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# Toward a semiotic account of practice-based mathematics teacher education

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*We sketch components of a descriptive theory of mathematics teacher education, aimed at raising empirical questions about practice-based mathematics teacher education (PBMTE) and the meanings that are transacted therein. We briefly describe PBMTE and pose the question of what meanings are transacted in this practice. We draw on our theoretical work on the practical rationality of mathematics teaching, bringing in the notion of instructional exchange to problematize the transaction of meanings in PBMTE and a multimodal semiotics perspective inspired by systemic functional linguistics to articulate how the meanings transacted in PBMTE can be accounted for.*

*Keywords: Teacher education, semiotics, instruction, practice.*

The practice of practice-based mathematics teacher education (hereafter, PBMTE) involves prospective teachers (hereafter, PT) in learning activities that call forth types of knowing, thinking, doing, and learning which are comparable to those an actual teacher engages in when teaching mathematics in real classrooms. These activities are very diverse in terms of the resources they make available for PT and what they require them to do but, as a rule, they rely on what has been called *representations of practice* (RoP; Herbst et al., 2016), which include representations of classroom interaction and of the mathematical work done therein. Important questions for which our field needs theory concern the opportunity to learn constructed through PBMTE: How can we describe the PBMTE curriculum, or the mathematical and didactical knowings available to be activated and learned in the context of PBMTE activities? Since PBMTE involves RoP which use multiple communication modalities, semiotic questions need to be addressed to support both the description and the design of improvements of those opportunities to learn. An analysis of the semiotic choices made in constructing the representation of practice can serve to unpack the meanings available for PTs to construe for the practice being learned. Herbst, Chazan, and Schleppegrell (2023) have proposed an adaption of Halliday's (Halliday & Matthiessen, 2004) *semiotic functional linguistics* to theorize RoP. And the theory of practical rationality (Herbst & Chazan, 2012) can be applied to the study of mathematics teacher education. This paper links both approaches by connecting questions about opportunity to learn to questions about the meaning potential of RoP.

Any RoP (a video of a lesson, a written case, a storyboard using cartoon characters, or a set of students' written work in response to a problem) used in PBMTE relies on semiotic choices to construe meanings about practice and its components (including the mathematics transacted). These semiotic choices involve expressive tokens such as words, symbols, icons, and indices; but they also involve the regular use of patterns of expressive tokens (e.g., when and whether to inscribe versus invoke meanings). Thus, in a RoP used in PBMTE one may see a mathematics problem presented at one moment, some work done on the problem at another time, and other work done on the problem at yet another time. Depending on what is inscribed, such snapshots of what avowedly happened in

response to the problem allow inferences about the development of the problem in that location, perhaps suggesting a single storyline, perhaps parallel storylines. Some other semiotic choices may point to the instructional context in which the problem and the work on the problem came to exist: For example, an image of the work on a problem may be projected on the board next to an image of a student, while another image of the work on the problem may be a shot from above the paper held by the student at their seat. Meanings concerning the management of the students' mathematical experiences in the classroom may be inferred from those semiotic choices. But there are still other semiotic choices that support the construal of meanings about teaching practice that such instructional context points to: The framing of the alluded choice of images as a case of problem-based instruction and the request to attend to the decisions the teacher made are examples of how the use of classroom artifacts as RoP often involves semiotic choices characteristic of the PBMTE context. In all such semiosis, the meanings intended by the designers of the RoP may not be the only ones made available: Other meanings (mathematical, didactical, and teacher educational) may become available too in the same way that uses of natural language may afford more than one reading. A description of the opportunity to learn RoP create needs the means to describe such meaning potential; these means are what we refer to as the *semiotic infrastructure of practice*.

## **Practice-based mathematics teacher education**

PBMTE emerged as an alternative to earlier forms of teacher education that emphasized the study of and reflection on material produced by disciplines that contribute to the scholarly understanding of teaching and learning. At one time, teacher educators drew the curriculum of teacher education from disciplines such as educational psychology or research in mathematics education. PT were to study, reflect upon, and figure out how to apply that disciplinary knowledge in their practice. But PBMTE proposed a radical reorganization of the teacher education curriculum which Lampert (2010) described as "learning in, from, and for practice." This approach promoted that PT study teaching practices (e.g., the providing of feedback to students' mathematical work) in the context of actual or simulated opportunities to do such teaching practices (e.g., while writing feedback to samples of student work) and for the purpose of developing competence in such practices. Clearly, research-based knowledge still supported teacher educators' design of activities for PT to learn in, from, and for practice. Furthermore, scaffolds for these activities might include readings from academic sources (e.g., articles that describe the conceptions students might activate in response to problems like the one they had responded to). Yet the goal was less one of developing academic knowledge and more one of enabling knowledgeable practice. Grossman et al. (2009) contributed the notion that pedagogies of practice (such as demonstration,<sup>1</sup> decomposition, and approximation) would support learning in, from, and for practice; these pedagogies of practice contrast with pedagogies of investigation in which PT develop skills comparable to those of a scholar.

