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Looping the Fibonacci sequence

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This paper is about an interaction between mathematics and a craft practice called “looping”. In particular, it elaborates on a series of pieces created by Geraldine Jones, an artist and collaborator, inspired the Fibonacci number sequence. The paper explores how the sensibilities and materialities associated with the practice of looping have the potential to be mathematically generative. To do so, I draw on the notion of “distribution of the sensible” (Rancière, 2004) to pay attention to the specific sensibilities involved in the practice of looping and partaking in a certain making sense of the Fibonacci sequence. This focus on the mathematical contributions of looping contributes more broadly to discussions opening new perspectives on mathematical dis/abilities (de Freitas, 2016; de Freitas & Sinclair, 2014) by opening a window on what can happen when loops, hands, fingers and metal wire are considered “able” in mathematics.

Keywords: Materiality, number sequence, pattern, Fibonacci.

Introduction

This paper is about an interaction between mathematics and a fibre technique called “looping”. In particular, it elaborates on the work of Geraldine Jones, an artist and collaborator, who created a series of pieces that embody diverse ways of thinking about and understanding the Fibonacci number sequence. The paper explores how the sensibilities and materialities associated with the practice of looping have the potential to be mathematically generative.

de Freitas (2018) invites us to turn our attention to the sensory dimension of mathematical activity and how this has historically been interwoven with important mathematical developments. She makes this case by visiting some examples of mathematicians who “partook in a radical reconfiguring of mathematics and, simultaneously, a reconfiguring of their own sensory engagement with the material world” (de Freitas, 2018, p. 50). Inspired by this attention to the material and sensory dimensions of mathematical invention, here I explore how the practice of looping (material used, technique, dexterity) can participate in a reconfiguration of the world of possibilities attached to the Fibonacci number sequence. In other words, I consider the practice of looping as involving a certain “distribution of the sensible” (Rancière, 2004) and partaking in a certain “making sense” of the Fibonacci sequence. In this sense, this paper explores a political reconfiguration of the Fibonacci number sequence at the socio-material level of the looping practice and its possible mathematical generativity.

This focus on the mathematical contributions of looping contributes more broadly to discussions opening new perspectives on mathematical dis/abilities (de Freitas, 2016; de Freitas & Sinclair, 2014) by opening a window on what can happen when loops, hands, fingers and metal wire are “able” in mathematics?

Conceptual Framework

Rancière's distribution of the sensible

Jacques Rancière's (2004, 2010) "distribution of the sensible" refers to a partition of the experience which allows it to become "sensible", meaning both part of a sensory awareness (sense-able) and meaningful (partaking in sense). In Rancière's (2004) words, it is a "system of self-evident facts of sense perception that simultaneously discloses the existence of something in common and the delimitations that define the respective parts and positions within it" (p. 12). With this notion he frames the sensibilities that define what "exists" against a background of meaninglessness. Or, to use a term that resonates with the education world, the distribution of the sensible defines that which is "able", that is that which is see-able, voice-able, say-able, smell-able, hear-able against a dis-able background.

Building on this idea, de Freitas (2016) argues that every action of sense making in mathematics is political since it re-affirms a certain "distribution of the sensible". This allows us to look at mathematics as a field where the intersections of different mathematical practices (algebraic, topological, arithmetic, geometric) are involved in re-affirming certain distributions of the sensible. By making "sensible" certain entities that participate in it and by relegating others to noise, or meaninglessness, these practices define ability in mathematics. And this re-affirmation of the mathematically "able" delimits what becomes mathematically capable and possible.

This idea of distribution of the sensible and its political implication for what is mathematically able is of interest for this paper because it raises questions about what can become possible when looping - with its own distribution of the sensible - participates in mathematics? Without stating that the looping practice admits a unique and stable distribution of the sensible, certain sensibilities - for the quality of the loops, their different ways of crossing, their sizes, the material used, its flexibility/resistance, fine movements of the hands and fingers to form the loops and hold them in place - seem to stand out in the practice of looping. The point here is that when these sensibilities are conceived as mathematically "able", when metal, hands, loops "exist" mathematically, new avenues for making sense can become possible and can, in turn, be mathematically generative.

Looping

Looping is a basketry technique commonly practiced in Borneo and Southeast Asia where it is part of a making tradition of baskets and basket parts used in the context of rice cultivation, hunting and domestic work (Mashman, 2006). The looping technique mostly stands out for its strong geometrical patterning and for the malleable structures it can form when made of soft and flexible material (Jones, 2021). The explorations reported in this paper make use of stainless steel metal wire which is a very responsive material that allows an easy flowing weave with enough friction to keep the loops in place, but also enough flexibility to form fluid and malleable structure.

