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# Limits and transforming power of didactics

Pedro Nicolás<sup>1</sup> and Josep Gascón<sup>2</sup>

<sup>1</sup>University of Murcia, Faculty of Education, Murcia, Spain; [pedronz@um.es](mailto:pedronz@um.es)

<sup>2</sup>Autonomous University of Barcelona, Faculty of Mathematics, Barcelona, Spain

*Didactic science, like all sciences, has limits. It cannot tell teachers what they should teach, with what means, nor can it tell them what they should teach for. Nevertheless, didactics has an enormous transforming power. It can provide us with well-founded criteria for choosing the most appropriate means in order to achieve certain previously determined educational goal. And it can tell us what consequences (intended or unintended) will be obtained if certain didactic means are used. In short, it can evaluate the functioning of the current mode of study in an educational institution, explain its possible inconsistencies and study the viability of certain changes.*

*Keywords: Educational institution, value judgement, normative prescription, didactic paradigm.*

## Introduction

Social sciences study the working of social institutions. A *social institution* consists in a set of constitutive<sup>1</sup> rules, stated by convention, which fix the following: A series of institutional positions; allowed, compulsory and forbidden actions for each position, and rewards and punishments for certain actions (Searle, 2010). Examples of social institution are chess, language, marriage, paternity, authorship, law, jobs, religions, etcetera. To specify a social institution, it is necessary to indicate, at least, its main constitutive rules, as well as the conditions of all kinds (cultural, social, economic, religious, etc.) that affect it and that depend to a large extent on the history of the country (and perhaps even the region) where the institution in question takes place.

A social institution is *purpose-oriented* if it has a declared aim (Searle, 2010). *Educational institutions* are purpose-oriented social institutions that have been established with the aim of bequeathing future generations the (procedural and propositional) knowledge and beliefs required to keep or even improve our present standard of living (Postman, 1996). In an educational institution there are, at least, two clear positions, teacher, and student. Among the compulsory actions of teachers, we find to teach a certain piece of knowledge, K. Hence, to provide a meaningful description of an educational institution, one has to describe K, even if only vaguely, with the degree of specification depending on the aim of the research. Examples of educational institutions are teaching primary education in a country C, teaching mathematics in primary education in a region R of the country C, and the teaching of geometry in primary education in the region R of the country C. It seems reasonable to say that *didactics of mathematics* is the science that studies the working of the current and the possible mode of study of those educational institutions devoted to the bequeathing of mathematical knowledge and beliefs concerning mathematics.

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<sup>1</sup> When we say that those rules are ‘constitutive’ we mean that they are not just rules that hold in the institution, but rather that the institution consists of those rules, the rules make the institution. Thus, for instance, rules and good manners at the table do not constitute the act of having lunch, whereas the rules of chess constitute chess itself.

In what follows, we will examine, in the framework of the Anthropological Theory of the Didactic (Gascón & Nicolás, 2019), some limits (connected to ethical, epistemological, and ontological issues) and strengths of didactics of mathematics as a scientific discipline.

## **What results can didactic science legitimately state?**

### **Neither value judgments nor normative prescriptions**

Sometimes, one finds in didactics certain value judgements and normative prescription presented as if they were results of research in didactics. However, we agree with Weber (1917/2011) when he says that, although value judgements and normative prescriptions play a crucial role in scientific activity, they cannot be regarded as scientific results. For example, when we choose a topic of research, when we formulate a hypothesis, when we interpret a phenomenon, and even when we choose a theoretical framework to investigate, we are making decisions based on value judgments. And when we follow the norms stipulated by a journal or a conference, then we are following normative prescriptions. So yes, value judgements and normative prescriptions are essential parts of scientific activity. But it does not imply that they can be scientific results.

