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# Lower secondary school students reflecting their two-week experience with quadrilaterals and GeoGebra

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*In this paper, we introduce a two-week long dynamic-geometry based series of geometry lessons, and investigate how lower secondary school students reflected their experience with this kind of learning. For a class of 25 7th-grade students, the series served as a full-fledged alternative to standard lessons on quadrilaterals, so it covered all required learning objectives. At the end of the series, the students reflected their experience, and we provide a qualitative study of their reflections. The results show that students mainly reflected on geometric topics instead of on working in dynamic environments. It indicates that dynamic geometry can stand in the background and at the same time play an important role in geometry education.*

*Keywords: Dynamic geometry environments, lower secondary school students, plane geometry.*

## Introduction

In the past decades, geometric constructions have appeared in a new context thanks to the development of dynamic geometry environments (DGEs) such as Geometry, Geometer's Sketchpad or GeoGebra software (GeoGebra, 2023) that provide us with the possibility to design and solve geometric tasks on electronics devices (personal computers, tablets, mobile phones). A DGE geometric task takes the form of a dynamic constructions created as an ordered combination of free and dependent objects (Mackrell, 2011). DGEs allow modifying these constructions retroactively by dragging the free points to new positions but, at the same time, maintaining invariant properties of all the objects dependent on these points (Mariotti, 2012).

The meta-study on recent research in geometry education conducted by Sinclair et al. (2016) reported seven major threads of contributions including advances in teaching and learning of definitions and in understanding of visuospatial reasoning. Among others, these threads cover investigations of students' understanding of geometric pictures, various approaches to defining geometric objects, or the role of digital technologies in identifying the features of a shape. What remained to be uncovered were suitable concepts of geometry lessons, teacher practice associated with task assignment in DGEs, and explorations on students' experience with DGEs. We intend to address these issues in our contribution. We introduce a series of plane geometry lessons for lower secondary school focusing on quadrilaterals and their features. The series takes the form of a two-week long set of educational activities that reach current trends in geometry education, especially the use of DGEs. We implemented the series with a class of 25 seventh-grade students and then let the students reflect on their experience in written form. Our research investigates the reflections; the research question is as follows: *How do lower secondary school students reflect their two-week dynamic-geometry based experience with quadrilaterals?* Considering the number of respondents and the fact that the reflections are subjective in nature, we analyse our data qualitatively.

## **Theoretical background**

Geometry education at the level of lower secondary school aims to shift from recognising and naming geometric objects typical for primary school mathematics to understanding the structure and the system of features of objects in a logical way. While having quadrilaterals in the centre of our intervention, the challenge here is to lead students from comprehending quadrilateral shapes as unique objects (*partition classification*; de Villiers, 1994) to comprehending them inclusively (*hierarchical classification*; *ibid*). In other words, to let students understand that specific quadrilaterals can be considered subsets of more general ones and, for instance, perceive the term *parallelogram* as the set of all quadrilaterals with opposite sides parallel. Within the frame of the parallelogram, a square, a rhombus, a rectangle and a rhomboid are understood by the transitive relation between them (Fujita & Jones, 2007).

Across all school levels, the teaching of geometry consists in gradually deepening the understanding of geometric objects. Such a process can be structured by van Hiele's five levels of geometric thinking (van Hiele, 1986). The recognising and naming that appears in primary school, is labelled as the first level (*recognition*). The shift from primary school to lower secondary school geometry corresponds with the movement from the second level (*analysis*, identifying the properties of objects) to the third level (*order*, sorting the objects according to their properties). Regarding quadrilaterals, this shift represents the transition from employing visual prototypes of the shapes to comparing properties of a set of such shapes, i.e. the movement from partition to hierarchical classification of quadrilaterals (Battista, 2002).

The outlined aims of studying the properties of quadrilaterals can be realized in mathematics classes through various situations. It is considered stimulating to provide students with activities that show the features of the shape while representing it by a composition of points, circles and lines (Duval, 2006). Such a situation corresponds with the classical geometric construction based on the use of a straightedge and a compass. It enables to determine geometrical object which properties are ensured by the basic features of a line – straight line passing two points – and a circle – a set of points in the plane that are at certain distance from a point, its centre. However, it can be achieved also through DGEs and dynamic constructions. For instance, DGEs can encourage users to identify properties of individual quadrilateral shapes (Bebernik et al., 2022) and, moreover, allow users to study the hierarchy of quadrilaterals dynamically, by comprehending them as a dragging family of objects (Forsythe, 2015).

