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Embodiment at the crossover of sensuous cognition and linguistics: a possible bridge between physics and mathematics education

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In this article, we propose a tentative path for the encounter of physics education and mathematics education at the crossover of Corni's and Fuch's imaginative and figurative approach and Radford's notion of sensuous cognition in mathematics education. We conceive cognition as the outcome of the interaction between the body and cultural environment via activity with signs and artefacts which shapes our senses into theoretical organs able to recognize patterns of actions (image schemas) that trigger a figurative and imaginative thinking based on conceptual metaphors. Physics as a cultural-historical environments provides a realm of activities that shape sensuous cognition into modes of theoretical perception that broaden mathematical meaning. We address the case of fraction in the context of prospective primary school teachers.

Keywords: Embodiment, sensuous cognition, image schemas, metaphors.

Introduction

In this article, we present a possible bridge between physics education and mathematics education. We envisage their encounter at the crossover of the imaginative and figurative approach in physics education (Corni & Fuchs, 2021,2022) and Radford's notion of sensuous cognition in mathematics education. Corni's & Fuchs's model for physical science education draws on existing frameworks in physics, narratology, cognitive linguistics, and a theory of the development of cognitive tools, creating a framework for an imaginative (fundamentally metaphoric and narrative) approach to the teaching of physical science. Thinking is the imaginative and figurative product of an embodied mind that unfolds as fundamental conceptual metaphors relying on image schemas, and narratives.

On the other hand, sensuous cognition considers mathematical thinking and learning as the dialectics between the individual's perception and the cultural-historical environment via activity carried out with signs and artifacts. The interaction between perception and the cultural-historical domain moulds human senses into "theoretical organs" able to perceive mathematically. In this theoretical stance, mathematical concepts are fixed patterns of activity that emerge in the cultural-historical development of mathematics. Learning is a process of objectification that allows students to transform, in sensuous cognition, mathematical objects into objects of consciousness.

The two approaches mentioned above hold a potential incommensurability, which we exploit to root embodied thinking in the dialogue between mathematics and physics. In fact, both theories tackle the same issue from different points of view, but they agree in considering ideal and material aspects of learning constitutively interlocked. The meeting point is in the ideal forms emerging from action, called *fixed patterns of activity* by Radford's Theory of Objectification and *image schemas* by cognitive linguistics. The two approaches look at different sides of the same coin. Sensuous cognition describes the dialectics between individual perception and cultural knowledge that opens to a

phenomenological approach to learning, overcoming the mind-body, ideal-material and conceptual-sensible opposites. Such a dialectics unfolds in activity connected to the use of signs and artefacts. The imaginative approach brings to the fore a complementary outlook on how cultural objects are transformed into objects of consciousness via sensuous cognition. Image schemas and conceptual metaphors outline how knowledge (as fixed patterns of activity) is recognized as an object of consciousness and evolves into broader and deeper meanings triggered by imaginative and figurative forms of embodied cognition. In turn, imaginative and figurative embodied thinking can trigger new forms of action sustained by image schemas and metaphors that materialize knowledge in sensuous cognition. In this study we focus on a range of activities allowed for by physics that give rise to effective image schemas to metaphorize the meaning of fractions. We take into account a series of fractional relationships of the type $a=b/c$ that express physical meanings based on the image schemas of container, flow, substance, verticality and resistance. This outlook roots the meaning of fractions in the embodied experience of physics, giving rise to new forms of sensuous cognition in mathematics and objectification of fractions enhanced by imaginative and figurative processes.

Sensuous cognition

Activity is at the core of the Theory of Objectification (TO), and mathematical thinking, knowledge and learning are inseparable from the social practices of individuals. In this perspective, *mathematical objects* are *fixed patterns* of reflexive human activity. Mathematical knowledge is a system of ideal archetypes of reflexive activity reified in the cultural-historical dimension (Radford, 2021). According to the TO, the issue of *learning* is rooted in the dialectics between the individual and their culture. Learning is a movement pushed by the intrinsic differential between individual and cultural knowledge. In fact, in attending to knowledge the student has to cope with something that in the beginning is different from him, an alterity that challenges, resists and opposes him. Learning is the process of objectification that erases such a difference to make sense of knowledge and transform it into something familiar, allowing new forms of action, thinking, imagination and feeling:

the social processes of progressively becoming aware of cultural-historical systems of thinking and doing—something we gradually notice and at the same time endow with meaning. [...] those acts of meaningfully noticing something that reveals to our consciousness through our bodily, sensory, and artefactual semiotic activity. (Radford, 2021, p. 78)

The signs and artefacts that mediate the objectification processes are called *semiotics means of objectification*.

