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Semiotic Process Cards as a tool to analyse learner's mathematical interactions

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The semiotic of Charles Sanders Peirce offers a fruitful theoretical perspective to consider mathematical activities of learners in the frame of diagrammatic reasoning and the use of signs of different kinds. Within two research projects, this theoretical approach is used to develop and adopt an analytical tool, the so-called Semiotic Process Cards. These cards are graphically designed and allow for detailed tracing of the sign process that occurs during learners' mathematical interactions. The focussed analytical tool uses the sign concept of Peirce to show relations between signs, highlights mathematical insights of learners and different representations. In this paper, the Semiotic Process Cards are theoretically rooted and methodically described by two examples. While the first project mainly considers written representations, the second one analyses gestures of second graders.

Keywords: Semiotic Process Cards, semiotics of Peirce, learners' interactions, sign process.

Introduction

A description of the analytical tool of the Semiotic Process Cards (SPC in the following) developed by Schreiber (2013) and further adopted by Huth (2014; 2022) is the central concern of this paper. This method of analysis offers the possibility to integrate theory in empirical research methods in line with the tradition of qualitative social research (Krummheuer, 1992). It is furthermore adaptable for different research foci as shown in the following. In a first step, the triadic sign concept of Peirce is outlined to offer a theoretical framework of this method of analysis and to describe the constituent element of the SPCs. In a second step, methodological considerations concerning the connections of Peirce's semiotic and the interactionist theory of mathematics learning are presented. Following Krummheuer (1992), mathematical learning is seen as a social process of common meaning construction. Meaning is not given in advance but is rather the dynamic result of a negotiation process. Learners gain autonomy in socially constituted mathematical argumentations and can be a role model for others in these interactions. As part of this process, and consistent with a semiotic approach, the multimodal use of signs and the construction, manipulation, and observation of diagrams are seen as the core of doing mathematics (Dörfler, 2015; Peirce, 1931, CP 1.54). After these theoretical considerations, an example of Schreiber's (2013) original version of the developed SPCs is described. Therein, a common inscription-based mathematical interaction of primary school pupils is shown. A second example, focusing on gesture-speech relations in learners' mathematical interactions, illustrates the multimodal extension of the analysis through a gesture triad (Huth, 2022).

Peirce's triadic sign concept – theoretical ground of Semiotic Process Cards

Peirce's semiotic perspective is used in various mathematics education research activities (e.g., Hoffmann, 2006; Dörfler, 2015; Presmeg, 2006; Billion, 2021). Eco (1976) highlights the advantages of this approach and emphasises the extensiveness of the Peircean semiotics (pp. 14–16). Within a

linguistic view, Fricke (2007) also underlines the “extensive semiotic concept of signs” (p. 182) after Peirce, referring to the possibility of “representing arbitrarily complex sign configurations and processes of semiosis” (p. 183). Thus, it is a powerful approach for the analysis of commonly accomplished inscriptions in different modes of expression in mathematical interactions of learners. It accentuates the subject as an interpretive instance and opens up the possibility of combining this approach with an interactional theory of mathematics education (Huth & Schreiber, 2017). Both theories focus on meaning or sign developmental processes and take the interpretation or interpretant into account. Signs do not have meaning by themselves, but gain meaning through the use of a sign producer and reader, which fits well with the idea of negotiation of meaning in interaction theory (Krummheuer, 1992). Peirce’s triadic sign relation consists of a “triple connection of sign, thing signified and cognition produced in the mind” (Peirce, 1931, CP 1.372). The three correlates in this triadic relation can be described as in the following and are shown in Figure 1.

A sign, or representamen, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object. It stands for that object not in all respects, but in reference to a sort of idea, which I have sometimes called the ground of the representamen. (Peirce, 1932, CP 2.228)

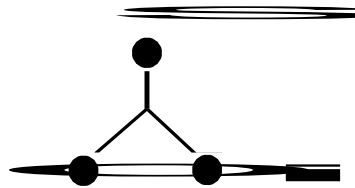


Figure 1: The triadic sign relation according to Peirce (Schreiber, 2013, p. 56)

According to Peirce, we use the concept of *sign* or *representamen* for anything which is external, visually, aurally, or otherwise perceptible (e.g., a written or a gestural utterance). The *interpretant* is a kind of inner sign which is associated with the external perception of the observer. The term *object* includes what the observer of an external sign believes the sign creator intended the sign to mean. It is not necessarily a material thing but can also encompass thoughts or emotions. For Peirce, the three correlates—representamen, interpretant and object—are integral parts of a sign and none of the three is superfluous. The sign itself only becomes a sign when it is perceived as a sign (Peirce, 1932, CP 2.228). In the paper, we will use the concept of representamen, interpretant and object for the correlates and the term *sign triad* for the triadic relation described above (Schreiber, 2013).