## **Design of the study**

### **Participants**

Participants of our research study were 25 students from the same seventh-grade class (age 13 to 14 years) at a suburban school in the Czech Republic. They had already passed through the topics of Euclidean planar geometry belonging to the grades 1 to 5 (primary school level) and grade 6 (the first grade of lower-secondary school level). So, they had discussed the nature and features of elementary geometric objects such as line and its parts, circles, angles and triangles. They were used to working with straightedge and a compass while constructing these objects.

Students were taught to recognize quadrilaterals by their shape during their studies in grades 1 to 6, namely they were familiarized with a square, a rectangle, a rhombus, a rhomboid, and a trapezium. GeoGebra as an example of DGEs was introduced to them prior to our intervention, however, they have worked with it rather randomly and sporadically.

## Lessons

Our intervention consisted of ten consecutive mathematics lessons, a series of educational activities on quadrilaterals conducted in the time span of two weeks. The series covered all learning objectives required by the curriculum documents. Having in mind the above-indicated aims of studying quadrilaterals at the level of secondary school and contemporary directions in geometric education to achieve these aims, we designed activities in which the properties of quadrilaterals were represented by geometric pictures composed of lines and circles. We performed such geometric constructions with students while using straightedge and compass and, moreover, we modelled the pictures in DGEs. We used the GeoGebra Classroom environment for assigning the tasks. Each student worked on a school tablet, i.e., a device with touchscreen and internet connection.

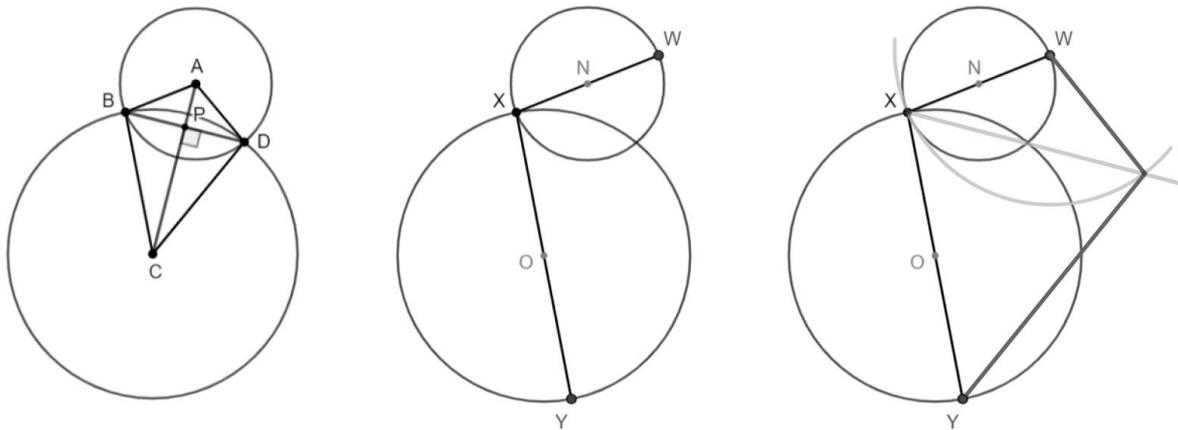
In order to promote mathematical discussion in the classroom, selected problems were solved repeatedly using different methods, with the purpose to mediate a suitable environment for developing students' mathematical creativity in geometry (Gridos et al., 2019). We targeted the development of students' knowledge of multiple solution methods. Such an approach corresponds with the framework of *flexibility* in solving mathematical problems (Rittle-Johnson et al., 2012).

Two new terms were introduced during the lessons – the above-mentioned hierarchical version of parallelogram, representing a certain group of quadrilaterals, and deltoid, a quadrilateral having adjacent sides of equal length, also called a kite. The series of lessons is summarized in Table 1 which connects discussed topics and performed activities with related literature and with Figures 1 to 4 representing selected tasks and constructions of quadrilaterals that were included in the series.