The technique of looping centres on the progressive addition of individual "loops" into a growing woven structure. At first this forms a "row" of interconnected loops, which at some point may be ended or brought to a close on itself, with additional rows able to be woven into and added above or below the first. It is therefore possible to look at a looped structure as an arrangement of successive

rows, with each row containing a certain number of loops. It is possible to increase, decrease or keep constant the number of loops from one row to another which generates different tensions in the material used and contributes to the formation of different types of curvatures.



Figure 1: Looped structure - full and increase loops

Figure 1 shows a looped structure made with 3 individual pieces of metal wire, each composing one of the 3 rows of loops. Two types of loops can be distinguished in the structure: *full* loops and *increase* loops. Full loops are extensions of the loops in the previous row, they are fully connected to them and ensure that they preserve the number of loops from one row to the next. In contrast, increase loops are loops that - as the name suggests - correspond to an addition of loops in the structure. In Figure 1, the top row contains 3 loops which are preserved in the middle row by the 3 full loops. On the bottom row, the second loop is an increase loop which results in an increase in the number of loops going from 3 in the middle row to 4 in the last row. The reader is invited to follow with a finger or pencil the rows of loops from left to right in order to make sense of the trajectory of the metal wire when a row is looped: the full loop completely traverses the loop above while the increase loop solely goes through the space between two loops (or lag).

Methods and episode

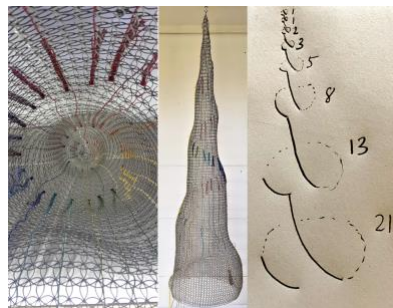


Figure 2: Stretched Fibonacci made by Geraldine

The episode presented in this paper comes from a series of filmed online and in-person meetings of a study group consisting of a basket weaver (Geraldine), a math educator (Ricardo) and the author (Charlotte). Geraldine works particularly with this looping technique and creates compositions often reflecting on different forms of growth and curves. Our regular meetings began around a common fascination for a piece looped by Geraldine called “Stretched Fibonacci” shown in Figure 2.

Stretched Fibonacci is a 150cm x 50cm circular piece that grows in a spiral along the coloured ribbons that run down and around the structure. One of the links of this structure with the Fibonacci sequence

is around the growth of this spiral which develops according to rules associated with the Fibonacci sequence. Growing from the top down, the growth phases of this spiral - the number of rows taken to return to the origin - are increasingly longer, following a relation to the Fibonacci number sequence. As illustrated on the diagram on the right of Figure 2, the first spiral extends over 1 row of loops, the second over 1 row of loops, the third over 2 rows of loops, then over 3, 5, 8, 13, 21, ...

One aspect of this process that was of particular interest to us was the set of rules that Geraldine created to make this structure without “counting” or “calculating” in the traditional senses. Decisions on where to add a loop in each row were made on the basis of certain readings of the loops and on the addition of certain material guidelines, a system that Geraldine has developed so that she does not have to count or calculate each time. For us, this raised several questions about the possible ways of generating the Fibonacci number sequence and the actors that can participate in it such as the looping technique, the hands, the metal wire, or the loops.



Figure 3: Fibonacci made by Geraldine

We used our weekly meetings to investigate these rules and also to explore related and new questions. The short episode detailed below comes from an online session in which Geraldine presents a piece that inspired the making of the Stretched Fibonacci piece. This is a row-by-row piece called Fibonacci (Figure 3) where the number of loops in each row grows according to the Fibonacci number sequence. The extract begins when Geraldine returns to the screen to show us the Fibonacci piece which, although it chronologically came after in our conversations, was the beginning of the exploration of the Fibonacci looping for Geraldine. The verbatim presented below is accompanied by a frame-by-frame series of images of Geraldine foregrounding, touching and stretching the structure to bring out for the reader the importance of the material, visual and haptic dimensions of our conversations even though they are often online. Some passages that will form the basis of comments are in bold.