In the present paper, we consider graphical and mostly written or drawn signs of learners while they are occupied with an arithmetic problem. On the other hand, we take a look at gestures as signs in mathematical interactions of second graders, while solving a volume measurement task. In our understanding of Peirce, a written or a gestural utterance can be perceived and interpreted as a sign by a sign reader (a counterpart or the sign reader himself). Thus, a written or gestural sign can be described in its triadic relation of representamen, object and interpretant. The examples described in the following illustrate the flexibility and adaptability of the chosen analytical tool rooted in Peirce’s

approach. In the ongoing use of signs, the interpretant of a sign can be uttered again by the sign reader and become a new representamen. Thus, the interpretant is only accessible through this new representamen uttered by the sign reader. Furthermore, the sign process is a constant and endless evolution of meaning. The process cannot be brought to an end but can only be interrupted, there is no first or last sign (Peirce, 1934, CP, 5.284). In turns of mathematical learning, Schreiber (2010; 2013) describes the potential complexity of this process of meaning construction. For example, one representamen can lead to different interpretants by different sign readers, and groups of sign triads can serve as representamen in a new triad. It follows that there is an ongoing process of interactional negotiation of a commonly accepted “taken as shared meaning” (Krummheuer, 1992, p. 18). With the consideration of gestures in Huth (2022), the complexity of the sign process is additionally extended on the level of multimodality. Due to the non-linear alignment and this extension, we use the concept of the “complex semiotic process” (Schreiber, 2013, p. 148f).

Methodological considerations and the development of Semiotic Process Cards

As a first analytical step, we always conduct a text-based interactional analysis according to Krummheuer (1992). This text-based procedure is ascribed to the qualitative social research and focuses on the development of the mathematical topic in interactions. It takes the development of (mathematical) meaning into account as a result of a highly social process of coordination and negotiation. Due to the concern of our paper, only the semiotic analysis, as a second analytical step, is shown which is based on an already conducted interaction analysis according to Krummheuer (1992). The interactional approach assumes that interactants act in a negotiation process of meaning according to a common ‘framing’ (Goffman, 1974) of the situation. For Peirce, this kind of a common interpretation of a situation is also present. The sign reader always interprets the sign based on a ‘ground’ which can be described as his experiences, concepts, habits, competencies, etc. Moreover, in Peirce (1998, EP II), he describes a so-called ‘commens’ that “consists of all that is, and must be, well understood between utterer and interpreter” (p. 478). It can be assumed as the social dimension of his sign concept. With this in mind, the two methods of analysis match perfectly according to the reconstruction of sign or negotiation processes with the focus on interpretations or interpretants of the interlocutors. The common meaning construction of learners based on a ground is central.

Example 1: Graphical interaction (Schreiber, 2013)

The empirical example presented in this paragraph is about an internet chat episode (Schreiber, 2013). It shows the first version of the development of the analytical tool and states a starting point of this method. Two pairs of pupils, one on either side of the internet chat connection in front of a computer, solve mathematical problems. The communication between the two sides of the internet chat setting was restricted to graphical communication by the technical environment. The two pairs of pupils could only communicate via internet chat with a whiteboard in the written and graphical mode. The example is based on an analyzed episode by using a transcription and an interaction analysis (Krummheuer, 1992). The second step, the semiotic analysis, which is demonstrated in the following, uses the SPCs. During the episode, the following mathematical problem was dealt with:

A snail sits in a well which is 3.2 m deep. It begins to climb up the wall. During the day, it climbs up 80 cm. During the night, it slides down 20 cm. How many days does it take for the snail to climb to the top of the well?