These objects, tools, linguistic devices, and signs that individuals intentionally use in social meaning-making processes to achieve a stable form of awareness, to make apparent their intentions, and to carry out their actions to attain the goal of their activities, I call semiotic means of objectification. (Radford, 2021, p.100)

The outcome of learning as a process of objectification is the encounter with mathematical cultural objects and their transformation into objects of consciousness. From this standpoint, learning has a strong phenomenological nature where noticing occurs in an enlarged notion of mind and consciousness, termed by the TO *sensuous cognition*, that intertwines ideal and mental in perception feelings and kinesthetic activity. Within this enlarged notion of mind and consciousness, the

encounter with knowledge and its transformation into an object of consciousness occurs within the features of *sensuous cognition*. In the light of the dialectic-materialist approach underpinning the TO, the basic tenet behind sensuous cognition is that the body, the senses, and the objects of sensation are not a priori entities but are mutually transformed by cultural-historical activity *entangled with the use of signs and artefacts*.

From the standpoint of sensuous cognition, human perception is a cultural artefact shaped by our own cultural-historical practices. In this regard, perception deploys cultural forms of seeing, touching, hearing etc. that characterize our relationship with the world. How do students change their perception from “spontaneous” forms of attending to objects to a mathematical and theoretical one? To answer this question, we must consider learning as a “domestication of the eye” (Radford, 2021), a long process that allows students - in cultural-historical activity intertwined with the use of signs and artefacts - *to transform the eye (and other senses) into sophisticated theoretical organs* able to notice and make sense of certain things in mathematical manners - for example, recognizing numerosity, algebraic structures, geometric invariants etc. We remark on the co-variational nature of sensuous cognition in that learning processes entail a transformation of perception and the perceived cultural object into an object of consciousness.

We underline the multimodal nature of the “domestication of the eye” in the various sensorial channels and the richness of signs and artefacts interwoven with cultural-historical activity (Radford, 2021).

Image schemas and conceptual metaphors

We believe that it is advantageous from an educational point of view building a bridge between the fixed patterns of activity, transformed in objects of consciousness in objectification processes, and the use figurative and imaginative according to cognitive linguistics. It allows students an effective and meaningful use of language and signs when encountering mathematical objects and triggers new forms of activity, endowing mathematical thinking with a stronger ability to interpret the situations and act accordingly, with versatility and adaptability in blending knowledge and knowing. Cognitive linguistics considers cognition as the outcome of the interaction between the body, and the physical and sociocultural environment. Thus, the relationship between elements of language and their referents stems from *human embodied action and experience* (Johnson, 1987, 2007). A fundamental tool in cognitive linguistics is the notion of image schema. They are foundational forms of embodied imaginative thought, sensorimotor gestalts that act as basic schematized perceptual units that develop through our interaction with the environment; among perceptual units forming in our mind, they are particularly simple (primary) and abstract forms. Image schemas are “recurrent, stable patterns of sensorimotor experience . . . [that] preserve the topological structure of the perceptual whole . . . having internal structures that give rise to constrained inferences” (Johnson, 2007, p. 144).

Corni and Fuchs (2020, 2021) call image schemas small-scale perceptual gestalts that exhibit limited structure (aspects) with an internal logic that is used in reasoning. Examples of image schemas (Table 1) are PATH, CONTAINER, IN-OUT, UP-DOWN (verticality), SUBSTANCE, AGENCY or CAUSATION, SCALE, CYCLE, PROCESS, and many more. Figure 1 presents the most common image schemas used in meaning making processes that Johnson (1987, 2007) identified in their

studies of cognitive linguistics. It is advantageous to bind image schemas with the notion of *cognitive tools*. They characterize the anthropogenic and ontogenetic development in the mastery of forms of language use and forms of embodied imaginative thought; for example, polarity (binary opposites), metaphors, analogical thinking, and narratives. For the scope of our study, we focus on conceptual metaphors, that is, figurative structures that are created by projecting an embodied understanding of a domain (source domain) into another domain of experience (target domain). A possible and potent source domain can be an image schema in the encounter with mathematical knowledge. They allow us to produce concrete metaphoric expressions.