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<sup>1</sup> Grossman et al. (2009) use *representation* as the name for the first pedagogy of practice. Because our semiotic use of the word *representation*, which refers to the work done by artifacts (or events) in all occasions in which PT engage in learning in, from, and for practice, Herbst et al. (2016) use *demonstration* to refer to the first pedagogy of practice, in which PT consider a case of practice.

The notion of pedagogies of practice has enabled much creativity from the community of mathematics teacher educators (MTE). Some MTE have used their university classes as spaces to represent practice, demonstrating practice for PT while having PT play student roles (e.g., Hodgson & Wilkie, 2022). And approximations of practice (i.e., involving PT in enacting practice in environments of reduced complexity) have been developed in face-to-face settings, for example, by having PT rehearse practices (e.g., carrying out a classroom discussion with their peers playing student roles; Lampert et al., 2013). Pedagogies of practice have also utilized digital environments to represent practice. For a long time, video records of practice have been used to demonstrate practice (Lampert & Ball, 1998) and animations and storyboards have been added in the last two decades (Herbst et al., 2016). Approximations of practice have also called for the development of technology-based simulations of practice, where PT can practice teaching simulated students.

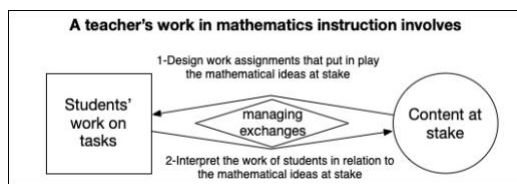
The notion of representation of practice has been fundamental to the implementation of all those pedagogies of practice, though its theorization for PBMTE is a work in progress. The word *representation* has its origins in semiotics (Hall, 1997) and applies just as well to the work done by artifacts such as a computer simulation as to the work done by events such as the rehearsal of a classroom discussion. While these two examples are different in terms of the material resources PT use to engage with it (e.g., in one, computer peripherals are used to manipulate graphics on a screen, in the other, embodiment, inscriptions, and physical props are used to interact with others), they are semiotically comparable: Both rely on signifiers (specific things and actions in the artifact or event) to point to signifieds (meanings of the practice being studied). Moreover, they do not do that in a univocal sense; rather, they profit from the context to support meaning-making. Thus, nondescript drawings (e.g., blue round shapes as in Figure 3) might stand for (i.e., represent) the heads of students in a computer simulation because the simulation tells the user that the screen shows a class which the user is, avowedly, teaching. Indeed, a semiotic system, an array of graphics, actions, and symbols, and of ways of creating patterns with them in the context of PBMTE, enables such artifacts or events to point to the meanings of mathematics teaching practice. We refer to that semiotic system as the semiotic infrastructure of practice. After elaborating how PBMTE supports meaning-making with such semiotic infrastructure we raise empirical questions about PT's opportunity to learn.

### **Instructional exchanges in PBMTE**

A particular type of instructional exchanges takes place in the context of PBMTE and under the gaze of MTE; we call these instructional exchanges *transactions of practice*. While PT engage with RoP, MTE have to interpret that engagement in terms of PT's having had the opportunity to learn about practice: For example, PT engagement in a simulation in which they play teacher in a class which is learning about tangents to circles and can choose among specific mathematical statements that support or confront the student contribution, prolong or curtail the discussion, keep the same student engaged or invites another student (Milewski & Strickland, 2020), may be interpreted by the MTE as having given PT an opportunity to learn to respond to students' contributions. The suggestion that there is an instructional exchange (of practice) points not only to the interpretive act of the MTE but also to the fact that two dissimilar representations of practice are being considered in terms of their possible equivalence (and, if so, exchanged) from the perspective of the MTE. These instructional

exchanges of practice resemble instructional exchanges in mathematics classes, a notion that has been key for us in describing the practical rationality of mathematics teaching (Figure 1).

