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# Preservice teachers' perceptions on the role of the geometric working spaces model in learning and teaching geometry

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*This study was conducted as a part of a project, where a teaching geometry course offered at a public university was designed based on the Geometric Working Spaces model. This course aimed to improve preservice teachers' content and pedagogical content knowledge related to geometry. 18 preservice mathematics teachers attended the course. The model was introduced to preservice teachers. At the end of the course, preservice teachers were asked to report if the model had any impact on them as a learner and as a teacher candidate, and whether they would use it in their own teaching practice. Participants' written responses to these questions were analysed. Findings revealed that preservice teachers found the model useful both for learning and teaching geometry. The model was claimed to be instrumental for conceptual understanding of geometry and designing lesson plans and course materials.*

*Keywords: Geometric working spaces model, teacher education, geometry education.*

## Theoretical background

Geometry, an essential topic of school mathematics, is a challenging learning area for students in many countries. Preservice mathematics teachers are also often observed to struggle in geometry content knowledge and preparing effective lessons. As an attempt to overcome these difficulties, a framework called Geometric Working Spaces (GWS) was used to design a teaching geometry course in the context of this study. It was thought that the GWS model would provide a useful tool for learning and teaching geometry.

In the GWS model, Houdement and Kuzniak (1999, as cited in Kuzniak, 2018) defined three different geometric paradigms that involve practices, beliefs, values and techniques related to geometry. The geometric paradigms are named Geometry I (Natural Geometry), Geometry II (Natural Axiomatic Geometry) and Geometry III (Formal Axiomatic Geometry). These paradigms are not organised hierarchically, making one more advanced than the other. Rather, their scopes of work are different and the choice of a path for solving a problem depends on the purpose of the problem and the solver's paradigm (Kuzniak, 2018).

In Geometry I, valid claims are generated by using arguments based on perception, experimentation and deduction and they generally involve instrumentation. There is a high similarity between a model and reality, and any argument is allowed to justify a claim and convince the audience. Dynamic and experimental proofs are acceptable in Geometry I (Kuzniak, 2014; Kuzniak & Rauscher, 2011). Geometry II is Euclidean geometry, it is based on axioms, theorems, and proofs. Once the axioms are set up, proofs have to be developed within the system of axioms to be valid. Even though it is based on an abstract system, the connection with real world applications is important.

The axiomatic system and deductive reasoning is also at the centre of the Geometry III paradigm. In this case real-world applications are not important, the main concern is to have a consistent axiomatic system (Kuzniak & Rauscher, 2011).

The three geometric paradigms are supported by the model of Geometric Working Spaces (GWS) that was proposed by Kuzniak (2018) based on Mathematical Working Spaces (MWS). Researchers from various countries worked together on MWS for over 15 years (Kuzniak et al., 2016). One of the domains where the MWS approach was used is geometry and the corresponding model is called Geometric Working Spaces (GWS). GWS is also referred to as Mathematical Working Spaces in Geometry (MWSG) in some studies (e.g., Jiménez & Ärlebäck, 2018).

The model explains the epistemological and cognitive aspects of geometric work on two metaphorical planes. The epistemological plane is related to the mathematical content of the subject studied. The cognitive plane is related to the individual's thinking system during a mathematical activity.

In order to explain the geometric work carried out in the epistemological dimension three interacting components are defined: (i) Representamen; tangible objects, (ii) Artefacts; drawing tools or software, and (iii) Referential; a theoretical reference system based on definitions, properties, and theorems (Kuzniak & Nechache, 2021). Three cognitive components that are closely related to the components in the epistemological dimension were introduced in order to clarify geometric work in the cognitive dimension: (i) Visualisation; related to deciphering and interpretation of indicators; (ii) Construction; depending on the works used artefacts and related techniques; and (iii) Proving, based on a theoretical frame of reference communicated through validation processes (Kuzniak & Nechache, 2021).

Three particular geneses - semiotic, instrumental, and discursive - were identified as connections between the epistemological and cognitive planes in order to comprehend the interactions in the GWS. Semiotic geneses are non-material tools that are for semiotic representations of geometrical objects. Instrumental geneses are material-like drawing instruments, algorithmic calculators, or dynamic software. Discursive geneses are also non-material and they are based on the mathematical/logical properties of geometric objects (Kuzniak et al., 2016). The circulation of the mathematical knowledge and usage of these tools while solving problems are specified with the dual connection of these geneses. These connections were identified as three vertical planes: semiotic-instrumental [Sem-Ins], discursive-instrumental [Dis-Ins], and semiotic-discursive [Sem-Dis] (Gómez-Chacón & Kuzniak, 2015; Kuzniak et al., 2016).

