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# Visualisation and spatial manipulation in argumentation: Creating a hypothesis

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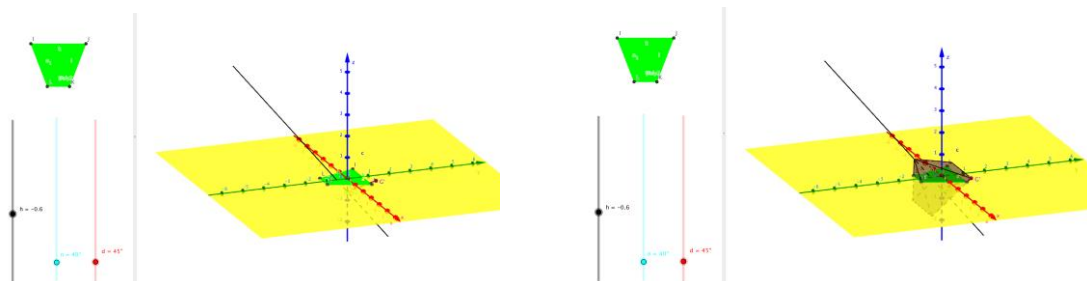
*In this paper, I discuss the significance of non-iconic visualization and spatial manipulation in students' argumentation. The findings presented here are part of the results of my doctoral research (completed in 2021). I introduce the five roles of non-iconic visualization and spatial manipulation in argumentation that I have identified through the analysis of the gathered data. I then focus on one of those roles, namely that of supporting the creation of a hypothesis or a claim. I present this role using a specific example from students' work.*

*Keywords: Visualization, spatial manipulation, argumentation, geometry, DGE.*

## Introduction

As a mathematician and researcher, I find that geometry's appeal lies (amongst other factors) in the plurality of the processes that take place while solving a geometric problem, amongst them: reasoning and argumentation (Douek, 2002), the association of properties, the engagement of visualization and spatial manipulation of the geometric object (Duval, 2002), the construction (or deconstruction) of the geometric object (Mithalal & Balacheff, 2019). These processes have been researched in the field of mathematics education and also addressed in this TWG, especially since 2015, in various teaching contexts. It has been shown that with properly designed tasks, the students have the opportunity to employ their visualization and spatial skills in collaboration with geometric argumentation, thus contributing to their learning of geometry (Palatnik & Abrahamson, 2022; Fujita et al., 2020; Papadaki, 2017). As a result, the role of visualization in students' argumentation in geometry has gained increasing attention.

In this paper, my aim is to move one step further, focusing on the ways in which visualization and spatial manipulation take place in students' argumentations. By analyzing the continuous interplay between students' visualization and argumentation, I try to unravel how exactly visualization contributes to students' argumentation and how its contributions can be depicted in argumentation structures. To do this, I created some tasks in which students can manipulate an invisible solid in a 3D coordinate system of a Dynamic Geometry Environment (DGE), using three sliders (h, n and d).



**Figure 1: a (left) and b (right). Task 3B – Pyramid at lowered and tilted position ( $h=-0,6$ ,  $n=40^\circ$ ,  $d=45^\circ$ ) as seen by the students (left) and as it appears when the solid is visible (right)**

## Visualization and argumentation in geometry

Visualization in geometry may be seen as a cognitive process (Duval, 2002; Mithalal & Balacheff, 2019) or as a type of spatial ability (Battista, 2007). In the first approach, (non-iconic) visualization (Duval, 2002) is about the comprehension of drawings, and dimensional deconstruction<sup>1</sup> is linked to it as an analysis-process of the drawings. In argumentation, dimensional deconstruction can be an observable phenomenon during students' discussions. In the second approach, visualization is described mainly as the ability to create and manipulate mental images of geometric objects in space. In my work, I adopt Duval's (2002) approach and its link to dimensional deconstruction, because it provides me with the tools that reveal it in argumentations.