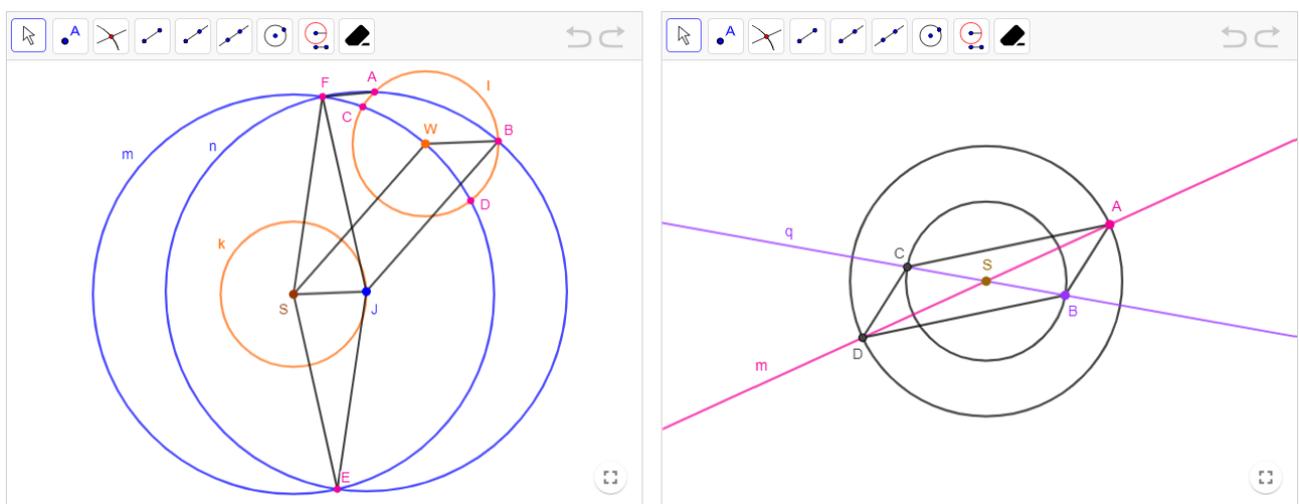
No.	Topics and activities (quadrilaterals are abbreviated as QL)
1	Introduction, partition classification of QL – repetition from primary school (de Villiers, 1994), properties of sides and angles of QL, whole class discussion.
2	Sum of angles of QL, constructing QL according to properties of sides and angles, working with straightedge and compass in school notebook, deltoid.
3	Free and dependent objects in GeoGebra (Mackrell, 2011), constructing QL using line segments with the endpoints at the given points in GeoGebra Classroom, discovering the term parallelogram (Fujita & Jones, 2007), discussion – which QL is a parallelogram?
4	Constructing QL on the basis of the sequence of constructional steps, working with straightedge and compass in school notebook. Ex.: Figure 1.
5	Discovering the properties of the diagonals of QL, working with prepared QL in GeoGebra Classroom, QL and circumscribed and inscribed circle, discussion.

6	Constructing QL in multiple methods (Rittle-Johnson et al., 2012), working with straightedge and compass in prepared worksheets, identifying QL in a picture composed of lines and circles (Duval, 2006). Ex.: Figure 1.
7	Repeating the selected task of Lesson 6 in GeoGebra Classroom, constructing QL in multiple methods according to properties of the sides and the diagonals (Rittle-Johnson et al., 2012). Ex.: Figure 2.
8	Hierarchical classification of QL (de Villiers, 1994), relationships between properties of QL, discussion.
9	Identifying QL in dynamic picture composed of lines and circles (Mariotti, 2012), inclusive relations between QL (Fujita & Jones, 2007; Forsythe, 2015), work in GeoGebra Classroom. Ex. Figure 3.
10	Conclusion, feedback

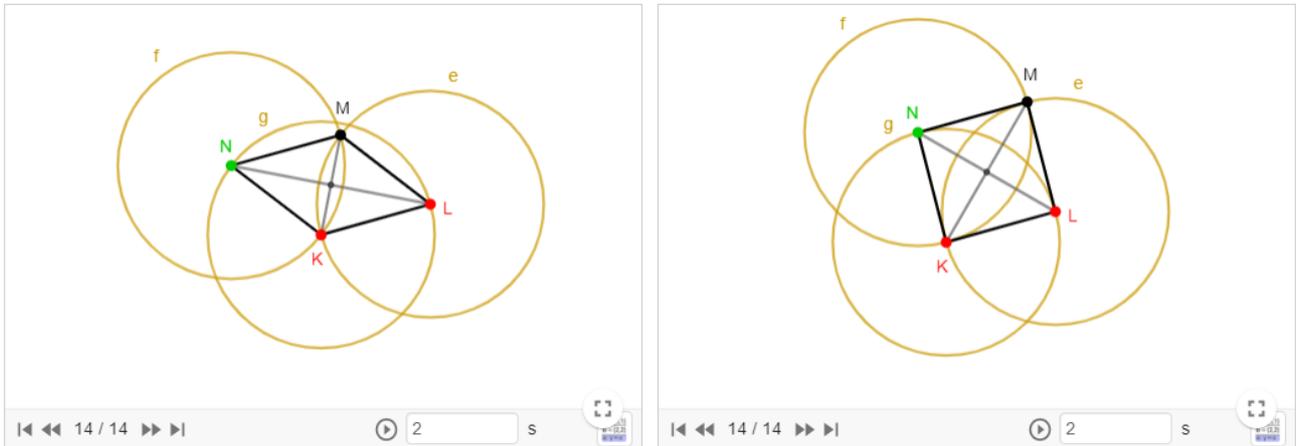
**Table 1: Summary of the series of ten lessons on quadrilaterals, with references to sample examples**