The presented SPC (Figure 2) should be read from top to bottom and in general from left to right. In order to make orientation easier, the triads are chronologically numbered. The letters “R” for representamen, “I” for interpretant, and “O” for object indicate the parts of the triads. Words, text and images are used to display and demonstrate the three correlates. In some cases, the interpretant of a triad will be supplemented by further aspects and thus become a new representamen (Figure 2, triad 5). The frame, which is reconstructed through the interaction analysis, is referred to in the semiotic analysis and indicated in the SPC. The complex semiotic process is represented by the configuration of the triads. The process can progress differently: Where the progress is linear, the subsequent triad with the correlate representamen is positioned at the correlate interpretant of the next triad (Figure 2, triads 5 and 6). Where two parts of the process relate to the same representamen, the representamen in question is assigned to two triads (Figure 2, triads 1 and 2). If the representamen of a triad corresponds with the entire previous process, then the correlate representamen is placed on a line of the box which underlies the hitherto existing process (Figure 2, triad 9).

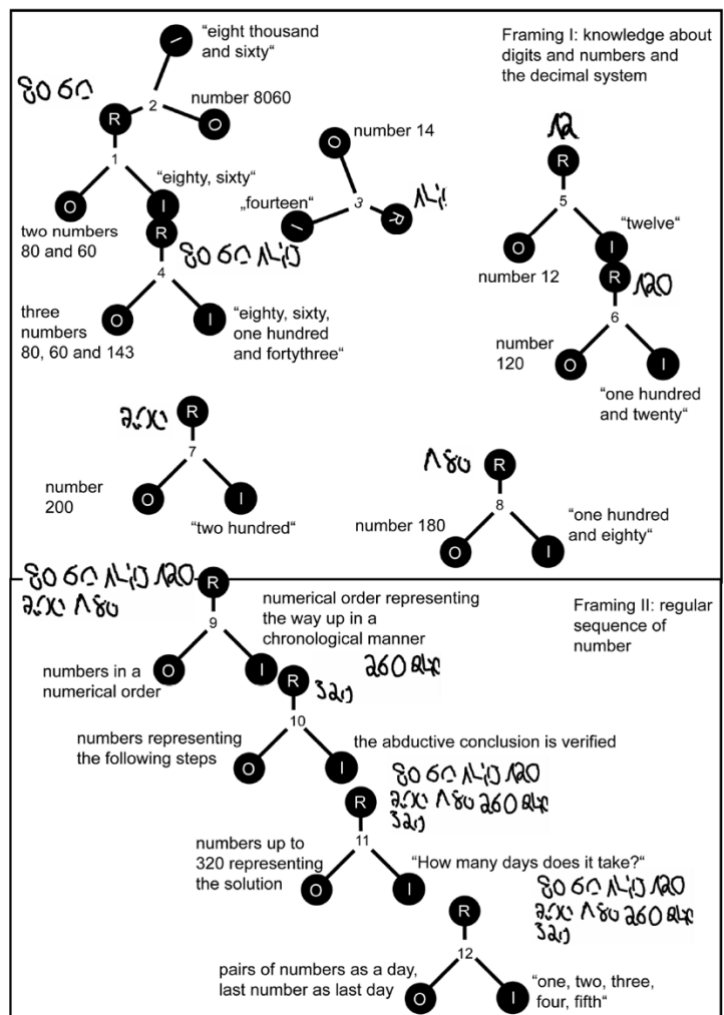


Figure 2: Semiotic Process Card (Schreiber, 2013, p. 64)

The summarized interaction analysis serves as a basis for the reconstruction of the complex semiotic process. The triads are used within the SPC (Figure 2), in which this analysis is presented graphically. The semiotic analysis of this scene is presented here from the perspective of one pair of pupils. It is described in detail in Schreiber (2010; 2013). Here, only the structure of the example is explained, since this is exactly what is relevant for this article. This specific SPC (Figure 2) shows a process in which two learners receive the developing inscription of two other learners and comment on it as it develops. In the process, it is not immediately clear what the signs - in this case numbers that have two and three digits - are supposed to mean. This ambiguity is then overcome by an abductive conclusion: One of the two learners interprets the numbers noted as a regular sequence of numbers representing the steps of the snail and thus leading to the solution. He recognizes that these numbers represent in pairs the state after the day and the night, and that by counting them off, the number of days can be determined which is asked for in the task. Thus, it can be seen in the SPC, that the process of interpreting the graphical representation was not yet purposeful for the solution until triad 8. From triad 9 onwards, the abductive inference made it possible to interpret the inscription of the other learners and thus to read off the result. Further examples were chosen specifically due to their partially analogue development or, in other cases, precisely due to their oppositional development. They were presented as SPCs and the complex semiotic processes were compared. The abductive conclusions the pupils reached and the associated adjustment of the frame were of particular interest. These conclusions and the adjustments of framing proved to be central to the utilization of inscriptions which were fundamental in this study.