Due to the design of the tasks the students worked with in my research, I hypothesized that during their argumentation they may describe the manipulations they imagine being performed on their mental image (Presmeg, 2006) of the invisible solid. I use the term *spatial manipulation* in order to refer to students' processes of mentally manipulating the invisible solid in space (or rather, the mental manipulation of the mental image of the invisible solid). *Spatial manipulation* comprises processes distinguished in the literature by various spatial abilities, such as "spatial visualization", which is the ability to mentally create and manipulate a geometric object in space (Battista et al., 2018). Therefore, I prefer to use the term *spatial manipulation* in order to avoid any notions (such as spatial visualization) that might cause confusion with that of visualization (Duval, 2002). Since I do not wish to speculate on what is happening in students' minds, I use *spatial manipulation* to include, under a single term, any kind of description that students give regarding their mental manipulation of a mental image.

In this paper, argumentation is related to the process of expressing one's reasons to support or reject a statement or an opinion, through verbal articulation, gestures or actions. Argumentation may take place between two or more participants, or a single student may perform it when he/she expresses his/her reasons out loud (based on Douek's, 2002 definition). In order to explore the role of students' visualization in their argumentation, an insight into the finer structure of students' argumentation is necessary. To do this I use Knipping's (2008) method for the reconstruction of students' argumentations, which is developed based on Toulmin's (1958) idea of the structure of arguments and how this structure could be symbolized and depicted graphically. Knipping (2008) extended Toulmin's model, creating connections between one-step individual arguments. The result of this reconstruction is what she calls *the argumentation structure*.

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<sup>1</sup> Dimensional deconstruction refers to the breaking down of a figure in its figural units: lower-dimension parts which compose the figure, i.e. faces or cross-sections of solids (2D figural units), sides, diagonals, circles, ellipsis (1D), or points (0D). In this operation, the figural units of the figure are linked with each other through geometrical properties.

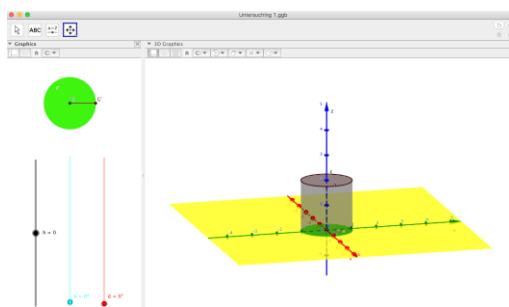
## Methodology behind the study and the data analysis

### Tasks

The findings presented here are parts of the results of my doctoral research. I conducted a qualitative research in the frame of two *experimental lessons* in geometry classrooms in Germany. The term *experimental lessons* is used to describe lessons during which the students work collaboratively on tasks specifically designed for the purposes of the learning goals set by their teacher and the researcher. In this study the students engaged in explorative geometric tasks, the aim of which was to encourage students to produce conjectures and examine their validity, going beyond iconic visualization (Duval, 2002) and towards non-iconic visualization, the use of geometric properties and argumentation.

The tasks have been constructed with the aim to promote students' visualization and dimensional deconstruction of visible 2D cross-sections on invisible 3D geometric objects. I call these tasks *D-transitional tasks*, because they involve transitions from two-dimensional to three-dimensional geometric objects (and vice versa). In such tasks the correlation of properties between geometric objects of different dimensions is vividly present. The specific tasks used here are designed in the Dynamic Geometry Environment (DGE) GeoGebra 5 (in 3D Graphics).

