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# Two inquiry approaches to STEM: The role of mathematics

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*Teaching STEM is often described as closely related to inquiry-based teaching. This paper describes two STEM projects that tested two approaches to inquiry-based teaching in two 7<sup>th</sup>-grade classes. One project focused on the 5E model and the other project focused on study and research paths, an inquiry approach developed in the paradigm of questioning the world. In the analysis, classroom observations were transformed into question-and-answer diagrams and combined with a praxeological analysis, which formed the basis for the interpretations. The results showed that students alternate between making inquiry and studying mathematics in two very different ways. However, both projects had challenges in respectively creating coherence between the mathematical studies and inquiry and in making the students perform an in-depth mathematical study.*

*Keywords: STEM, inquiry, paradigm of questioning the world, praxeology*

## Two different approaches to STEM

There is almost a consensus today that an inquiry-based method, which places students at the centre of investigations and provides them with an active role in their own learning process, is an important and effective approach when teaching STEM (Bybee, 2013). An inquiry-based approach is often described with a large degree of openness, where students can ask questions and there is room for them to pursue the paths they find interesting (Artigue & Blomhøj, 2013). There are, however, many different approaches to inquiry-based teaching (Artigue & Blomhøj, 2013). In this paper, we describe, analyse and discuss two projects of STEM inquiry (project 1 + 2). In the first project, the students worked with the 5E model (Bybee, 2018) to address the problem of building a hydroelectric plant. The idea was that the teachers should use the 5E model to plan their course, so that the students through the course worked in all the phases in the model. The 5E model is an instructional learning model that promotes collaborative, active learning in which students work together to solve problems and investigate new concepts by asking questions, observing, analysing and drawing conclusions. This cyclic model involves the following five phases: engage, explore, explain, elaborate and evaluate (Bybee, 2018). In the second project, the students worked in the paradigm of questioning the world (PQW; Chevallard, 2015) and explored the question ‘Is it possible to affect climate changes if we modify our eating habits?’. The PQW, developed in the anthropological theory of didactics (ATD; Chevallard, 1992, 2015), is another inquiry-based approach that differs from the traditional paradigm of ‘visiting monuments. In the PQW, the curriculum’s focus is on important questions, such as those about the world. The students are motivated and driven by these open generating questions, and both the issues and the context are taken seriously rather than being considered just a pretext for visiting a certain area of mathematics (Chevallard, 2015). In the inquiry process, the students are required to do more than just provide answers; they are also expected to pose related but smaller questions, find answers in books or other media, explore and discuss and validate findings (Chevallard, 2015). In the PQW the teacher used a study and research path (SRP) as a way to design this course. SRP is a method

for designing projects used in the PQW, with a focus on the dialectic of students' studies and research (Bosch, 2018). The concept of SRP states that students follow many unknown paths in the process of study and research. An SRP begins with generating a 'big' question  $Q_0$  (Bosch & Winslow, 2015). The students pose derived questions ( $Q_{1...}$ ), study established answers and verify them. In an SRP, the study is not determined in advance but has to be found in accordance with  $Q_0$ .

In both inquiry approaches, the intention is that the students' investigations must guide the process. In this paper, we describe two STEM cases in which the intention is for mathematics to be the focal point of the projects. We are interested in determining how students develop mathematical understanding in the two cases. **RQ: What mathematical knowledge comes into play in the two inquiry-based STEM projects, and what consequences does this have on the role of mathematics in the two projects?**

## Methodology

In both cases, design research (Cobb et al., 2017) was used, which involves four cycles of *identifying* either the PQW or an inquiry-based STEM approach; *design of the activity*; *implementation* and *observation* and *reflections* upon students' learning and different revisions of the activity. The two cases originate from two research projects and are defined as critical cases (Flyvbjerg, 2006); they were developed and designed differently. In what follows, the two cases are described and analysed separately.