Episode



Figure 4: Presenting the Fibonacci structure

- Geraldine: Yeah, this one (Figure 4.a)
 Geraldine: This is what happens if you join...
 Ricardo: Fibonacci rows

Geraldine: hmm 3,5,8, ... da, da, da ... in each consecutive row (Figure 4.b)
 Geraldine: **So that's too much** ... to kind of work out really (Figure 4.c)
 Geraldine: Yeah, it goes to whatever row that is ...121 (Figure 4.d - *showing the last row*)
 Charlotte: Where is the initial row? In the, where is the beginning?
 Geraldine: Here (Figure 5.a - *showing the first row*)



Figure 5: Questioning the Fibonacci structure

Charlotte: Ah, right!
 Geraldine: So then you've got 1,1,2,3,5,8 (Figure 5.b and 5.c - *counting/stretching the rows*)
 Charlotte: So that is a flat ... well "flat", but it's not ... (*making circular gestures*)
 Geraldine: Yeah, its not spiral.
 Geraldine: It's a row by row one (Figure 5.d - *stretching the rows to see it is not circular*)
 Ricardo: And where did you decide where to add the new loops?



Figure 6: Describing the Fibonacci structure

Geraldine: **Oh just easy** (Figure 6.a - *reading on the loops*). Just where the, the... If you've got 3, you've got 1, 2 and 3 (Figure 6.b - *pointing to the row with 3 loops*). The first one is a big loop and a little loop. The second one is growing bigger. The second one is a big loop and a little loop at the end. So this row of 5 now, now I know that I will put the extra loop at the end. **So you can see immediately above you, you can see that you've got a big one or a small one. So you know when you got a big one, you add the one after it.** (Figure 6.c - *showing the increase and full loops*)
 Ricardo: Yeah
 Geraldine: When you got a small one... I'm not adding it before it.
 Ricardo: Yeah, I see that. The diagram is like collapsing.
 Geraldine: **When you stretch it** you just add the first one in on the first row, the second one in on the second row and the third on on the third row. Until you get to the next number. (Figure 6.d *pointing to the loops on the structure*)
 Charlotte: So every big loop is doubled and the baby ones just grow big.
 Geraldine: Yeah.

Discussion

On the looping Fibonacci method

The system developed by Geraldine to generate the Fibonacci sequence with loops echoes the well-known rabbit story for talking about the Fibonacci growth. According to the story, the sequence begins with one pair of rabbits who, through sequential growth stages, generate pairs of rabbits that will, in turn, generate more pairs of rabbits, in the end increasing the number of pairs of rabbits

according to the Fibonacci sequence (1,1,2,3,5,8, ...). Inspired by this story, Geraldine created a similar pattern of growth to generate a series of interconnected rows each containing a Fibonacci number of loops. Geraldine's method, however, instead of focusing on the rabbit's reproduction, approaches the problem from a looper's perspective or with looping sensibilities. This means that the Fibonacci sequence is generated by being sensitive to the looping technique and the nature of the loops formed - which are either full loops or increase loops. What Geraldine explains at the end of the episode in terms of big (full) and small (increase) loops is that, from one row to the next, each big loop becomes a big loop followed by a small loop and each small loop becomes a big loop as shown in Figure 7 below.

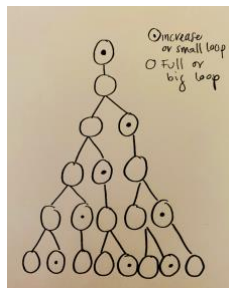


Figure 7: Big and small loop system to generate the Fibonacci sequence

This simple set of rules allows her, by being sensitive to what the loops tell her along the way, to go from one number in the Fibonacci sequence to the next, to go from a row of 5 loops to the next one containing 8, and so on. It starts with a small loop, which becomes big in the next row, and which generates a big loop and a small loop in the next row, and so on. The first row containing 1 loop, the next 1 loop, then 2, then 3,5,8,

What is worthy of attention about Geraldine's way of generating the Fibonacci sequence is that the method and the sequence generated is rooted in the looping technique. This means that Geraldine does not just impose the Fibonacci number sequence on the loops. Rather, the Fibonacci sequence emerges *with* the looping and *in terms* of loops. Her technique and the loops themselves work with her in this generation. Or, to frame it in Rancière's terms, this particular distribution of the sensible where the metal wire, the loops, their connections and their structures are voice-able and see-able allows for the generation of the Fibonacci sequence. As she says in the episode, it's "easy", she can "see immediately above", what the loops - being full or increase loops in the previous row - are telling her and what she has to do below. This political reconfiguration of ability - by giving voice to loops and metal wire - generates a mathematical capacity to form the Fibonacci sequence.

