich insight into their conceptualizations of inquiry.

Transcriptions of the interviews were thematically analysed (Braun & Clarke, 2006), guided by the research question. Critical events were identified where a teacher expressed thoughts on what inquiry in teaching and learning mathematics entailed or on its purpose (e.g., motivating students). Instances were labelled based on the facet of inquiry they addressed (students, teachers, problems), and coded according to their underlying meaning with codes deducted from theory (Table 1). An utterance could be given multiple codes within one label, and it could be given multiple labels if it addressed multiple facets. Further, themes were extracted from recurring codes within and across transcripts and from links between codes (Braun & Clarke, 2006).

Table 1: Coding guide, codes deducted from the theory section

Label	Code
Students	<i>Actively engage; responsibility; connect existing knowledge; use knowledge in new situations; reinvent mathematics; share ideas; listen to ideas; build on ideas; assess; question; justify.</i>
Teachers	<i>Structure fruitful activities; few prescriptions; guide; support; ask for justifications; inquire into student thinking; illuminate student inquiry; bridge student ideas and formal mathematics.</i>
Problems	<i>Open; few prescriptions; multiple solution paths; LFHC (low floor high ceiling); engaging/relevant; balance known and unknown.</i>

Results

The teachers expressed that they felt unsure of either what inquiry really was, how to realize it in their mathematics classroom, or how to match inquiry approaches with curriculum goals. This highlights how teachers need support in understanding and realizing inquiry. Their limited experience with inquiry in mathematics – both as teachers and as students – was repeatedly expressed, underlining the challenges when expecting teachers to implement pedagogies that they themselves do not feel familiar with. Nevertheless, the teachers did share several ideas and views throughout the interviews.

Table 2 outlines their first responses when asked directly to share their conception of what inquiry in mathematics is. Alex, Cameron, and Darcy seemed to focus on how *inquiry separates from copying*

mathematical prescriptions. In a way, also Eli touched upon this view by focusing on persistence and exploring solution methods. Moreover, Alex and Darcy talked about students connecting knowledge, while both Alex and Eli emphasized multiple solution methods to mathematical inquiry problems. Neither collaborative actions nor the teachers' role were mentioned (beyond not giving away methods), and only Eli mentioned argumentative and evaluative features (analysing, being critical).

Table 2: The four teachers' first expressed thoughts when asked what inquiry in mathematics is

Alex	<i>The main idea is to not use tasks that are already structured. So that [the students] must connect a lot of knowledge to solve them. But it must be a low entrance point so that all students can accomplish something. (...) So, a low entrance point but an infinite number of ways to find an answer. And it's not even an answer, it's a process.</i>
Cameron	<i>Instead of giving [the students] sets of rules or algorithms, or a recipe, and then they follow that and find an answer (...) you inquire more on what [the students] think of solving a problem, how relevant this is to different curriculum goals, and then it can become more exiting and relevant.</i>
Darcy	<i>To me, inquiry in mathematics is that you don't explain the method or how it's specifically connected (...) Simply, that they aren't served a solution but must give some effort (...) and find out how it's all connected and use everything that they know.</i>
Eli	<i>To me, inquiry is to dare to bang one's head against the brick wall. And to keep on trying, (...) to explore (...) alternative solution methods (...) and to analyse your own answers, see where it went wrong, and dare to be critical to yourself.</i>

To further illuminate their conceptions of inquiry in teaching and learning mathematics in terms of the role of the students, teachers and problems, the teachers were asked follow-up questions addressing the three facets. In their elaboration on **the students' role**, emphasis was put on students' *active engagement*. The teachers reinforced that the students should use their existing knowledge (Alex, Cameron, Darcy), make mathematical connections (Alex, Cameron, Darcy), and discover that there are multiple ways to solve a problem (all teachers). These can be considered explorative actions. However, it was sometimes unclear what was really meant, as illustrated in this interaction:

Researcher: What's the difference in the students' role in inquiry versus traditional teaching forms then?

Alex: (...) That you can learn that more ways lead to Rome [sic]. (...) So, I try to implement that, but some students get confused: "Now you're doing it in a different way every time". And I'm like: "Yeah, but that's the point. It's not just one way, but you can pick. And then I'll try to repeat the one that you liked the best".

Such utterances point towards the students choosing an algorithm from an "inner catalogue" provided through teacher-centred methods, rather than combining mathematical knowledge to grapple with unfamiliar problems and create something new to them. Nevertheless, thoughts in the direction of students using their knowledge to develop something new were also shared by some of the teachers.