In the case of building a hydroelectric plant (project 1), the STEM project was designed and implemented in a research and development project called LabSTEM. Organisationally, in LabSTEM, temporary communities of practitioners and researchers are established in the form of STEM laboratories. The first author of this paper has been leading one of these laboratories, wherein a group of three teachers developed a STEM project for a 7<sup>th</sup>-grade class at a nearby hydroelectric plant. In this laboratory, all participants were primary or lower secondary teachers. Two were mathematics teachers, one was both a science- and mathematics teacher. They met 12 times for 3 h workshops. In these workshops, among other things, the teachers were introduced to the 5E model, which all were supposed to include in their STEM activity. In the hydroelectric plant project, the three teachers and the leader were involved in testing the developed STEM project, which lasted an entire school day and was recorded with a handheld camera. When there was group work, the camera followed a specific group, which was selected by the teachers based on their capability and confidence. Before the project, the students had no knowledge of the hydroelectric plant but had been working a little with the concept of energy.

In the PQW (project 2), three 7<sup>th</sup>-grade mathematics teachers participated in a research project about changing the teaching paradigm from 'visiting works' to 'questioning the world'. The three teachers developed, together with the second author of this paper, an SRP focusing on the following generating question: 'Can we affect climate changes if we modify our eating habits?' The aim was also to include the statistical program TinkerPlots, as the idea was, that this dynamic computer program could help students to be able to make inquiry with their data. Both the researcher and teachers actively participated in the processes of planning, implementing, discussing and revising the SRP. Because TinkerPlots was new to the teachers, a teacher workshop was arranged at the beginning of the study. The three teachers participated in the study, which lasted for approximately 80 h and included

interviews; workshops and design, implementation, reflection and revision of the SRP in three classrooms. The SRP was tested in a Danish 7<sup>th</sup>-grade class where video observations, screenshots and lesson notes were collected. Before the project was implemented, the students in this class had no knowledge of how to work on mathematics projects and only little knowledge of statistical analysis.

## **Analytical framework**

To analyse the two inquiry-based approaches, we use methods developed in the theoretical framework of ATD (Chevallard, 1992, 2015). We use both the *SRP concept* (Winsløw et al., 2013) and the notion of *praxeology* (Chevallard, 2015). According to Winsløw et al. (2013), an SRP can be used as both a planning tool and an effective method for analysing instructional procedures. The notion of *study* in SRP is defined as the activity of accessing existing knowledge, and the concept of *research* is defined as the process of autonomous inquiry (Bosch & Winsløw, 2015). According to Winsløw et al. (2013), the aim is for students to engage in a dialectic between study and research in which new answers are generated by the students, and the study is used to uncover the answers. The students' studies and research can then be unfolded in a tree-like diagram in which the students' and teachers' questions and answers (Q&As) refer to the paths they have chosen. *Praxeologies* are known as models of human behaviour (Chevallard, 2015). A praxeology represents knowledge in terms of a praxis (know-how) and a logos (know-why) block. A praxis block is composed of different types of tasks and techniques, while a logos block is composed of a combination of technology and theory. The goal of technology is to make techniques understandable so that people can comprehend why certain techniques are necessary and how they relate to other techniques. Theory is related to the justification and unification of technologies. Praxis and logos are mutually constitutive (Chevallard, 2015).

## **Analysing the two inquiry-based approaches**

In the following section, we use Q&A diagrams as the analytical tool, as they provide a visualisation of who actually poses questions and who finds the answers in the two projects. An analysis of the praxeologies of mathematics in the two cases provides an overview of mathematics knowledge to reflect on the role of mathematics in the projects.