Instructional exchanges describe a crucial aspect of the work of a mathematics teacher in instruction, suggested by Brousseau (1997): To recognize relationships between two different manifestations of mathematics. These are, (a) mathematics manifested in the items of content designated to be learned and (b) mathematics implicit in the work that students do in the context of classroom tasks (Herbst & Chazan, 2012). Figure 1 depicts this exchange of manifestations of mathematics that a teacher needs to manage when teaching a particular course of study. Chazan and Herbst (2023) elaborate how instructional exchanges can also be observed in teacher education, with MTE being responsible to operate an exchange of sorts between (1) competence in teaching practices that are part of the work of teaching and (2) the work that PT do in the context of teacher education activities. In PBMTE, these exchanges involve transactions of practice in two different directions. In one direction, the various practices that compose the work of mathematics teaching and are the target of PT learning need to be instantiated in activities for PT to work within a RoP. In the other direction, the work that PT do within those RoP needs to be inspected for indications that PT are engaging in work that is comparable with the work they would do when enacting the teaching practice to be learned. Figure 2 attempts to represent these exchanges.



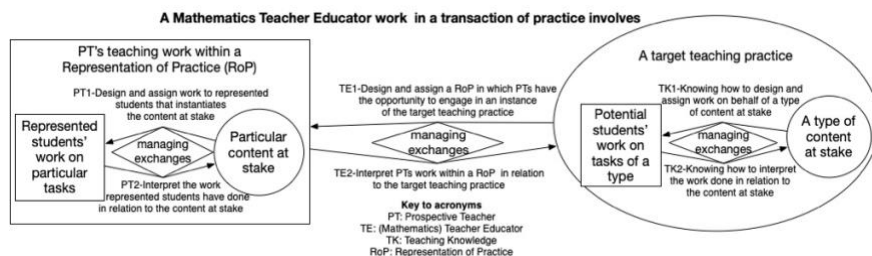
**Figure 1: Instructional exchanges**

The exchanges represented in Figure 2 are second-order instructional exchanges in which the knowledge at stake are the practices of instruction (represented by the larger oval on the right in Figure 2) and the work the prospective teachers (PT) do to lay claim to that knowledge involves actions within RoP in activities that might be described using one of Grossman et al.'s (2009) pedagogies of practice (represented by the large rectangle on the left in Figure 2). However, these second-order exchanges are not simple ones; they nest instructional exchanges in them. Furthermore, while each of these nested exchanges refers to exchanges like the one in Figure 1, they are also different from it in that they are both removed from actual mathematics teaching practice. The exchanges nested in the rectangle on the left of Figure 2 happen not within actual practice but within a RoP.<sup>2</sup> The exchanges nested in the oval on the right of Figure 2 represent generically each of the professional practices that form part of the work a mathematics teacher does as the management of exchanges between types of tasks done by potential students and types of knowledge at stake.

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<sup>2</sup> We are not considering here the case of student teaching, when PT not only construe a representation of practice in which they might be observed by their supervisors but also teach actual content to actual students. This more complicated case needs to be examined separately, with the benefit of combining the models offered by Figures 1 and 2.

Figure 2 depicts PT in two positions. As learners (left side), they do the actual work of managing specific instructional exchanges within a RoP. The MTE manages an exchange in which that work of PT is interpreted in terms of the practice whose competence is at stake. The right-hand side locates PT as professional teachers, potentially being adjudicated competence in a specific teaching practice.



**Figure 2: The work of teacher educators managing transactions of practice**

Figure 2 also allows us to pose some questions that concern how representations of practice used in PBMTE and the mathematics handled in such activities relate to each other in enabling the learning of practice. We conjecture that in the context of these transactions, there is a structural tension between the subject-specificity of the actual work of teaching and the relative genericity with which the practice of teaching is often decomposed to be learned in PBMTE. These questions and tension bring us closer to problematizing the semiotic infrastructure of practice.