Finally, *collaborative features* were addressed by three out of four teachers; however, mostly in shallow ways (e.g., "the students can collaborate" or "they use each other") without further

explanations to what this entailed. Where the teachers did offer more elaborative thoughts, they mentioned (without any depth) sharing or listening to ideas (Cameron, Darcy, Eli), with the goal of seeing alternative methods to solve mathematical problems. Only one teacher (Eli) talked about students questioning each other's contributions and welcoming critique on their own contributions.

Conceptualizations regarding **the teachers' role** were vaguely expressed but seemed to circle around *not prescribing methods*. Words such as 'guide', 'hint', 'support', and 'motivate' were used, but when asked to elaborate, the teachers seemed unsure. Two teachers (Cameron, Eli) directed their focus to finding motivating tasks. Although the teachers didn't share many thoughts on teacher-student interactions, Darcy and Eli did express some thoughts in line with illuminating and prompting student explanation:

Darcy: It's about asking critical questions, maybe. Like, if I see that something's wrong, I can ask: "How did you get to that answer, could you explain?" (...) Basically, to guide and ask open questions and get them to think.

Eli: I'm not looking for the answer. I want [the students] to explain their method.

These types of teacher (inter)actions promote communicative processes through encouraging students to verbalize their reasoning. They can also promote argumentation and evaluation given that the students explain 'why' (justification) and not just 'what' (description); however, the teachers did not go into this distinction. Contrasting these teacher reflections, and conflicting with the teachers' principal conception to not prescribe mathematical methods, some utterances went in the direction of funnelling the students. For example:

Darcy: (...) I could give them a small hint, like: "You could look at this particular thing, 'cause that's relevant if you want to move on. So, focus on that".

Alex: (...) Imagine you're at the woodwork room (...) [The students] come to the teacher and ask: "Do you have anything so that I can do such and such?". "Yes. What about this hand plane, it will make it smooth". (...) That you at least point them in the right direction of what they can look for to accomplish the task they are doing.

The teachers' thoughts on **mathematical inquiry problems** were, just like their thoughts on students' and teachers' roles, centred around *providing freedom of methods*. They focused on tasks with few prescriptions (Alex, Cameron, Darcy), approachable through multiple methods (Alex, Cameron, Eli), and having multiple answers (Alex, Eli). Nevertheless, the teachers also expressed concerns regarding either finding or using inquiry problems, and that they would like help with this. This means that they might not yet have rich catalogues of mathematical inquiry resources or even know where to find them. The teachers used words such as 'exciting', 'cheerful', 'creative', and 'wide-ranging' to describe inquiry in mathematics, and often *contrasted it to traditional teacher-led approaches*. At the same time, some of them expressed concerns regarding how to combine inquiry and curriculum goals. Darcy even suggested that inquiry approaches could lead to "double work", as the problems might be so open that one must teach the formal mathematics again through traditional approaches. Contrasting this, Eli did share reflections on inquiry approaches being beneficial for deeper learning.

Table 3 summarizes key elements from the four teachers' verbalized conceptualizations of inquiry in mathematics throughout their interviews, in terms of the student and teacher roles and inquiry problems. The depth of their utterances addressing these elements varied.

Table 3: Key elements from the four teachers' conceptualizations of inquiry in mathematics

Students' role: <i>Discover (multiple) solutions or solution paths (Alex, Cameron, Darcy, Eli); give an effort / actively participate (Cameron, Darcy, Eli); share or listen to ideas (Cameron, Darcy, Eli); use existing knowledge (Alex, Cameron, Darcy); make mathematical connections (Alex, Darcy); collaborate (Darcy).</i>
Teachers' role: <i>Guide / hint (Alex, Darcy, Eli); few prescriptions (Cameron, Darcy); choose "good" problems (Cameron, Eli); direct the students on the right path (Alex, Darcy); motivate (Cameron, Eli); request explanations (Darcy, Eli).</i>
Inquiry problems: <i>Few prescriptions (Alex, Cameron, Darcy); multiple solutions or solution paths (Alex, Cameron, Eli); variation to meet different needs and competences (Alex, Eli).</i>

Discussion and final reflections

This study contrast suggestions that teachers do not know what inquiry is (e.g., Capps et al., 2012). Throughout the interviews, several characteristics were brought up that are emphasized in research and in the new Norwegian mathematics curriculum, e.g., multiple solution paths, minimal prescription teacher guiding, and students searching for mathematical connections (e.g., Artigue & Blomhøj, 2013; Laursen & Rasmussen, 2019; Norwegian Directorate for Education and Training, 2020). The four teachers' initial descriptions (Table 2) indicate conceptions of inquiry promoting student thinking (cf. Maaß et al., 2017) as the students are offered intellectual space for mathematical exploration. Further, their conceptions value inquiry as a student active learning approach (cf. Kuster et al., 2018) as the students grapple with and build on their own knowledge to explore and develop mathematical ideas (e.g., Laursen & Rasmussen, 2019). Their focus on mathematical discoveries links inquiry to mathematical learning; however, looking at the teachers' concerns about combining inquiry and curriculum goals, one might get the impression that they see inquiry as a recreational activity to boost student motivation more than to learn mathematics (cf. Maaß et al., 2017).