In the first task (Task 1), a visible cylinder is designed in a three-dimensional Cartesian coordinate system in the DGE (Figure 2). In each of the other four tasks (Tasks 2, 3A, 3B and 3C), an invisible solid is designed in the three-dimensional Cartesian coordinate system (see example in Figures 1a and b<sup>2</sup>). The purpose of Task 1 was to introduce the students to the DGE and to the set-up of the main tasks. The tasks I focus on for the purposes of my research are the ones with the invisible solids (Tasks 2, 3A, 3B and 3C). In these tasks, the main question set to the students was: "Which solid do you think this could be, judging by its cross-sections?". The identification of the solid's form is based on the cross-sections emerging by the intersection of the (invisible) solid with the (visible) plane  $xOy$ . The students can move the solid in space, using three sliders:  $h$  (stands for Höhe, meaning height),  $n$  (stands for Neigung, meaning tilt) and  $d$  (stands for Drehung, meaning rotation) (details on tasks in Papadaki, 2017).



**Figure 2: Task 1 – Visible Cylinder: Position of the solid at initial position ( $h=0$ ,  $n=0^\circ$ ,  $d=0^\circ$ )**

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<sup>2</sup> These figures demonstrate examples of  $(h, n, d)$ -positions, for the task of the invisible pyramid. The students only worked with the solids being invisible during their explorations. Here, I present the position also with the solid being visible, for the purposes of task presentation.

## Implementation, participants and data analysis

The study was conducted in three schools in Germany, in which participated students and their teachers from one class in each of the schools. The total number of participants is seventy-two students and three teachers. The results and the episode presented here are from one class of 9<sup>th</sup> grade students. First the students worked on the tasks in pairs and then they discussed their findings in whole class discussions.

Students' work was recorded with cameras. Their work on the computer was also recorded with a screen-recording program. The students were given worksheets, where they could keep notes about their explorations and observations. In this paper, I present findings from the work of one of those student pairs. I analyze the collected data by reconstructing the logical structure of students' argumentations (Knipping, 2008). I examine the structure of the overall argumentation in detail and determine the functions and roles of students' visualization in their argumentations. To illustrate the specific points, in students' arguments, in which visualization and spatial manipulation take place, I introduced two additional elements in the argumentation reconstructions: *non-iconic visualization* (NIV) and *spatial manipulation* (SpM). For the identification of non-iconic visualization, I examine how the students use the properties of the cross-sections, whether they identify figural units of the solid and how they relate these figural units with the solid itself. As Mithalal and Balacheff (2019) comment "Non-iconic visualization is a necessary condition for dimensional deconstruction to be operational" (p. 165). Hence, the use of dimensional deconstruction is a clear *indicator* of non-iconic visualization. I identify students' use of spatial manipulation by examining their verbal description about the movement and orientation of the invisible solid in space, as well as the accompanying gestures or metaphors. Those are the *indicators* of spatial manipulation.

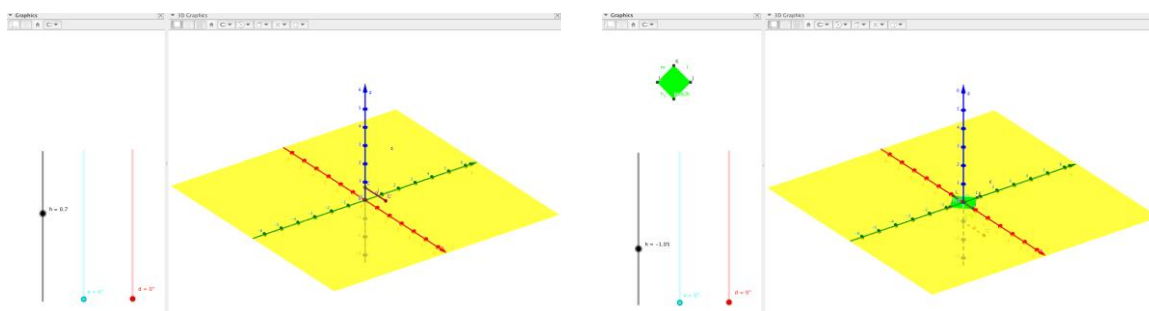
## Findings

Here, I focus on one of the roles of *non-iconic visualization* and *spatial manipulation* in argumentation: Supporting the creation of a hypothesis or claim. I present and explain it in detail through an example from the pair-work of students Axel and Dave. After that I mention all the roles I identified through the analysis of all the data.