In literature, the model has been used for examining geometric work of students (e.g., Gómez-Chacón et al., 2016) and teacher candidates (e.g., Kuzniak & Nechache, 2021). In addition, it has been used to evaluate course materials and curricula (Jiménez & Ärlebäck, 2018). This study was conducted as a part of a project, where a teaching geometry course offered at a public university was designed based on the Geometric Working Spaces model. In a previous study, content of the course was analysed using the model (İmamoğlu et al., in press). Geometry tasks used in the course were evaluated and according to the findings, most of the existing tasks were in discursive genesis and geometry 2 was predominantly used. To help preservice teachers to see how transition from one paradigm to the other can be possible, tasks that are based on geometry I paradigm were added. In

addition, tasks targeting semiotic and instrumental geneses were also included in the course design. During the course, the model was also introduced to pre-service teachers. The aim of this study is to examine preservice teachers' perceptions on the role of the GWS model in teaching and learning.

## Method

Teaching geometry course was designed with the basic aim of improving preservice mathematics teachers' content and pedagogical content knowledge in geometry. Preservice teachers have basic factual knowledge of Euclidean Geometry from high school. Since there is no other course in their program related to Euclidean geometry, preservice teachers' content knowledge was limited to their high school knowledge at the beginning of the study. In this course, axiomatic structure of geometry, justification and proof were emphasised in addition to discussing pedagogical issues on teaching geometry. The course is designed in a manner to provide both content knowledge (Euclidean 2D) of geometry and the methodology for teaching middle and high school geometry. Hence the flow of the lesson was based on a series of geometry tasks to provide the opportunity for the preservice teachers to develop their geometric understanding and to familiarise them with the different types of activities and tools that can be used in school settings. The tasks used in this course were targeting all geneses of the model, but the emphasis was mostly on discursive genesis. The GWS model was also introduced to the preservice teachers, to be used as a framework for evaluating geometry activities and their own geometric work. As a part of the course, students were asked to evaluate tasks by specifying the considered geometric paradigm and geneses activated by the task.

The participants of the study were 18 third year preservice mathematics teachers in a state university, enrolled in the teaching geometry course offered in 2022-2023 fall semester. To understand participants' characteristics and to ensure that the course tasks appropriate for the participants, their geometric reasoning levels before taking this course were analysed through van Hiele test (Usiskin, 1982). The results showed that majority of the participants (66%,  $n=12$ ) were at the *informal deduction level* (see Table 1).

**Table 1: Frequencies of participants' geometric reasoning levels**

Levels	Frequency (Percentage)
Level 1-Analysis	1 (6%)
Level 2- Informal Deduction	12 (66%)
Level 3-Deduction	1 (6%)
Level 4- Rigor	4 (22%)

Data of this study was collected at the end of the semester. The participants gave written responses to the following open-ended questions:

- How does the model help you as a learner and as a teacher candidate?
- Would you use this model in the future? How?

The research team coded the student responses. The codes were categorised under emerging themes.

## Findings

### Impact of the model

Preservice teachers' responses to the question “How does the model help you as a learner?” were categorised into the following three themes: cognitive aspect, metacognitive aspect, and attitude. Cognitive aspect refers to the process involved in knowing, learning, and understanding of the GWS model, metacognitive aspect indicates understanding of one’s own thinking about the GWS model and reflecting on the process of thinking, and attitude is perception and emotion about learning the GWS model. The frequencies of preservice teachers’ responses are shown in Table 2.

**Table 2: Utility of GWS model for learning**

Theme	Frequency
Cognitive Aspect	12
Metacognitive Aspect	4
Attitude	4
Total	20

Under the theme *cognitive aspect*, some responses were formed around the structure of the GWS model. Preservice teachers reported that understanding the relationship between the components of the model enabled them to see geometry as a whole. For example,

- PT11: This model helps us as a learner to understand and identify the relationship of semiotic genesis, instrumental genesis and discursive genesis.  
 PT3: This model shows me that geometrical understanding is a whole concept.  
 PT9: With GWS model, I don't look at geometry with a narrow mind. It helped me a lot.

Preservice teachers also mentioned that the model was helpful for understanding geometry conceptually:

- PT3: ...visualization, construction, representation, and proof are needed to increase conceptual understanding in geometry.  
 PT17: As a learner, I understand the idea behind the tasks.  
 PT18: This model helped us to develop our geometric thinking process. Visualisation, construction, proof are connected elements and strengthen our skills so as a learner it helps us to build our knowledge.

Responses categorized under the theme *metacognitive aspect* refer to the organisation of thoughts and learning, selection of tools, and evaluation of the learning process:

- PT1: It helps me to organise my thoughts.  
 PT12: As a learner, I can analyse how I am learning and also I find what I need.  
 PT13: I can compare myself how much I learn and grow academically.

In terms of attitude, novelty and usefulness of the model was emphasized.

- PT9: I did not have an idea about theorems, axioms etc., geneses, or paradigms. But now, I can see that way thanks to this...  
 PT12: ... I can compare my performance on how much I learn and grow academically.

Responses of preservice teachers to the question “How does the model help you as a teacher candidate?” were also categorised into three themes such as *cognitive aspect*, *metacognitive aspect* and *attitude leading to teaching practices*. The frequencies for the question are shown in Table 3.