The hydroelectric plant (project 1): This STEM project was divided into four exercises, all of which were performed at the hydroelectric plant. The aim of the first three exercises (parts 1–3) was for the students to gain knowledge of building praxeologies so that they could design and build an efficient hydroelectric plant themselves (part 4). (Q<sub>0</sub>: How to design and build an effective hydroelectric plant). Part 1 was an introductory exercise aimed to **engage** the students. The exercise involved making the students discuss and decide whether they would choose to expand the plant to produce more green energy, even though this could disturb nature and negatively impact biodiversity. Part 2 was an orienteering photo-race in which the students had to take pictures of natural science phenomena to be discussed afterwards (turbines, generator, water supply, pipe and Eel box). The aim of the exercise was to make the students **explore** the hydroelectric plant. Part 3 comprised four workshops of 25 min each. These workshops aimed for the students to examine the individual parts of the plant in detail and to use mathematics to **explain** and **elaborate** on how the efficiency of a hydroelectric plant could be improved. Each workshop had an overall question to be solve. In what follows, the four workshops are described; however, the mathematics content was only discussed more in-dept in workshop 1+2.

Workshop 1: Q1: How much water can you transport from the top of the reservoir down to the bottom and at what speed? Here, each student received a bucket to fill in water, and then they were asked to run down the hill with the filled bucket. At the end of the hill was a tub shaped like a truncated cone, which they were asked to fill. The following Q&As were addressed (to clarify who is speaking the students' Q&As are coloured red, while the teachers' Q&As are in black):

Workshop 1 was linked to Q2: "What is the speed at which the water is transported?"

Q2.1: Don't we first need to find the volume of water?

A2.1.1: You need to use the formula for a truncated cone.

A2.1.2: Measure both the radius at the top (where the water stops) and at the bottom of the tub.

A2.1.2.1: I get the radius as 52.

Q2.1.3: You have to calculate in litres what unit you need?

A2.1.3.1: Metre

A2.1.3.2: Decimetre

Q2.1.4: You also need the height of the water?

A2.1.4.1: 24.5

Q2.1.5: Do we then have all the numbers in the formula?

A2.1.5.1: Yes - G then becomes 26.42 (calculator).

Q2.1.6: What about small g in the formula?

A2.1.6.1: It is 24

Workshop 2: Measure the velocity of the water stream going out of the hydroelectric plant.

Q1: How fast does the water move away from the hydroelectric plant?

Q1.1: Do you know the unit of velocity?

A1.1.1: Yes, it is how strongly something moves in time. Km/h.

A1.1.1.1: It's with length per time. Make different measurements and find the average.

Q1.1.2: Is it time divided by length, or is it the opposite?

A1.2: Ok, let's start by measuring 1 metre.

A1.3: Try saying 1 metre divided by the measurement.

A1.3.1: 0.056 metres per second.

Q1.4: Shouldn't we try with a measurement of 2 metres?

A1.4.1: It was 26 seconds.

A1.4.2: 1 divided by 26; this is 0.0374.

A1.4.2.1: But shouldn't we divide by 2? Otherwise, it will be the same as before.

A1.4.2.1.1: Then we just need to say 1 divided by 13.

Q1.5 Ok, we should try 1.5 metres.

Q1.5.1: Don't we need to say 1.5 divided by the time then?

Q1.6: What is the average of your measurements?

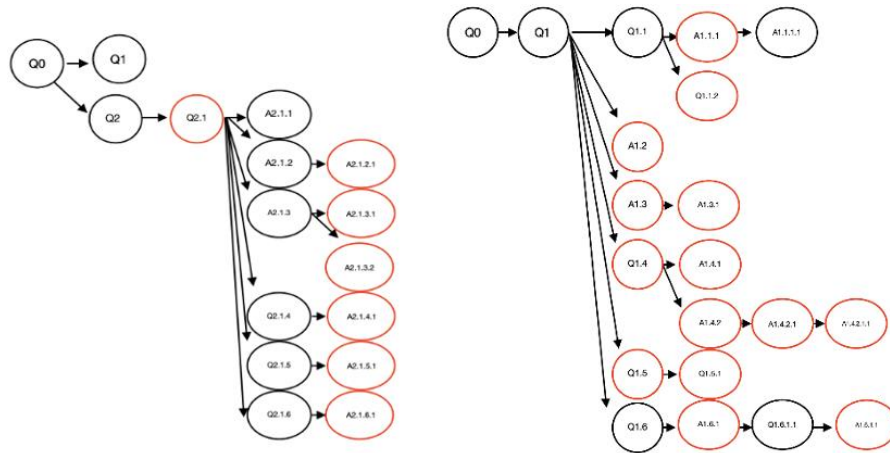
A1.6.1: 0.131.