In the context of didactics, what are the typical value judgements and normative prescriptions? Let K be something to be taught, and let T be a type of teaching. A typical value judgement is a statement of the following form: “The teaching of K of type T is good/bad,” and a typical normative prescription is a statement of the form: “The teaching of K must (not) be of type T.” Notice that this kind of value judgements and normative prescriptions are the two sides of the same coin, as one can be translated into another. Thus, from now on we will only speak of value judgements.

As has been argued in (Gascón & Nicolás, 2017), value judgements should not be regarded as scientific statements as they do not satisfy some indispensable requirements for scientific statements<sup>2</sup>. Let us see which are those requirements. A typical scientific statement is of the form “Every event of type A is an event of type B,” where typically an event of type A consists in the description of a certain system and an event of type B consists in a statement of a certain law satisfied by that system. Think, for example, in Newton’s laws of motion. Part of the interest of scientific statements is that they satisfy at least two requirements. First, A and B must be objective types of events, where “objective” means susceptible of being confirmed or denied in an intersubjective way. Second, the statement “Every event of type A is an event of type B” must be supported by a valid, rational, argument, susceptible of being shared and confirmed or rejected in an intersubjective way. Of course, if the types of events A and B are not objective, then they statement cannot be confirmed or rejected in an intersubjective way. Notice that the value judgement “The teaching of K of type T is good” says that an event of type A (namely, “the teaching of K of type T”) is an event of type B (namely, “something good”). But the event of type B is not objective.

Moreover, in Gascón and Nicolás (2017) we also present some ethical reservations concerning value judgements. For instance, we record Michèle Artigue’s and Guy Brousseau’s views according to

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<sup>2</sup> Someone could think that it is a contradiction to claim that value judgements and normative prescription are not scientific statements, and, at the same time, to claim that scientific statements are statements that should satisfy certain valuable norms. But there is not contradiction here, because the fact that scientific statements are statements satisfying certain valuable norms does not make these statements themselves into value judgements or normative prescription.

which didactics, in its present state, does not seem prepared even to venture in a precise way how things really work in educational institutions. Also, they wonder where the moral legitimacy of researchers in didactics is to say teachers how they must do their job. Hence, there seem to be both scientific and ethical limits to accept normative prescriptions and value judgements as part of the scientific result of didactics.

### **Possible results of research in didactics: a first approach**

Didactics does not need to use either value judgements or normative prescriptions to offer valuable insights into educational institutions. On the contrary, given an educational institution, I, didactics of mathematics can provide substantiated scientific results on at least three different interesting issues concerning I:

- *descriptive analysis*<sup>3</sup>: How does the current mode of study of mathematics in I work? What is taught? With what means? What are the educational goals pursued? Are the means and the goals coherent? And what are the didactic phenomena that become visible when we carry out this analysis from a certain perspective?
- *backwards analysis*: Why does the current mode of study of mathematics in I work as it does? Where does this mode of study come from? To answer these questions, in the Anthropological Theory of Didactic we take into consideration, among others, transpositional constraints (Chevallard, 1985/1991; Bosch & Gascón, 2006).
- *forwards analysis*<sup>4</sup>: What should be done to change the current mode of study of mathematics in I in a certain direction? Statements corresponding to forwards analysis are not normative prescriptions, because they do not have the logical form: “The teaching of K must be of type T”, but the form of a conditional statement: “If the teaching of K is of type T, then the educational ends EE will be achieved”.

Notice that backwards analysis depends on a previous descriptive analysis. Notice also that forwards analysis depends not only on the descriptive analysis of what is currently going on, but also on the description of a possible situation. Therefore, the power and eloquence of the description of what happens and what could happen in educational institutions determines the relevance of didactics in a crucial way. To carry out a *conscious description* of the mode of study in a certain educational institution, didactics of mathematics needs to emancipate from the current, spontaneous, point of view prevailing in that institution.