The teachers' ideas of their own role in inquiry in mathematics were surprisingly vague. They emphasized supporting, guiding, and motivating students, but even with probing questions they had few thoughts on teacher (inter)actions and orchestrations of the classroom that would support, guide, or motivate the students. Choosing "good" problems, which was emphasised by two of the teachers, is important in structuring fruitful inquiry activities (e.g., Artigue & Blomhøj, 2013; Laursen & Rasmussen, 2019); however, it paints a narrow picture of the teachers' role. Further, collaborative features were also vaguely described, which is interesting considering the massive emphasis this has received in research (e.g., Artigue & Blomhøj, 2013; Laursen & Rasmussen, 2019; Naalsund et al., 2022). Vague conceptions are hard to bring to concrete actions supporting inquiry in the classroom.

Exploration was emphasized more than argumentative, communicative, and evaluative features of inquiry by the teachers in this study, as has also been the case in science research (e.g., Romero-Ariza et al., 2020). Some of the teachers mentioned wanting their students to explain, share and listen to mathematical ideas; however, what was majorly emphasized was a wish for their students to find multiple methods and answers – to explore. They rarely expressed (apart from Eli) thoughts related to the students providing mathematical arguments to support or evaluate their exploration. Also, the

thoughts shared on collaborative actions (sharing and listening to ideas to see alternative methods) addressed communicative processes but were still focused on exploration (discovering different methods) rather than argumentation and evaluation (challenging and justifying, negotiating strategies, assessing reasoning) or the goal of shared understanding emphasized in the curriculum (Norwegian Directorate for Education and Training, 2020). Analytical and evaluative processes are important for the “higher-order thinking” required for mathematical inquiry (Laursen & Rasmussen, 2019).

This study provides findings on four teachers’ conceptualizations. They are not generalizable. Rather, they spotlight ways that teachers might view inquiry in teaching and learning mathematics. The conceptions shared in this study might also reflect the social context the teachers participate in, i.e., taken-as-shared views on inquiry in teaching and learning mathematics that are circulating in the four teachers’ respective environments (schools, teaching teams, classrooms, etc.). The findings can inform inquiry-inspired PD courses that might support the development of teachers’ PCK. I suggest the following for the design of out-of-school inquiry PD courses in mathematics for in-service teachers, addressing the conceptions outlined in this study:

Portray inquiry as a daily approach to meet curriculum goals, not as something ‘different’ or recreational. From the findings in this study, I suggest connecting inquiry and mathematical learning, for example through nuancing the current view of inquiry as very open, by promoting that inquiry problems also have a mathematical aim and direction (e.g., Kuster et al., 2018; Bråtalen et al., 2022).

Balance inquiry examples and experiences. The teachers wanted help to find fruitful mathematical inquiry problems. Exposing teachers for a wide range of problems with different structures, degrees of openness and mathematical aims (cf. the above suggestion), which they can adapt to their own practices, can be a first step towards realizing inquiry in the classrooms. However, there is a difference between using some inquiry-based approaches and developing a deeper inquiry mathematics teaching orientation (Buell et al., 2017). PD courses should therefore not reduce inquiry to “using some problems” but allow teachers to experience inquiry in various forms from both a learner and teacher perspective. Further, the teachers should be encouraged to reflect on their experiences regarding students, teachers, and inquiry problems, to refine their conceptualization of inquiry in mathematics.

Promote teacher (inter)actions that support the teachers in developing concrete experiences and conceptualizations of their own role in the inquiry mathematics classroom. I suggest promoting reflections on for example questioning techniques to illuminate and prompt students’ mathematical inquiry processes, promote argumentation, elicit student thinking in individual work, group work and whole-class situations, and invite the students to contribute to justifications and evaluations of mathematical methods and suggestions. Through this, several aspects of the students’ role in inquiry are illuminated as well, possibly elevating the teachers’ conceptualizations of inquiry from inquiry as exploration to inquiry as “higher-order thinking” (cf. Laursen & Rasmussen, 2019).

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