Q1.6.1.1: That cannot be true. Look at the numbers. Have an idea of what it might be?

A1.6.1.1: Then it is 0.05.

Workshops 3 and 4: In workshop 3, the students had to investigate the question 'How much water can the water supply pipe hold?', which means that the students had to measure the circumference and length of the pipe and calculate its volume. In workshop 4, the students had to investigate the question 'What is the volume of the water reservoir on top of the pipe? This reservoir is box shaped'. Part 4 was linked specifically to Q0: How can we build the most efficient hydroelectric plant? (by using only these materials—different pipes, jugs of water, small turbines and voltmeters). This part can be considered an engineering project and an **evaluation** of the focus of the day. The intention is that the students use all the knowledge they have learned throughout the day to build their own mini hydroelectric plant. It is, however, limited to the extent to which we see these things explicitly included in the students' Q&As throughout this part.

The Q&A diagrams in Figures 1 show that almost all questions were posed by the teachers (black), while the answers were developed by the students (red). In workshop 1, the students answered the questions posed by the teachers (all Qs are written in black); however, the further questions were generated on a question from a student (Q2.1). In workshop 2, there were slightly more questions from the students; however, the questions were mostly about the calculation (the methods) and the different ways of measuring velocity, as well as the way to use the formula (A1.4.2.1). Overall, it was not the students' questions that generated the study in the two workshops.



**Figure 1: Q&A diagram of workshop 1 (to the left) and workshop 2 (to the right)**

In a praxeological analysis of the mathematics discussion, we observed that, during the process, there were situations in which the students explicitly developed different mathematical praxeologies. In workshop 1, the students used a specific technique to measure the amount of water in the truncated cone. The technique involved inserting the measured numbers into a formula (the volume formula). Here, we almost see what Brousseau called the topaz effect (Brousseau, 1997, s. 25). In this situation, the students and teachers did not talk explicitly about the technology of this technique, but the theory was the volume formula. In workshop 2, the students measured the length and time by using a tape measure and a stopwatch, respectively, and calculated the velocity using the division technique. When using the velocity formula, there is also a discussion about the appearance of the formula (technology). This is the only instance in this project in which we explicitly observed a mathematical ‘technology’ at play. The theory in this discussion is the theoretical formula of velocity. In part 4, there was no explicit mathematical praxeology; instead, the students adopted the technique of trying different approaches, such as using a voltmeter and acquiring ideas. However, they discussed neither the reason why these techniques were required nor any theories behind them.

If we look more generally at the process, the project is structured such that there should be some workshops in which students together undertake the more inquiry-based research task of building a hydroelectric plant themselves (Figure 3). However, there is not much explicit alignment between the different praxeologies used in the studies and the more inquiry-based research task of building the plant (as denoted by the dashed lines in Figure 3). This means that the students did not necessarily see the connections and possibilities in the intentions of these studies.

Questioning the world (project 2): This project started with the generating question Q0: Can we affect climate changes if we modify our eating habits? This question was posed by a teacher and led the

students to pose and answer different Q&As in an interaction between group work and classroom discussions. The teacher asked many questions (Q<sub>1</sub>–Q<sub>9</sub>) (e.g. Q<sub>2</sub>: What contributes to global warming? Q<sub>5</sub>: Is climate change ‘Fake News’? and Q<sub>9</sub>: What do we know about grade seven? The students answered these questions through their own immediate reflections, searching the internet or doing some research. In Q<sub>4</sub> and Q<sub>9</sub>, the students used internet data in an attempt to answer the questions. The use of data is coloured green.