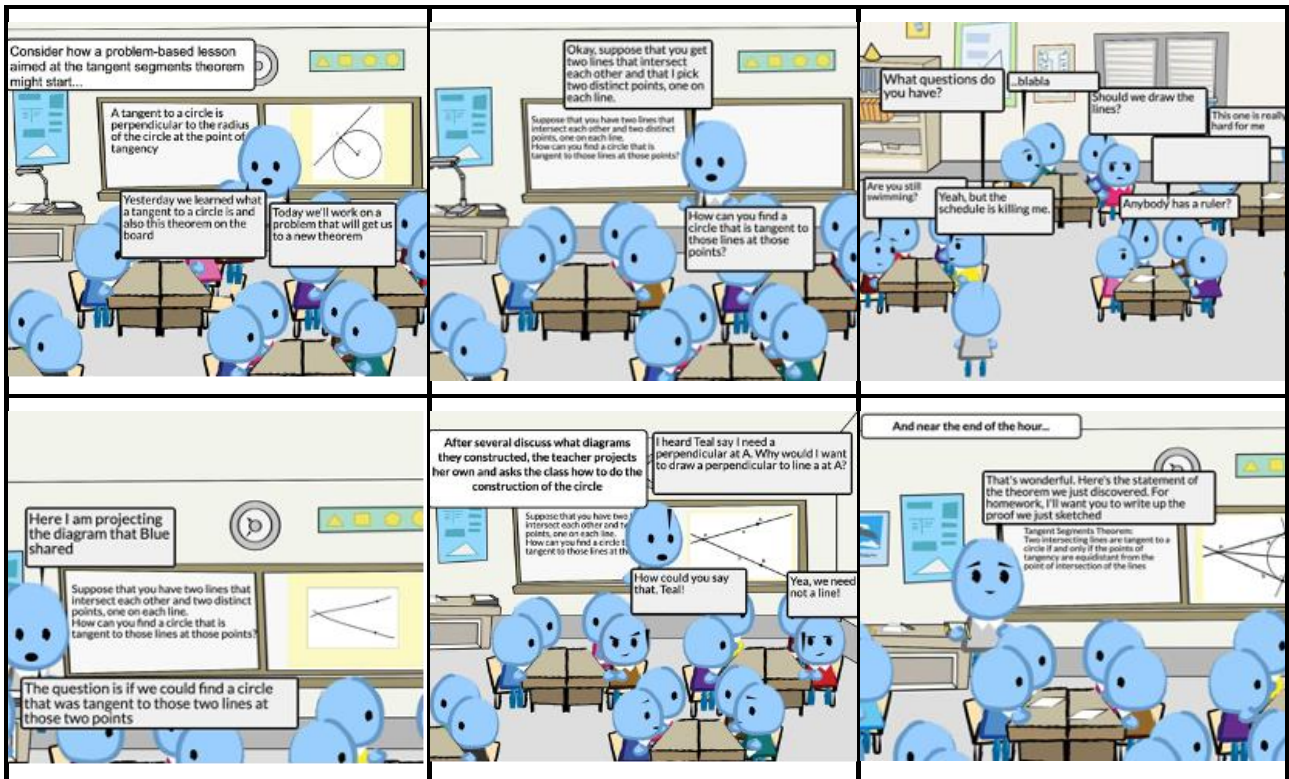
### **The meanings potentially construed in transactions of practice**

Just as the content at stake in instruction is an institutional object, transposed from disciplinary knowledge (Chevallard, 1991), the target practices to be learned by PT are institutional objects. They are not extractions from actual practice, but abstractions and reductions of practice often done by combining knowledge generated from research, policy, and practice. “Teaching a problem-based lesson,” “launching a task,” “managing classroom discussions,” or “responding to students’ contributions” illustrate how these practices are diverse in grain size and often stated in generic terms. The incorporation of such practices into the PBMTE curriculum often involves providing them with more context that still maintains them at some level of genericness—for example, “launching students’ work on a task” may be especially important to learn when students’ work in such task is to be used to feed a discussion that will lead the class to a new item of knowledge. We consider this case to raise questions about subject specificity and subject genericness in these transactions of practice.

Consider a transaction in which the MTE seeks to create an opportunity for PT to learn how to manage a classroom discussion. The MTE provides the storyboard in Figure 3 and asks students to add at least 7 more storyboard frames to script a dialogue that represents the actions that took place between the events in Frames 5 and 6. To do such scripting, students use a storyboarding software such as *LessonDepict* ([lessondepict.org](http://lessondepict.org)) which allows them to duplicate frames and edit their contents (selecting characters and expressions, adding text and graphics, etc.). Three semiotic registers are employed in realizing this transaction of practice and visible in the artifact provided to the PT.