Opposition	<i>binary opposition, polarity (*)</i>
Scale	<i>path, gradient</i>
Space	<i>up-down, verticality, high-low, front-back, left-right, near-far, center-periphery, contact</i>
Process	<i>process, state, cycle, change</i>
Container	<i>containment, in-out, surface, full-empty, content</i>
Force/Causation	<i>balance, counterforce, compulsion, restraint, enablement, blockage, diversion, attraction, manipulation, tension</i>
Unity/Multiplicity	<i>merging, collecting, splitting, iteration, part-whole, mass-count, link</i>
Identity	<i>matching, superimposition, difference</i>
Existence	<i>removal, bounded space, object, (fluid) substance</i>

Figure 1: The most fundamental image schemas outlined by Johnson (1987): on the left image schema category, on the right its components

Fractions at the crossover of mathematics and physics

We propose an interpretation of fractions based on figurative and imaginative thinking that stems from the realm of physics. We consider a series of physical situations that metaphorize the meaning of fractions taking as source domain a network of image schemas intertwined with sensuous activity mediated by physical apparatus.

Based on the *container, substance, and verticality* image schemas we metaphorize the following physical quantities:

- $\Delta h = \Delta V/S$, the change of height of a fluid substance in a Becker depending on the change of volume and on the (vertical wall) container section;
- $\Delta T = q/C$ the temperature of a body depending on the exchanged quantity of heat and the body thermal capacity;
- $\Delta V = Q/C$ the change of electric potential of a body depending on the exchange of electric charge and the electric capacity;
- $\Delta v = p/m$ the change of velocity of an object depending on the exchanged momentum and the mass.

Based on the *flow, verticality, tension, and resistance* image schemas we metaphorize the following quantities:

- $I_v = \Delta P/R$, the current of a fluid depending on the pressure gradient and the resistance of flow;
- $I_q = \Delta T/R$, the heat flow (current) depending on the temperature gradient and the heat resistance;

- $I_Q = \Delta V/R$, the electric current depending on the electric gradient and electric resistance.

In the case of the metaphor based on the *container, substance, and verticality* image schemas, perception is moulded to recognize the fixed pattern of activity based on perceiving the increasing/decreasing height of a liquid in a container as we pour in/spill out it. The fraction expresses the layers of liquid that deposits on the section of the container. This activity mediated by physical apparatuses such as a Becker, indexical use of natural language allows perception within sensorimotor activity to recognize the image schema of container as a fixed pattern of activity that gives a metaphoric meaning to the fraction $\Delta V/S$. The symbolic expression $\Delta h = \Delta V/S$ formalizes the physical meaning based on the embodied metaphoric thinking but at the same time it opens to new and broader meaning of the fraction beyond the one-whole interpretation of the numerator and denominator. We can say that the first relation and the ensuing physical meaning serves as first source domain and the other physical relations as target domain for their embodied imaginative understanding. Heat, electricity, and momentum are not fluids in a container, but the metaphor outlined above allows the embodied physical meaning of the fractional relations.

In the second group of fractional relationships based on *flow, verticality, tension, and resistance* image schemas, the interaction between the body and the environment is based on activities where a liquid flows in a conduit under a certain push that encounters resistance from the nature of the liquid and of the conduit. The fraction expresses the push the liquid receives against its resistance. The activity mediated by physical apparatuses such as a vessel filled with water, a pipe, indexical use of language shapes perception within sensorimotor activity to recognize the image schemas of *flow, verticality, and resistance* as a fixed pattern that gives an embodied meaning to the fraction $\Delta P/R$. Such image schemas allow us to metaphorize the other physical quantities of the second group mentioned above.

A teaching-learning episode

We share data taken from the physics lab activities carried out by prospective primary school teachers of the Faculty of Education of the University of Bolzano, attending a course in physics education based on imaginative and figurative thinking. The experiment combines the variation of height of a liquid depending on the volume and section $\Delta h = \Delta V/S$ and the current of a fluid depending on the pressure gradient and the resistance of flow $I_V = \Delta P/R$. The experimental apparatus is made of two plastic cylinders connected by a tube with a tap (Figure 2a). In one cylinder we pour water, and we analyse the current of fluid as the water goes from the first to the second cylinder. The level of the water in the first cylinder is constant to maintain the pressure stationary. On the filling plastic cylinder there is a scotch paper tape (Figure 2b) where the students mark with a pen the level of water every 10 seconds. The students understand the meaning of $\Delta h = \Delta V/S$ and $I_V = \Delta P/R$ based on the image schemas mentioned above acquired in previous experiments. Based on their metaphoric understanding of the formulae they construct the conceptual map that describes the evolution of the physical system obtained by the interaction of the two cylinders (Figure 2a). In fact, their imaginative and figurative embodied thinking triggers new forms of action related to the interaction between the two cylinders.