Q4.1: What to eat?

A4.1.1: Analysis, minimum/maximum value

A4.1.1.1: Diagram and climate – converted into new models

Q4.1.1.1.1: What are the benefits of eating Danish and local food?

Q4.1.1.1.1.1: How do we construct a new pie chart?

A4.1.1.2: Frequency table of carbon footprint and different kinds of foods?

Q4.1.1.2.1: What is the carbon footprint (meat)?

Q9: What do we know about grade 7?

A9.1: Data on 7<sup>th</sup>-grade survey about food and eating habits

Q9.1: What do 7<sup>th</sup>-grade students eat?

A9.1.1: Frequency tables – mode: Most students eat meat.

Q9.2: Where is the food grown?

A9.2.1: Frequency tables – mode: the food is grown in glasshouses.

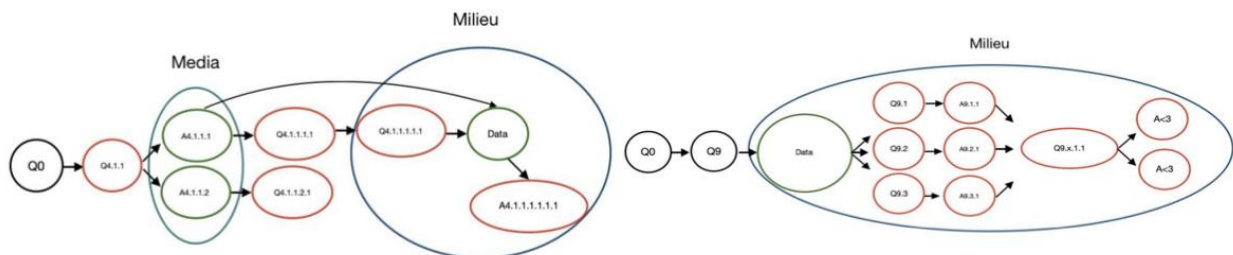
Q9.3: Does the food come from Denmark or abroad?

A9.3.1: Frequency tables – mixed

Q9.x.1.1: Is it possible to cut down the amount of meat and eat more vegetables?

A9.x.1.1: Yes

A9.3.x.1.2: No



**Figure 2: Q&A diagram Q<sub>4.1</sub> (to the left) and Q<sub>9</sub> (to the right)**

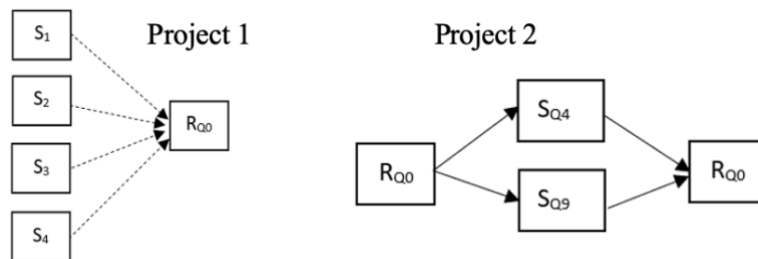
During the project, the students answered Q<sub>4</sub> and Q<sub>9</sub> by studying the data in two different ways. In Q<sub>4</sub>, they found answers by reorganising data from the internet or other media (e.g. newspapers and science papers) into new diagrams (Figure 2). In Q<sub>9</sub>, a group of students studied a homemade survey about food and eating habits. These students worked in TinkerPlots, where they elaborated on their data. The answers found in media and the new raw data resulted in the students posing smaller questions, developing new answers and comparing the new answers with established answers from media. In general, it is the teachers who often posed the questions, but the students also felt that they had to conduct some research and study to answer these questions. If we analyse which mathematical praxeology is present in answering Q<sub>4</sub>, the type of task the students undertook was ‘how to (re)model data to answer the question’. Their technique involved using Google to find data and then using the TinkerPlots program to (re)present the answers. Neither the technology nor the theory was explicit in the students’ studies and research.