Example 2: Gestural interaction (Huth, 2011)

In the following, an excerpt of an example of the mentioned gesture-study in mathematics learning is described. Whereas, in the example above, mainly written mathematical representations of pupils were in focus, the analytical tool is now extended to a pictorial and seemingly fleeting mode of expression, the gestural mode. However, in both cases, visual expressions are of central interest. Gestures are understood in the sense of Kendon (2004). They are characterized by “features of manifest deliberate expressiveness” (p. 15) in the interpretation of the counterpart. As parts of the multimodal system of speech, they are by no means purely unconscious creations or a mere accessory of speech, arbitrarily exchangeable. They can rather be integrated in the language structure, and often fulfil a syntactical function. Their systemic relevance for language is reflected in their co-expressiveness with speech in form, function and syntax (Fricke, 2007; McNeill, 2005). Huth (2011; 2022) could find evidence that gestures also have a subject specific role in mathematics learning. In this research, the gestural activities of learners were contrasted with Peirce's idea of diagrammatic thinking. It became clear that learners show and develop their mathematical ideas in their gestures, construct mathematical relations, and perform manipulations on mathematical diagrams. Sometimes they even create purely gestural diagrams which can be manipulated by gestures again. In this sense, gestures reach the status of reproducible and stable forms in mathematical interactions. Their visual form can activate well-known framings in mathematical interactions and enable potentially new mathematical insights.

Similar to the first example above, the now presented SPC (Figure 3) is based on an interaction analysis according to Krummheuer (1992). The example is described in detail in Huth (2011). The

SPC (Figure 3) can also be read from top to bottom and in general from left to right. Triads are chronologically numbered and include the name of the sign creator. Again, letter “R” stands for representamen, “I” for interpretant, and “O” for object. As an extension of the complexity on the multimodal level, every mode has its own triad: A co-expressive utterance in gesture and speech is pictured by two sign triads, one for each mode (speech on the left, gesture on the right). They are connected via a common interpretant. Simultaneous utterances are mapped with a and b in one row.