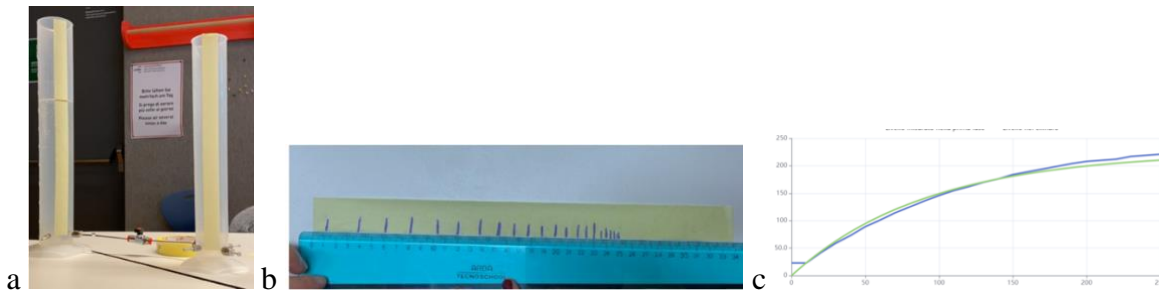


Figure 2: a) The pump and the filling cylinder; b) scotch paper tape to measure the level of the water; c) the experimental graph height-time (blue) and the graph of Insight Maker (yellow)

The students produce 2 graphs with software Insight Maker (Figure 2c). After building the conceptual map in Figure 3, based on the flow and container image schemas, and inserting the appropriate equations in the ellipses of the map, Insight Maker carries out the calculations and provides the graph showing the height of the liquid as a function of time. The first is the theoretical plot obtained inserting in the software the appropriate equations. The second is obtained by inserting the experimental data, height in mm of the level of water in the cylinder every 10 seconds. The students are asked to predict the outcome of the experiment and interpret the theoretical and the experimental plots - based on their metaphoric understanding of the relationship between the physical quantities involved (Figure 3) in the experiment.

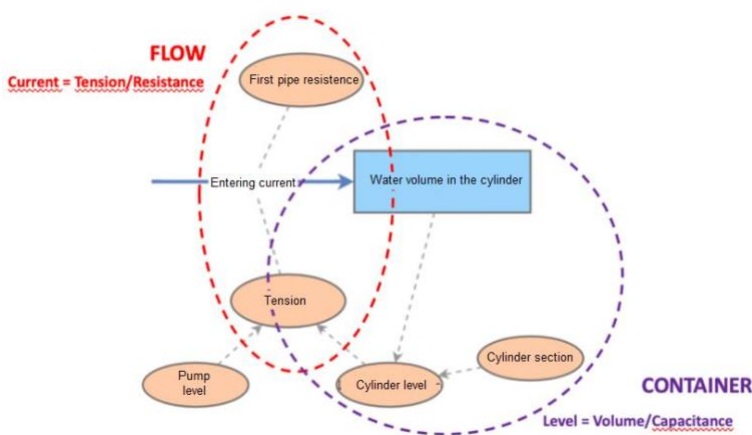


Figure 3: Conceptual map produced by the students with Insight Maker based on the flow and container image schemas

We provide below some excerpts of the students taken by their laboratory reports that show their conceptual control of the physical phenomenon based on their figurative and imaginative reasoning.

Group 1 remarks that: From the observation made during the experiment, we could see that in the first intervals of time, the water flowed from the pump cylinder to the experimental cylinder with great speed. Towards the end of the experiment, however, we noticed how the water flowed more slowly and consequently how the rise in level in the experimental cylinder slowed down. This can be observed by looking at the table with the data collected, as well as the scotch tape paper: from 10 to 20 seconds, the water level rose by 27 millimetres, while in the last 10 seconds it rose by only 2

millimetres. Consequently, it can be seen from this analysis that the difference in cylinder pressure is directly proportional to the water current. During the experiment, we were able to observe the achievement of dynamic equilibrium of the system, which is given by the balance of the two currents.

Group 3 remarks that the amount of water in the two tubes is not important: since the tubes had two different diameters, the amount of water they contained was different, but the important thing was the level of water in the two cylinders: in fact, the water only stopped flowing from one cylinder to the other when the level of water in the two cylinders was the same and consequently the pressure difference was cancelled. We noticed that initially the water flows very fast, as the pressure in the two cylinders is different, whereas as the experimental cylinder fills up with water and the pressure balances, the water flows more slowly.