The linguistic notion of *register*, according to systemic functional linguistics, identifies patterns of instantiation of an overall semiotic system associated with given types of context, or a variation within a semiotic system that serves a particular use (Matthiessen et al., 2010). Christie (2002) has

distinguished two registers in classroom discourse: the register in which the content of instruction is construed (instructional register) and the register in which the classroom experience is construed (regulative register). An instance of the instructional register is the inscription on the board in Figure 3 Frame 1 and an instance of the regulative register is seen in the first speech bubble in Frame 4.



**Figure 3: A storyboard of The Tangent Circle lesson given to prospective teachers**

Christie (2002) claims that the regulative register projects the instructional register—which means that the instructional register represents the object of the interactions between teacher and students which are construed through the regulative register. Along those lines, this object can be seen as evolving in its formulation from Frame 2 where it is present in writing and speech, to Frame 4 where it is also present in a diagram, and Frames 5 and 6 where the diagram is made more and more complex. That evolution in the representation of the problem (using the instructional register) is projected through a particular usage of the regulative register that includes the teacher writing and reading the statement of the problem, inviting questions about it, then asking students to work on it, and retrieving some of that work. One could observe here that similar use of the regulative register might project a different instantiation of the instructional register (e.g., a different problem). Yet, that does not mean that the regulative register is generic. Its dependence on the instructional register to construe the object that the regulative register projects results in a complex semiotic performance (e.g., the teacher's speech in Frame 4) which relies on the instructional register for its meaning: Frame 4 suggests the teacher wants students in the class to consider the diagram displayed as a step in the solution of the problem; if the paper shared by Blue had instead an unrelated diagram (e.g., a circle and no lines), the second speech bubble from the teacher might not be able to refer to it. In other words, while expressive tokens of the regulative register (e.g., "the diagram that Blue shared") may be used to project different tokens of the instructional register (e.g., different diagrams) the meanings construed by such complex

multimodal clauses would be different. In this case, where the display of Blue's diagram on the board accompanies a new formulation of the problem, we are led to infer that there is cohesion between the contribution displayed and the original problem. This hypothesized cohesion suggests that it makes sense that students draw the lines talked about in Frame 2, and not necessarily with straightedge.

The previous discussion has addressed the storyboard as if we were peering over a real classroom, but this is a fiction. Not only are the classroom participants too similar physically, but also their speech is written text rather than oral, and some inscriptions appear to address us directly (e.g., the captions on Frames 1, 5, and 6). Herbst et al. (2023) have elaborated on how both language and visuals in the storyboard multimodality attest to yet a third register, which they call *transactive* and which projects the classroom and mathematical meanings construed with the regulative and instructional registers. The transactive register is that with which the experience of the user of a RoP is construed—in particular, how the producer and reader of a RoP relate to each other and to the practice represented. One of the systems of the transactive register, what Herbst et al. (2023) called *visibility* highlights the possible tension between genericness and subject specificity of practice in transactions of practice. Briefly put, visibility refers in general to semiotic resources that enable the designer to open-up or close-down the practice to be inspected by the viewer. It can be exemplified in Figure 3 Frame 4 by the image on the board. The choice made is to inscribe an image that purportedly represents what Blue had shared; the alternative could be to only invoke such image (e.g., inscribing a placeholder) but leave the specificity of the diagram Blue had drawn unknown. As Frame 4's inscription is contraposed to the alternative of invoking a diagram, the viewer is led to construe the meaning of Frame 4 as saying not only that the teacher has shown the work of one student but also that in doing so, they have conveyed some expectations about the students' mathematical work. In this case, the possibility that acceptable work may not need to rely on precise construction.

## **Conclusion**

PBMTE involves an exchange between practice meanings that are often expressed with different semiotic resources. In spelling out the content of teacher education, the practices to be learned tend to use generic linguistic resources (e.g., managing classroom discussions). However, for PT to lay claim on such practices through engagement in practice, they often participate in interactions within RoPs that can vary in their subject-specificity. In these RoPs, the resources of the transactive register add to those of the regulative and instructional registers to provide for the possible construal of subject-specific meanings for the practice being learned. Different multimodalities (e.g., storyboarding, videography) may afford different choices. An analysis of the semiotic choices made in constructing the representation of practice can serve to unpack the meanings available for PTs to construe for the practice being learned. The three registers instructional, regulative, and transactive organize the various semiotic choices enabled by a semiotic infrastructure of practice.

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