**Figure 1: Deltoid and its construction (left), a different deltoid construction to be completed (middle), a solution of the different construction (right); pre-prepared worksheet, Lessons 4 and 6**



**Figure 2: Multiple construction methods of parallelogram, students' work in pre-prepared applet in GeoGebra Classroom, Lesson 7**



**Figure 3: Dynamic modifications of rhombus, comparing properties of angles and diagonals between rhombus and square, pre-prepared applet in GeoGebra Classroom, Lesson 9**

### Data collection and data analysis

During the last lesson of the series, we assigned the students a reflective questionnaire with both closed and open questions (Table 2). The participants had 20 minutes to complete the questionnaire.

Question(s)	Label
Which quadrilateral caught my attention the most? Choose just one, your favourite. Circle it. Square    Rectangle    Rhombus    Rhomboid    Deltoid    Isosceles trapezium    Trapezium	the name of the chosen quadrilateral
For what reason was this particular quadrilateral interesting to me?	QLP
What caught my attention the most about quadrilaterals and their properties...	CON
On geometric constructions I appreciate...	

**Table 2: Questions from the questionnaire and their labels used when presenting data excerpts**

To address our research question, we analysed students' answers qualitatively using open coding and constant comparison (Miles et al., 2014). During open coding, we monitored various aspects that the participants considered relevant. The constant comparison was applied from the overall perspective, from the perspective of individual aspects mentioned in data, and from the perspective of individual participants across all aspects.

### Findings

The five following code categories appeared as relevant during the analytic process: Novelty (codes *unknown*, *little known*, *many names*), Visual impression (codes *likable shape*, *interesting shape*, *unusual shape*, *shape resemblance*), Construction impression (codes *likable construction*, *difficult construction*, *multiple construction ways*), Shape properties (codes *many properties*, *properties vague*, *properties particular*, *properties & understanding*), and Multiple shapes perspective (codes *comparison of shapes*, *square in hierarchy*). Below, we describe the categories in detail and provide illustrative data excerpts. Each excerpt starts with an anonymous label of the respondent (C01 to C25), then the label of the question (as in Table 2), and finally the answer given by the particular respondent to the particular question. Some excerpts were assigned two different code categories; the other code category is indicated in square brackets where applicable.

## Novelty

Some of the respondents just commented on deltoids as something new or least familiar for them:

- |     |         |   |
|-----|---------|---|
| C11 | Deltoid | I learned that it exists (I didn't even know about it).                 |
| C06 | Deltoid | Because I didn't know there was such a name for a quadrilateral at all. |
| C23 | Deltoid | I knew the least about him, so I was most interested in him.            |

One of them provided a similar name-oriented response also to the QLP question:

- |     |     |                               |
|-----|-----|-------------------------------|
| C23 | QLP | How many names can they have. |
|-----|-----|-------------------------------|

## Visual impression

More than third of the responses commented on purely visual aspects of the quadrilateral shapes.

Some of them shared their subjective visual impression of the selected shape:

- |     |           |   |
|-----|-----------|---|
| C07 | Trapezium | I like its shape.   |
| C21 | Deltoid   | It looks good.  |
| C01 | Deltoid   | I liked its shape and name, it was also something new for me. [+ <b>Novelty</b> ] |
| C23 | Rhombus   | It is interesting because it is slanted.  |

Others reasoned more objectively and noticed a visual resemblance of the selected shape to another quadrilateral shape or to an everyday object:

- |     |         |  |
|-----|---------|--|
| C19 | Rhombus | Because I always thought it was just a rotated square. |
| C15 | Deltoid | Because it can look like a kite or like an arrow.      |

## Construction impression

Comments displaying a subjective impression appeared also in relation to the construction of the selected shape:

- |     |         |  |
|-----|---------|--|
| C12 | Square  | Because it draws well.   |
| C14 | Rhombus | Because I find it unusual and enjoy drawing it. [+ <b>Novelty</b> ]                    |
| C13 | Deltoid | It is the prettiest and the most difficult to construct. [+ <b>Visual impression</b> ] |

More objective responses in this code category focused on constructions or quadrilaterals in general:

- |     |     |  |
|-----|-----|--|
| C22 | CON | That a given quadrilateral can be constructed in multiple ways.                  |
| C13 | QLP | How many ways can they be made (and how difficult it is sometimes to make them). |

## Shape properties

Some of the respondents provided statements regarding the properties of quadrilateral shapes, of various precision and depth. We found some vague, indirect statements regarding the (existence of the) properties of the shape(s):

- |     |         |  |
|-----|---------|--|
| C10 | Deltoid | I was attracted by its shape and properties. [+ <b>Visual impression</b> ] |
| C11 | QLP     | That they have many properties.  |
| C06 | QLP     | How many properties they have and what all we can read from them.          |

Some direct connections between construction and (determination, understanding of) properties:

- |     |         |  |
|-----|---------|--|
| C08 | Rhombus | It has a lot of properties, especially in construction. [+ <b>Construction imp.</b> ]                                |
| C10 | CON     | That when I drew the given shape, I was able to determine its properties better. [+ <b>Construction impression</b> ] |

And also a list of particular properties of a given shape:

- |     |        |   |
|-----|--------|---|
| C05 | Square | Because it has all sides of the same length, 90 degrees, and looks interesting. [+ <b>Visual impression</b> ] |
|-----|--------|---|

## Multiple shapes perspective

The last code category belongs to excerpts that provide observations based on comparison of shapes. There were statements that can be considered preceding hierarchical understanding of quadrilaterals:

- |     |     |  |
|-----|-----|--|
| C08 | QLP | How much they have in common.  |
| C19 | QLP | That they are all very similar, but at the same time they are constructed differently. |
| C17 | QLP | That in each quadrilateral the diagonals have a slightly different property.           |
| C16 | QLP | How nicely they connect to each other...   |

And also, explicit references to the hierarchy of quadrilaterals:

- |     |        |   |
|-----|--------|---|
| C17 | Square | According to the hierarchy we did yesterday, I was interested in it because it exactly combines the properties of a rectangle and a rhombus. It's basically the very top. |
| C24 | Square | Position in the hierarchy + the reality that it is a combination of many quadrilaterals.  |

## Discussion and conclusion

In this contribution, we presented our series of geometry lessons on quadrilaterals, and investigated how lower-secondary school students reflected their experience with this series. The series included paper-and-pencil classical constructions as well as dynamic constructions in the GeoGebra Classroom environment. Our questions focused on quadrilaterals, their properties, and constructions.

In their responses to the questions, the students were able to reflect various aspect of the series. From the perspective of van Hiele' levels of geometric thinking (1986), we identified responses belonging to several levels. The code category *Novelty* belongs to the first level, since it is comprised of responses that just mention names of shapes. Also, the category *Visual impression* belongs to the first level of geometric thinking, here the responses reflect on purely visual attributes of the shapes and refer to visual prototypes (Battista, 2002). The code category *Shape properties* includes one response (C05) that might be considered belonging to the second van Hiele' level. All the other responses in this category refer about the properties more or less vaguely, rather in the form of awareness, so they can be considered a good basis for the move to the third van Hiele' level, they are supported by the ability to identify the properties of a geometric object based on the observation of its picture (Duval, 2006). The second and third levels are fully present in the *Multiple shapes perspective* category. From the perspective of classifications of objects (de Villiers, 1994), just the *Multiple shapes perspective* category aims outside the partition comprehension of shapes (Fujita & Jones, 2007).

The role of DGEs might be perceptible behind many responses: in *Visual impression* category, when students were able to visualize one quadrilateral in different positions (C19) or its different shapes (C15), in *Construction impression*, when students notified multiple possible ways of constructing one quadrilateral (C22, C13), and probably in *Multiple shapes perspective*. Nevertheless, none of the participants mentioned GeoGebra explicitly in their reflection. This fact corresponds with the findings by Bebernik et al. (2022). She concluded that DGEs plays important roles in studying quadrilaterals, but did not mention any reactions of the participants of her study that would directly appreciate DGEs.

Regarding the challenges for research in geometry education mentioned by Sinclair et al. (2016), we can conclude as follows. We integrated the principles of DGEs into the series of lessons on

quadrilaterals. Geometry topics were highlighted in students' feedback while explicit references to work in DGEs did not occur. Referring to (Mariotti, 2012), dynamic geometry can be situated in the background of the role of the teacher and the activities of the classroom collective during the mathematics lesson. Future research can focus directly on students' reflecting on work with dynamic geometry, i.e. not on reflecting on mathematics lessons as a whole.

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