**Table 3: Utility of the GWS model for teaching**

Theme	Frequency
Cognitive Aspect	2
Metacognitive Aspect	4
Translating into Practice	11
Total	17

For cognitive and metacognitive aspects, the focus was on the teaching rather than learning geometry. For the cognitive aspect, the reactions of preservice teachers shaped around knowledge of the geneses for the tasks and the overall GWS model:

- PT17: As a teacher candidate, I notice that there are cognitive and epistemological aspects for teaching geometry and they make teaching easier.  
 PT5: ...it is important for us to know the tasks require [which] genesis, so we can prepare the activities accordingly.

Some preservice teachers stated that the model might help them metacognitively as in categorising activities, analysing tasks for students, and selecting proper tools or theorems for students:

- PT8: As a teacher candidate, I can see how questions should be analysed and provided to students.  
 PT15: It makes easier to categorise geometrical subject.  
 PT1: It helps to categorise activities I have seen on external sources.

Majority of the responses were constructed around “attitude leading to teaching practices” theme. Here, they reported that the model would help them in preparing effective lesson plans, activities and tasks for students and also reflecting expectations from students:

- PT2: As a teacher candidate, it helps me to prepare more effective lesson plans.  
 PT12: I can provide more structured and organized plan.

Pt18: ...as a teacher it clearly reflects our expectations from our students when we are coming to the class, the model helps us to develop our idea of how to construct our lesson more effectively.

### Thoughts on the use of model in future practices

Preservice teachers were asked “Would you use this model in the future? How?”. They affirmed to use this model in their future practices in different aspects as the frequencies are given in Table 4.

**Table 4: Potential future use of the model**

Theme	Frequency
Cognitive Aspect	2
Metacognitive Aspect	1
Translating into Practice	13
Geometric Paradigm	2
Total	18

The responses were formed around instructional practices in general: designing activities, lesson plans, choosing tasks, etc.:

- PT8: I would try to use it in my work. I can choose tasks by looking this model. I can decide what I should do to be clearer in my objectives.
- PT10: ... I can use different methods for different topics. For example, I can use some visualisations like drawings or visual models for some geometric models and also some proof methods for explaining axioms or theorems. This model acts as a glue for cognitive and epistemological plane.
- PT15: I can teach any geometry subject according to this model. Thanks to this, I can select the appropriate method and tools again...Thus, I can improve [students'] skills in a better way.

In some responses, there were direct references to the model and evaluations regarding the use of the model. Some hesitations on the use of Geo II paradigm and discursive genesis in the middle school were observed.

- PT15: Yes, I will try to use it. I think it may be hard for middle school students to make proofs since they do not know enough theorems. So discursive genesis activities may not be useful. But other two geneses are necessary for preparing the tasks. Just giving the questions is not effective, we should try to use different resources and instruments.
- PT9: I want to use it because in this lesson our teacher uses it, and we understand it at the end. This model looks beneficial because it helped me a lot and I would like to help my students too. I [would] use this model in Geo 1 paradigm because my

students will be in middle school and by giving all axioms or theorems, I don't want them to hate this lesson, see this lesson as a burden. I might add Geo 2 paradigm, but Geo1 will be more.

PT12: I will use this model. While teaching geometry, students need to clear visualisation and explanation. I believe that they need mostly how to think and how to draw, in this step I will use semiotic genesis. If this step is well understood then I will move forward to instrumental genesis and the questions would be more complex, they need to connect their prior knowledge and what they will learn. After that I may use discursive genesis for the clever [students] to use proof. Because sometimes using proof could be confusing.

## **Discussion**

In this study, which is a part of a project on teacher education in geometry, we could specify preservice teachers' perceptions on the role of the GWS model in learning and teaching geometry. Kuzniak and Nechache (2015) claimed that introducing the model is more beneficial to teacher trainers rather than teachers. However, the results of this study showed that preservice teachers found the model useful both as a learner and as a teacher in different aspects. As a learner, preservice teachers seemed to make sense of the comprehensiveness of the model and value what is called "completeness" by considering all geneses (Kuzniak & Nechache, 2021). A similar result was observed in a study (Kuzniak & Nechache, 2015) where teacher trainers indicated "...it is possible to have a global vision of geometry thanks to the GWS model". In addition, preservice teachers repeatedly specified the impact of the model on their conceptual understanding of geometry. The model was also found to be a good monitoring tool for their learning process as a metacognitive prompting.

As teacher candidates, they expressed that the model contributed their knowledge on teaching geometry. They also pointed out the contribution of the model in organising and planning their instruction. The model guided them in evaluating their teaching experiences, providing a framework for a reflective practice. According to the preservice teachers' responses, the compact and structured nature of the model would lead to designing effective lessons and bring clarity to what is expected from students.

All teacher candidates displayed positive attitudes towards using the ideas of the model in their future teaching practices. Some preservice teachers reported that they were reluctant to use the Geo II paradigm and discursive genesis in middle school level. They found teaching proofs, axioms and theorems to students in that age group challenging and appropriate only for high achieving students in mathematics and geometry. Their reservations regarding the implementation of the Geo II paradigm and discursive genesis in middle school was noteworthy and needs to be studied further. In order to understand whether this model had a permanent impact on students' perceptions, evaluating whether teacher candidates could apply the ideas in the model in their own lesson plans and instructional practices would be beneficial in future studies.

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