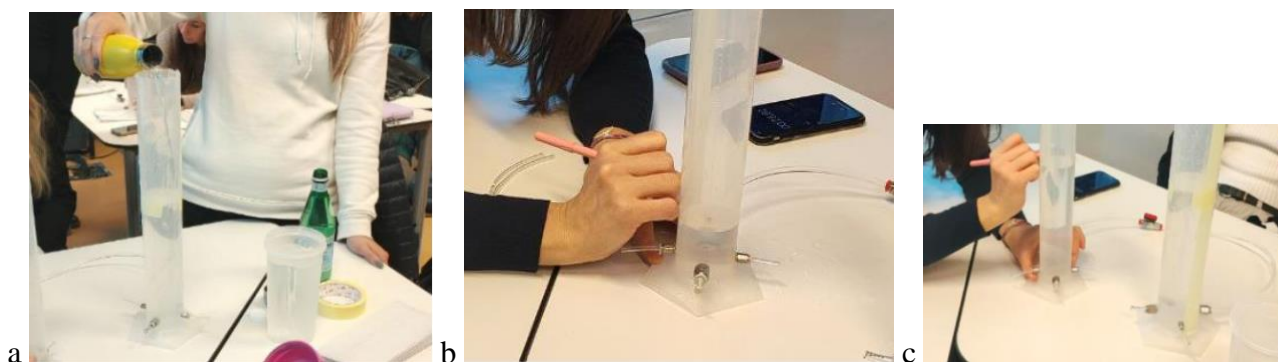


Figure 4: a) pouring water in the pump; b) and c) marking the level of the water in the cylinder

The students' interpretation of the physical phenomenon is based on conceptual metaphors to build the physical meaning. Given the complexity of the phenomenon, the metaphors involve a network of the image schemas we presented in the previous section: *container*, *substance*, *verticality*, *flow*, *tension*, and *resistance*. In the interaction with the physical apparatuses, Insight Maker, and the ensuing activity they perform, their perception is moulded to “see” according to the culture of physics and recognize the configuration of image schemas that trigger their figurative and imaginative thinking. For example, the action of pouring water in the pump cylinder (Figure 4a) and marking the water level in the cylinder shapes perception towards the recognition of the image schemas that ground their metaphorical understanding of the relation between the height variation of liquid in the cylinder, the current of the fluid and the pressure gradient.

In the setting of this activity, designed specifically for the learning of physics, mathematics apparently remains in the background. This is true in that it does not appear as explicit calculations that would be too difficult to handle by prospective primary school teachers and they are outsourced to Insight Maker software. Nevertheless, mathematical meaning is behind, qualitatively, the relations between the physical quantities involved. The height of the water in the pump cylinder depends on its section and the change of volume ($\Delta h = \Delta V/S$) and the flow of water in the cylinder depend on the gradient of pressure and the resistance of flow ($I_v = \Delta P/R$). Moreover, the students need to understand the meaning of the formulae they have to insert in Insight Maker via the conceptual map. The use of the formulae is bound to the intertwining of their mathematical and physical meaning controlled by conceptual metaphors based on the conceptual map they construct with Insight Maker. We remark that when the physical phenomenon changes shifting from fluid dynamics to other kinds of physical phenomena

based on the same image schemas that provide a metaphoric understanding of the physical behaviour and the fractional relations, the students easily reconstruct the conceptual map and insert the correct equations that allow Insight Maker to plot the dependence between physical quantities. The student's behaviour accounts for a broader and meaningful understanding of fractional relations as the outcome of the metaphorical thinking rooted in sensuous cognition.

Conclusions

According to Radford (2021) mathematical objects are both conceptual potentialities embedded in mathematical culture and actualized in objectification processes that develop our senses in theoretical organs capable of perceiving mathematically in suitable domains of activity with signs and artefact. A possible link between the potential and actualised nature of fractions is the figurative and imaginative nature of thinking carried out with image schemas and metaphors. In fact, image schemas and metaphors have a referential function towards general concepts, but they can also trigger sensuous cognition through the embodied experience they condense. Physics enlarges the possible domains of action in which fractions can be objectified with a broader range of interpretations and more robust meaning, fostered by conceptual metaphors, not confined to a void manipulation of signs. This perspective on the fractions, rooted in the embodied context of physics, fosters the students' ability to model physical situations tapping into a solid and versatile understanding of fractional relationships. Of course, the same holds for mathematics in general when learning according to this perspective.

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