### **Need to use a reference system**

Natural sciences provide descriptions of natural systems that do not assume a description of themselves. For instance, the system formed by a certain type of molecules, with which a biologist works, does not assume a description of itself. On the contrary, social sciences aim to describe social institutions that typically do assume a description of themselves. For instance, a researcher in

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<sup>3</sup> In the Anthropological Theory of Didactic, we call this type of analysis *economic analysis* (Gascón & Nicolás, 2018, 2019), in the sense of the economy of a system.

<sup>4</sup> In the ATD we call backwards and forwards analyses ecological analysis (Gascón & Nicolás, 2018, 2019), in the sense of the ecology of a system.

didactics of mathematics studies certain educational institutions which, in turn, have descriptions of important parts of themselves, recorded in textbooks, syllabi and many other official documents. Therefore, researchers in didactics risk to assume, uncritically, the description that educational institutions assume of themselves. To avoid it, researchers should be aware of and have reflected on the ontology and methodology they use. How could this be done?

Frequently, our descriptions consist in detecting similarities and differences between a certain state of things that are currently happening and a possible state of things, that play the role of a reference system. Therefore, for the sake of emancipation, it seems advisable to have an explicit description of that reference system through which we inspect the current state of things.

### **Emancipation tools: reference didactic paradigms**

In Gascón and Nicolás (2018, 2019), we introduced the notion of *didactic paradigm*. On one hand, it is a way of organising descriptions of important aspects of current or possible modes of study<sup>5</sup>. On the other hand, the use of didactic paradigms can help to emancipate from the self-vision of educational institutions. Next, we elaborate.

A didactic paradigm in an educational institution, I, is a 4-tuple,  $DP = [EM, EE, DM, D\phi]$ , where EM is the *epistemological model* is the choice of one of the possible kinds of organization of the knowledge K to be taught in I, EE are the *educational ends* pursued in I, which specify the purpose of learning K, DM are the *didactic means* used in I, which are the actions carried out in order to reach the EE, and  $D\phi$  is a *didactic phenomenon* that [EM, DM] try to avoid.

By definition, a DP is always attached to a certain educational institution I, and there are educational institutions at different levels of specification, according to the level of specification of the K taught therein. When K is just mathematics, with no further specification, we say that the corresponding didactic paradigm is a *disciplinary (mathematical) didactic paradigm*. When the knowledge to be taught in I is more specific (for example, decimal numbers), we say that the corresponding didactic paradigm is a *subdisciplinary (mathematical) didactic paradigm* (Gascón & Nicolás, 2018).

When we use the idea of didactic paradigm to describe the current mode of study of a certain educational institution I, we speak of a *current didactic paradigm (CDP)* in I. When we use the idea of didactic paradigm to describe a possible mode of study of a certain educational institution I, we speak of *reference didactic paradigm (RDP)* for I. In the case of the CDP, we put first the letter 'C',  $CDP = [CEM, CEE, CDM, CD\phi]$ , and in the case of the RDP, we put first the letter 'R',  $RDP = [REM, REE, RDM, RD\phi]$ .

An RDP can be regarded as the scientific hypothesis according to which the ends REE would be achieved and the phenomenon  $RD\phi$  would be avoided if the didactics means RDM were used based on the model REM. This hypothesis could and should be contrasted with reality through empirical studies.

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<sup>5</sup> The intuitive idea of modality of study used up to this point can be specified (and, in a certain sense, modeled) by the notion of didactic paradigm.

## Possible results in didactics: a second approach by using didactic paradigms

In what follows, we will use the idea of didactic paradigm to provide a more accurate description of some of the possible kinds of results that can be achieved in didactics.