### Axel and Dave: Creating claims

In this example, Dave and Axel work on Task 3B (invisible pyramid). At this point it may be helpful, while reading this text, to also look at the coded transcript in Table 1 and at the argumentation structure of the episode in Figure 4. The students manipulate the sliders  $h$ ,  $n$  and  $d$  trying to figure out the form of the solid. Dave starts by performing a height-exploration ( $h$ ,  $n=0^\circ$ ,  $d=0^\circ$ ), dragging the height-slider ( $h$ -slider) above and below zero. In the software, the cross-section disappears as soon as the  $h$ -slider is dragged over zero (Figure 3a), while when dragging the  $h$ -slider under zero, there appear square cross-sections that diminish in size (Figure 3b) until they converge to a single point. Dave and Axel's discussion during these events can be seen in the transcript in Table 1.

Dave and Axel’s argumentation structure (Figure 4) consists of two different paths. One path starts with the visual data  $VD_{2-3.1}$  where the cross-section disappears as soon as the h-slider is dragged over zero, while the square cross-sections diminish when dragging the h-slider under zero. From that, follows Axel’s claim  $Cl_{3.2}$  that the solid is a pyramid, to which Dave agrees (utterance 4). The other path starts with three conclusions, which are then used as data. These describe what happens with the cross-sections when dragging the h-slider down: the cross-section “is definitely a quadrilateral” ( $C_{6.1/D}$ ), the pyramid “runs up to a point when one goes to the negative area” ( $C_{7-8/D}$ ), and when the value of height is equal to minus two ( $h=-2$ ), the cross-section “is only a point” ( $C_{8.1/D}$ ). From these three data, Dave draws his claim  $Cl_{10}$  that the solid “is a reverse pyramid”.



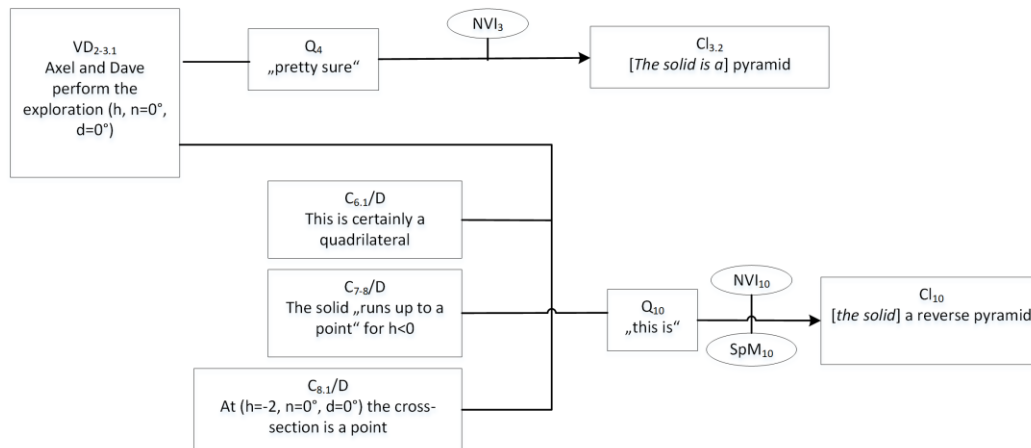
**Figure 3a (left) and b (right): No cross-section at position ( $h=0,7, n=0^\circ, d=0^\circ$ ), smaller square cross-section at position ( $h= -1,05, n=0^\circ, d=0^\circ$ )**