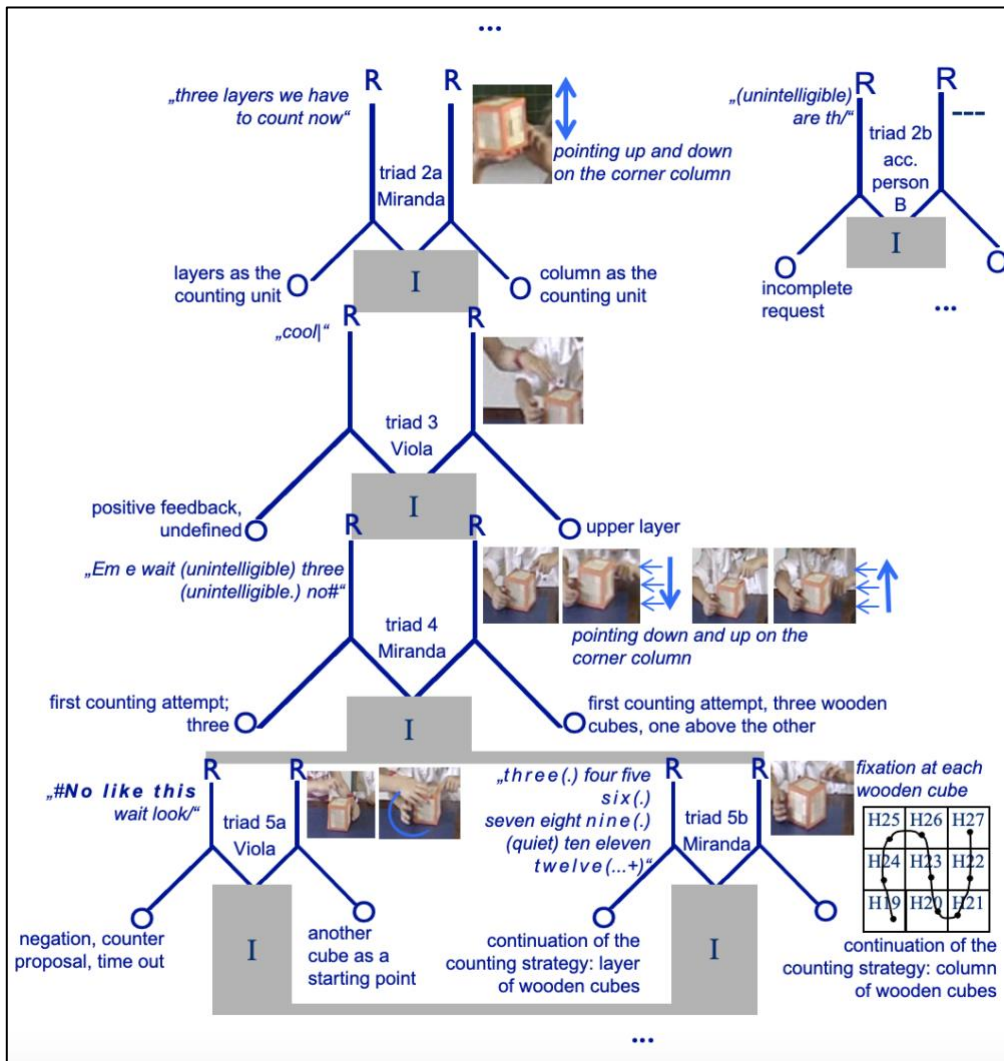


Figure 3: Semiotic Process Card under the light of multimodality (Huth, 2011)

In the depicted sign process, two second graders, Miranda and Viola, have to find out how many wooden cubes fit in an edge model of a $\mathcal{V} = 3^3$ cube. In the analysis, trajectories of signs can be reconstructed: Miranda utters in triad 2a in speech the idea of counting layers, whereas in gesture, the counting of column is more in focus. Viola takes up the layer-idea in her gesture in triad 3. Thus, different sign creators and readers use diverse focal points in their sign interpretation, reconstructable in different modes. Sign developments over the process can be observed. Different mathematical strategies come to the fore in different modes (counting layers and/or columns, triads 2a and 5b). We also see an abortion of some strands of the sign process, e. g., in triad 2b, where no sign interpretation

follows. In total, it seems like counting layers in column is an interconnectable strategy in speech and gesture for Miranda, whereas Viola utters counter-proposals (triad 5a).

Summary: Semiotic Process Cards as a theory-based analytic tool

In the present paper, the use of the analytical tool of SPCs in line with the semiotic theory of Peirce was considered. Based on two empirical examples, the method was explained and depicted. Whereas the cards are originally developed to focus on written externalizations of learners and their inscriptions in mathematical interactions (Schreiber, 2013), their adaptable use to the particular empirical data is possible in flexible ways. Therefore, speech and gesture utterances can also be analytically considered by using these cards (Huth, 2014; 2022). This feature shows its suitability for qualitative social research. The analytical method is rooted theoretically and concepts of this theory are used to describe the empirical findings. Expressions of learners can be tracked on a micro level to gain detailed insights into the mechanisms, structural manifestations, as well as sign uses and developments in interactional processes in mathematics. However, interaction analysis is required beforehand. With this first step, the reconstructions in the SPCs of representamens, objects and interpretants are not arbitrary but based on analytical results. Additionally, it is possible to look back and enrich the first analysis with findings in the SPCs. Thus, the semiotic analysis via SPCs is integrated in a reflexive analytical procedure: The cards are both, analytical tool in their construction and analytical source in the reconstruction. They allow to pursue different sign-based research foci on, for example, issues of mathematical language formation by learners, multimodal sign transfers between modes, or specific externalizations by participants. With this adaptability, it could be used for further empirical studies, e. g. to focus on other forms of expression or to investigate different age groups, teachers or mathematical experts and their activities with signs and diagrams.

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