### Descriptive analysis of the current mode of study in an educational institution

As we say above, a crucial part of research activity in didactics relies on the description of the current mode of study in an educational institution. For this description to not be a hostage of the description that the institution assumes of itself, one must try to ensure a certain degree of emancipation. How can this be done? Typically, a description is done by comparing what we find with what we could have found, by comparing what we have with something else that we take as a reference system. The more explicit this reference system, the greater the degree of emancipation. Ideally, this is done as follows: first one makes an explicit description of a possible RDP in I, and then one compares the working of I with the RDP. This contrast provides a representation of the CDP in I. The RDP is a lamp, and the corresponding CDP is the part of I lit by that lamp. Each possible RDP provides a representation of the CDP regarded from that RDP. The RDP states what could happen, and the corresponding CDP states what happens compared to the RDP previously fixed. In practice, the explicit descriptions of both the RDP and the CDP are done simultaneously to a degree. One starts with an implicit idea of a RDP, and, along the development of the description of the corresponding CDP, one makes more and more explicit both the RDP and the CDP.

Example of CDP: As explained in Gascón and Nicolás (2022), the current didactic paradigm of the institution “teaching of mathematics in the Degree of Mathematics at Spanish universities” is the following:

- CEM: mathematics is presented as it is regarded in current research in mathematics. In particular: it is presented as part of set theory, which is a first order formal theory; definitions are always axiomatic; the only allowed arguments to support theorems, propositions, corollaries, and lemmas are the so-called *deductive* arguments, that is to say, arguments (able to be) made of certain formal steps.
- CEE: students must understand how mathematics is the result of a modelling activity (which explains the role it plays in our civilization), and also students have to be trained in such a way that they are able to pursue any of the possible subsequent paths: research in mathematics or other related disciplines, mathematics teacher, and other professions requiring deep mathematical knowledge.
- CDM: students are trained in the use of deductive arguments and axiomatic definitions (and standard exercises) by imitating what the teachers and textbooks do. The presentation of mathematical knowledge has authoritative features.
- CD $\phi$ : the emphasis in axiomatic definitions and deductive arguments is the answer to the way of doing mathematics that led to the crisis of the foundations at the end of the 19th and the beginning of the 20th century.

Example of an RDP: the previous description of the current didactic paradigm is done in contrast with the following reference didactic paradigm, which can be called *paradigm of mathematical modelling*:

- REM: mathematics is regarded as a coherent family of models of different kinds of (intra or extra mathematical) systems. Those systems are not explicitly considered only in examples of the applications of mathematics, but also in the process of making mathematical notions, results and arguments themselves.
- REE: students must learn how mathematics guides our actions on a certain system by providing relevant information, and how mathematics is constructed along the modelling process of that system. In this way, students will learn not only what mathematics is and why it is useful, but also why it is the way it is.
- RDM: students are trained in modelling activities, not regarded as the use of previously existing models, appearing out of the blue, but rather as the reflexive and critical construction of models to increase our procedural knowledge and propositional knowledge of a certain system.
- RD $\phi$ : this didactic paradigm tries to avoid the forgetfulness of the problems that are at the basis of mathematics and of how these problems contributed to shape mathematics, which, according to this paradigm, would be the result of the modelling of certain systems of every kind (physical, biological, economic, mathematical, etc.) with the aim of mastering them.

### **Assessment of the current didactic means and the current epistemological model**

The use of an RDP is not only useful in order to provide a (perhaps partial) description of the CDP, but also to assess efficacy of the CDM and the CEM to reach the CEE. Notice that, in an RDP, both the RDM and the REM must be suitable for the achievement of the REE. This is so because an RDP is not a product of a historical process, but rather the expression of a virtual possibility that the researcher designs ensuring the internal coherence. However, a CDP is a description of a product of historical evolution and contingency, and it might be, after all, somehow internally incoherent. How could this incoherence be detected? We can construct an RDP with the same educational ends than the CDP. If the RDP is well constructed, then the REM and the RDM should be effective to reach CEE, and so they can be used to assess, by comparison, the CEM and the CDM.