Utterance	Codes	English translation
3	$VD_3, Cl_{3.2}$	Axel: Yes. Yes. <b>Pyramid</b> . [Axel says that as he watches Dave dragging the h-slider over and under zero, for the case ( $h, n=0^\circ, d=0^\circ$ )].
4	<b>Q4</b>	Dave: <b>Pretty sure</b> , right?
5		Axel: Yes
6	<b>C<sub>6.1/D</sub></b>	Dave: <b>This is definitely a quadrilateral</b> [Dave refers to the shapes of the cross-sections emerging on the screen when the h-slider is dragged under zero]. Draw a quadrilateral. Do you have B? B? [Dave talks to a different student]
7	<b>C<sub>7-8/D</sub></b>	Axel: Does <u>it run up to a point when one goes</u> to the negative area [for h under zero] or to the positive [for h over zero]?
8	<b>C<sub>8.1/D</sub></b>	Dave: <u>To the negative</u> . (...) And <b>h at minus two there is only a point there</b> .
9		Axel: The [Axel mumbles while he is keeping notes on the worksheet]..
10	<b>Q<sub>10</sub>, Cl<sub>10</sub></b>	Dave: <b>This is a reverse pyramid</b> .

**Table 1: Coding for argumentation reconstruction**

The synergy of *spatial manipulation* ( $SpM_{10}$ ) and *non-iconic visualization* ( $NIV_{10}$ ) occurs in the argument joining the conclusions  $C_{6.1/D}$ ,  $C_{7-8/D}$  and  $C_{8.1/D}$ , with the claim  $Cl_{10}$  (Figure 4). In more detail: all three data consider the cross-sections of the solid, which is claimed to be a pyramid. Dave and Axel first identify the shapes and properties of the cross-sections when Dave says that they are quadrilaterals (utterance 6). Dave uses the word “quadrilateral”, but Axel draws a square in the worksheet. I infer, that the fact that all the cross-sections are squares (they say “quadrilateral”,  $C_{6.1}$ ) that get smaller when dragging the h-slider under zero ( $C_{7-8}$ ), while they disappear completely as

soon as  $h$  is dragged over zero (VD<sub>2-3.1</sub>), leads the two students to the claim that the solid is a pyramid (Cl<sub>3.2</sub>), which is also part of the claim that the solid is a reverse pyramid (Cl<sub>10</sub>). This means that Axel and Dave identify the cross-sections as 2D figural units of the pyramid. This is a process of *dimensional deconstruction*, which is the *indicator* of the use of *non-iconic visualization* both in NIV<sub>10</sub> as well as in NIV<sub>3</sub> (Figure 4).



**Figure 4: Synergy of NIV<sub>10</sub> and SpM<sub>10</sub> in Dave and Axel's argumentation structure**

When Axel asks: “Does it run up to a point when one goes to the negative area or to the positive?” (utterance 7), Dave responds that the cross-sections get smaller when he decreases  $h$  under zero (C<sub>7.8</sub>), making the cross-section converge to a point (C<sub>8.1</sub> in utterance 8). After this question, Dave claims that the pyramid is reverse at its initial position ( $h=0$ ,  $n=0^\circ$ ,  $d=0^\circ$ ) (Cl<sub>10</sub>). Dave arrives at the claim Cl<sub>10</sub>, following a process, during which he combines the movement of the slider (dragging  $h$ -slider downwards under zero) with the diminishing of the cross-sections. Seeing that the square cross-section “shrinks” when the height-slider is moved downwards under zero, he claims that the pyramid is reverse. Axel's question, is relative to the position of the pyramid in space, since one can determine whether the pyramid is upward or reverse by finding out for which values of  $h$  (height) there are cross-sections with the plane  $xOy$ , and for which there are not. The mental process that takes place then is *spatial manipulation*. Dave's ability to imagine the solid and its orientation in space (regardless of its correctness) is the result of using *spatial manipulation*.