On the generativeness of the exploration

Another aspect that seems important to raise is related to the generative dimension of Geraldine's exploration of the Fibonacci sequence with looping. In the conversation Geraldine explains that her Fibonacci structure is "too much". This "too much-ness" here refers to the crowded aspect of the looped structure growing at a Fibonacci rate. The addition of a growing number of metal wire increase loops to each row causes it to become very dense, and to start curving rapidly and messily. This more and more drastic insertion of material contributes to the formation of a pronounced negative curvature as can be seen in Figure 3.

This too-much-ness generated new avenues of investigation for Geraldine who wanted to respond to it by “diluting” or “stretching” the Fibonacci growth alongside more rows. In response to this too-much-ness, the stretched Fibonacci structure presented in Figure 2 was developed. In this structure, only one loop is added per new row while retaining the “locations” where this growth occurs in the structure. For example, in Figure 7 we can see that from the row containing 5 Fibonacci loops to the row containing 8 Fibonacci loops, 3 increase loops are added at positions 2, 5 and 7. For the new stretched structure the question for Geraldine became about how to dilute or stretch this increase of loops so that only one loop is added per new row while retaining the locations (2nd loop, 5th loop, 7th loop) where these increases occur. Thus, what used to happen in a single growth phase from the row containing 5 loops to 8 loops for the Fibonacci to Fibonacci structure, now would happen in 3 successive growth phases for the stretched Fibonacci structure as shown below in Figure 8 below.

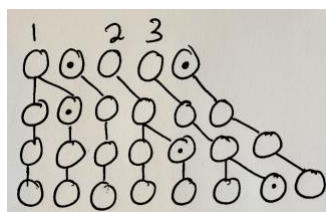


Figure 8: Stretched Fibonacci’s diluted growth

However, this problem is not obvious from the looping point of view because, as it can be seen in Figure 8, increase loops directly become full loops in subsequent rows making less visible (or even invisible) for the looper where the increase should occur in each new row. For example, to go from the in-between Fibonacci row containing 6 loops to the in-between Fibonacci row containing 7 loops, how do you know to add the increase loop at the 5th position based on a reading of the previous row? The relationship is more complex than in the Fibonacci structure where Geraldine could see directly in the previous row where to add the new loops.

Geraldine answered this question by developing a system of guidelines - coloured ribbons added to the structure - to mark and trace the lineage of each loop. Making the lineage of each loop visible allowed her to form a system to work with the loops and the new guidelines to “stretch” the growth. This stretching has triggered new mathematical speculations in its manipulation of the Fibonacci sequence. This re-distribution of the sensible, including guidelines tracing the lineage of each new loop brought to light several other questions, including one related to a number sequence that becomes visible in this process. This sequence of numbers, that we have called Geraldine’s sequence, appears when we pay attention to the guidelines or to position of the increases in each of the rows: this increases always seems to occur at the following positions: 2, 5, 7, 10, 13, 15, 18, The reader is invited to generate and continue Geraldine’s sequence with the help of the diagrams in Figure 7 and 8 above, paying attention to the position of increase loops. It is not the intention of this article to delve into the details of these insights and other investigations, but rather to point to the generativeness of Geraldine’s looping Fibonacci investigation. What seems important to note is the productivity of her investigation of Fibonacci with looping which has generated a rich new area of investigation to be explored.

Conclusions

de Freitas (2018) describes a fine intertwining of sensory engagement and innovation in mathematics. The case presented in this paper about the Fibonacci number sequence and looping is an example of an investigation where the sensibilities and materiality attached to the practice of looping have contributed to a new look at this well-known number sequence and the possible ways of understanding and generating it. The sensibilities to the metal loops, their connections, their flexibility/rigidity, their flatness or its “too much-ness”, have brought to light areas of investigation that mathematically contribute to enliven the theme of the Fibonacci number sequence, even though it may at times seem like a theme of the past that is finished and well understood. The practice of looping, by transforming what is “able” - what is voice-able, see-able, touch-able - has illuminated new links and patterns that enrich and enhance the world of possibilities attached to this mathematical theme. This example contributes to a growing discussion on the ways that sensibilities and materialities matter in mathematics. It raises questions about who and what is “able” in mathematics by opening a window on what can happen when loops, hands, fingers and metal wire contribute collectively to the making of new mathematics.

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