Let us illustrate it by using the examples of CDP and RDP previously presented. Among the CEE we find that students have to understand how mathematics is the result of a modelling activity. According to the RDP, in order to achieve this EE one needs an EM which presents mathematics as a family of models of explicit systems, and one needs DM devoted to train students in the construction (not only in the use) of models from the very beginning, starting with natural numbers and basic operations. However, neither the CEM nor the CDM satisfy these requirements. Indeed, the CEM presents mathematics out of the blue, axiomatically, instead of presenting it as the result of a modelling process starting with an explicit system. On the other hand, the CDM are devoted to train students in the understanding and making of deductive arguments, and only occasionally in the application of mathematics, but the real *construction* of mathematics (for example, integers, polynomials, groups, rings, modules, manifolds, vector bundles, chain complexes) along a modelling process is missing. Therefore, according to this RDP, although the CEM and the CDM may be useful in order to achieve some of the CEE, they do not contribute to the achievement of one of the CEE, namely, “students have to understand how mathematics is the result of a modelling activity”.

## **Forwards analysis: transforming power of didactics**

The fact that value judgements and normative prescription cannot be part of the scientific results of didactics of mathematics does not condemn us to a fatalistic conformism, it does not imply that didactics of mathematics cannot contribute to transform the teaching of mathematics. On the contrary, didactics of mathematics is perfectly able to propose transformations of teaching, once certain educational ends have been previously fixed. Didactics cannot judge educational ends but can assess the efficacy of certain means to achieve those ends and can even propose changes in order to achieve those ends. If we have an RDP which shares the educational ends with the corresponding CDP, then the REM and the RDM can be used not only to assess the CEM and the CDM, but also to set a direction in which both the epistemological model and the didactic means should be directed in order to achieve the educational ends. Thus, the use of an RDP sharing the educational ends with the corresponding CDP helps to propose changes in the current epistemological model (how to organise what is to be taught) and in the current didactic means (how to organise the way of teaching).

In practice, when a researcher uses a reference didactic paradigm to study the working of an educational institution I, she takes as  $RD\phi$  a didactic phenomenon which is currently happening in I and which is regarded as undesirable by the researcher. Then, the researcher makes a RDP devoted to avoid  $RD\phi$ , and this RDP might not share the educational ends with the CDP. As before, the RDP helps to propose changes in the current didactic paradigm. Or, better, as these times changes affect the whole current didactic paradigm, the RDP can be used to study what would be the conditions and the constraints offered by the educational institution for a complete change of paradigm, from the CDP to the RDP. This has been done in many doctoral dissertations carried out in the framework of the Anthropological Theory of the Didactic (e.g., Lucas, 2015).

## **Conclusion**

After explaining why value judgements and normative prescriptions should not be presented as scientific statements, we have used the idea of didactic paradigm to suggest several kinds of important results that can be achieved in didactics. In summary, we can use a reference didactic paradigm, which describes a possible mode of study in a given educational institution for several purposes: 1) to provide an emancipated description of a current didactic paradigm, 2) to assess the suitability of the current epistemological model and the current didactic means to achieve the current educational ends, 3) to study the required changes in the current epistemological model and the current didactic means in order to achieve the current educational ends, and 4) to study the required institutional changes in order to shift from the current didactic paradigm to a different one which would avoid a certain didactic phenomenon that takes place in the institution. However, didactic science is not legitimized to present as a scientific result that one didactic paradigm is better than another. This list of possible results in didactics are not a proposal for the community of researchers in didactics, but rather a description of what is really being done. However, we think that it is generally done without making explicit the reference didactic paradigm used. As said in (Bikner-Ahsbals et al., 2022, p. 4), “With vertical theorizing, researchers focus on meta-issues of theory use for the purpose of *understanding theories as entities in and of themselves*”. We still have gone deeper into the link between vertical theorizing and our description of research in didactics of mathematics in terms of DP. Our hypothesis is that each theory implicitly assumes a RDP, and to make explicit this RDP seems to be connected



to vertical theorizing. To make explicit the RDP used in each research would make it easier to have the scientific discussion within our research community.

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