To conclude, Dave's claim Cl<sub>10</sub>, is the result of the *synergy* of both *non-iconic visualization* (NIV<sub>10</sub>) and *spatial manipulation* (SpM<sub>10</sub>). *Non-iconic visualization* is behind the parts of Dave's and Axel's argumentation in which they use geometric properties (in the process of *dimensional deconstruction*) in order to connect the form of the cross-sections with the form of the solid. On the other hand, *spatial manipulation* is the process that helps Dave to connect his action on the  $h$ -slider with the orientation of the solid in space at its initial position ( $h=0$ ,  $n=0^\circ$ ,  $d=0^\circ$ ). It is this connection between the external factor influencing the movement of the solid (the movement of the slider) and the solid's movement that indicates the use of *spatial manipulation*. Dave's claim Cl<sub>10</sub> (that the solid is a reverse pyramid) may not be entirely correct, since the invisible solid is indeed a pyramid, but it is not reversed in its initial position. Based on the above description of how *non-iconic visualization* and *spatial manipulation* operate in Dave's argument, it seems that what constitutes a challenge for Dave is imagining the movement of the invisible pyramid in space. NIV<sub>10</sub> helps Dave

to shape the part of his claim  $Cl_{10}$  that is relevant to the form of the invisible solid, namely that the solid is a pyramid. Therefore, it is not an ignorance of the properties (which would be related to his *non-iconic visualization*  $NIV_{10}$ ) that challenges him. It seems that  $SpM_{10}$  leads Dave to the (incorrect) second part of his claim, namely that this pyramid is reversed. Knowing that the misconception lies in Dave's *spatial manipulation* can help the teacher to choose the appropriate arguments with which to negotiate Dave's claim.

### The roles of *non-iconic visualization* and *spatial manipulation* in students' argumentations

Overall, the data analysis revealed five roles that *non-iconic visualization* (NIV) and *spatial manipulation* (SpM) have played in students' argumentations (Table 2). In those roles, NIV and SpM function in argumentation in the same way that warrant-statements, backing-statements and refutations do (Toulmin, 1958). The *roles* of NIV and SpM describe the ways in which they enhance students' argumentations. Their *functions* describe their positions in the argumentation structure, which emerge when one analyses students' argumentation.

Roles of <i>NIV and SpM</i> in argumentation		Functions of <i>NIV and SpM</i> in argumentation
1	Creating a hypothesis or a claim	Warrant
2	Drawing a conclusion	
3	Explaining visual data	
4	Creating a refutation	Warrant, Backing or Refutation
5	Backing a warrant	Backing

**Table 2: Roles and functions of *NI-visualization* and *spatial manipulation* in students' argumentations**

There are cases in students' argumentations, where NIV and SpM support the students to create a hypothesis or a claim, originating from the data they observe (Role 1), as in the example of Dave and Axel. In this case, they function in argumentation as warrants. That means, that they stand between the datum (or data) and the hypothesis or claim, leading the argument from the former to the latter (see Figure 4). In other cases, NIV and SpM support the students in drawing a conclusion (Role 2) or explaining visual data that they perceive on the screen during their explorations (Role 3). In both those cases, they function in the argumentation just as warrants would. NIV and SpM have also been observed to help students create a refutation (Role 4) in order to reject a hypothesis, in which case they may function in more than one ways in the argumentation: in the position of a warrant, or a backing supporting a warrant or even a stand-alone refutation. Finally, there have been cases in which NIV and SpM have acted as individual backings of a warrant (Role 5).

### Conclusion

In this paper my aim has been to illustrate how *non-iconic visualization* and *spatial manipulation* can be identified in students' argumentations and show the roles that they can play in it. I have only focused on a small part of it, presenting the role they play in the process of creating claims. The distinction between what constitutes *spatial manipulation* and what constitutes *non-iconic visualization* at each point in the argumentation, can help us (as researchers and as teachers) to recognize whether it is *non-iconic visualization*, *spatial manipulation* or their synergy that operate, as well as the instances in students' argumentations in which they take place. Much like in the case



of Dave's misconception of the non-reversed pyramid, this knowledge can assist us to unravel the cause of students' misconceptions and help them to overcome them, thus enhancing their learning.

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