In general, the students took responsibility and undertook studies (S<sub>Q4</sub> + S<sub>Q9</sub>) closely related to Q<sub>0</sub>.

## Discussion and conclusion

If we look more generally at the process, in the first project, it is structured such that there should be some studies ( $S_{1-4}$ ) in which students together undertake the more inquiry-based research ( $R_{Q0}$ ) task of building a hydroelectric plant themselves (see Figure 3). However, there is not much explicit alignment between the different praxeologies used in the studies and the more inquiry-based research task of building the plant (as denoted by the dashed lines in Figure 3). This means that the students did not necessarily see the connections and possibilities in the intentions of these studies.

Figure 3 also include project 2 and it shows how the students started by conducting research on the generating question  $R_{Q0}$ , analysed data and made different studies ( $S_{Q4} + S_{Q9}$ ) and then applied this knowledge to their research related to  $R_{Q0}$ . It is not the teacher who ‘dictated’ what the students should study but the fact that the students experienced a need to be able to answer  $Q_0$ . However, in this case, the students’ study was not particularly thorough in relation to the mathematical content and the mathematical praxeology analysis showed that it neither included technology nor theory, which means that the mathematical discussions were limited in answering both  $Q_4$  and  $Q_9$ .



**Figure 3: The dialectic of study and research in the two projects.**

In the two analysed projects, the students worked with mathematics content in two ways. In the first project, the students’ study and their mathematical immersion were placed before the inquiry-based research. Here, the students performed different mathematical exercises without any connection to the later, more investigative process. In these small exercises, mathematics was the objective - students measured the volume of water in the pipe and velocity of water in the stream as the aim of those exercises, however, during the engineering exercise, mathematics was intended to be more of a means to construct an efficient hydroelectric power plant. Here the students, did not use any mathematical concepts to discuss the design. Instead, the students became more practical and tested the different tubes, which obviously had different radii, or created different slopes on the tube without explicitly talking about the velocity or speed of water. In this way, there was no explicit alignment among the 5Es in the process. In this case, the role of mathematical praxeologies became somewhat invisible in the students’ engineering research.

In the second project, the students’ mathematical studies were based on their own (or the teachers’) questions and, consequently, were an explicit part of the inquiry to answer the generating question; however, the students’ study carried the hallmark of being without any mathematical technology and theory and only included praxis. In this project, the students needed some mathematical/statistical calculations to be able to address the broader question. In other words, mathematics here once again becomes a means to substantiate their final answers.



Although the teachers and researchers in the two projects intended to carry out inquiry-based teaching in STEM with a focus on mathematics, in both cases, we observed challenges. First, it was not the students who asked the questions; therefore, it was not the students' questions that generated the process, and the students could not pursue the paths they found interesting (Artigue & Blomhøj, 2013). Second, the students' research must naturally include some mathematical studies that involve both praxis and logos; however, if the students can choose their own studies, they must be guided by a teacher to gain depth in the study so that they do not become superficial investigations.

Note that the specific use of the 5E model or SRP model in this context does not mean that this is how these models always are or should be implemented. However, the case here shows an example of how the models were interpreted and implemented in practice. Thus, this paper points to what teachers must pay attention to when undertaking inquiry-based projects. In general, we find the use of Q&A diagrams and praxeologies as relevant approaches to analyse and study the dialectic of inquiry-based teaching between both the teachers' and students' Q&As; however, it also provided a visual overview that helped create Figures 3 and 6, which shows the dialectic between the students' study and research. This facilitates an interesting angle on how the different inquiry-based models influence the role of mathematical praxeologies in STEM teaching.

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