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► **To cite this version:**

Alessandro Gambini, Giada Viola. Doubling the side, doubling the area: Managing representation with a Geoboard tool. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04419259

**HAL Id: hal-04419259**

**<https://hal.science/hal-04419259>**

Submitted on 26 Jan 2024

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# Doubling the side, doubling the area: Managing representation with a Geoboard tool

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*In this paper we analysed the ability of high school students to handle the representation of the squares and to construct new figures in Geoboard software. We proposed two questions to the students, in which they were asked to construct squares under certain conditions. We analysed the difficulties encountered and the strategies implemented framing them within Duval's theoretical framework. Moreover, we focused the attention on the presence of an intuitive rule that guided the resolution process compromising the solution of the tasks and which processes are involved in the handling of the representation of geometric figures.*

*Keywords: Geometry, representations, area, squares.*

## Introduction

Students frequently equate area to simply being the product of length and width. This can be a sign of a limited conceptual understanding of the concept of area, as well as a lack of multiple strategies for finding the area of geometric figures, even if the students have learned about counting units as a method for finding area (Freudenthal, 1983). It is important to promote teaching methods that encourage the development of flexible strategies for measuring area.

In an active learning environment, teachers should facilitate student understanding and application of concepts, rather than solely expecting students to listen and memorize information (Cattaneo, 2017). Bjørkås & Heuvel-Panhuizen (2019) conducted a small-scale classroom experiment on teaching area measurement using a digital environment geoboard. A geoboard is a tool consisting of nails driven halfway into a board with elastic bands used to create different shapes.

The teaching sequence that was developed and tested by Bjørkås & Heuvel-Panhuizen (2019) was conducted over two lessons and aimed to teach students the use of measurement strategies in a flexible manner. The results of the pre- and post-tests conducted on both the task and student level showed that most students developed more flexible strategies, but there were some who did not apply them accurately. The results indicate that education can support not only the ability to find correct answers, but also the flexible use of strategies.

Wheeler and Champion (2016) explored Probability Explorations with Geoboards in secondary school students aged 11-13, while Uhlřřova & Laitochova (2020) summarize the results of a different educational experiment, GPME2, that was conducted with 5th-grade primary school students and focused on developing basic geometric concepts and mathematical thinking through activities on a geoboard. The impact of using geoboards in teaching Euclidean geometry to 11th-grade mathematics students was studied by Sibiya (2020), using a qualitative research design. The study aimed to provide insight into the teaching methods used by teachers and to identify ways in which Euclidean geometry could be taught more effectively. Sibiya (2018) argues that many teachers simply follow the textbook

without adding any new perspectives or explanations, which can lead to difficulties for students when solving more advanced problems in exams.

The findings of the study revealed that using a geoboard allows students to learn at their own pace and in small groups, with the teacher providing guidance through geometric theorem worksheets. The geoboard also promotes collaborative learning and a learner-centered approach. Research has also been conducted in students' geometrical thinking, with Papadaki (2016) presenting theories of the evolution of geometrical reasoning. Our study aims to investigate the management of the representation of the geometric squares and the concept of area and the presence of an intuitive rule.

## **Theoretical framework**

Managing different representations of mathematical objects is the key in mathematics teaching and learning, and difficulties in doing so can impact student learning. To grasp students' comprehension difficulties in mathematical activities, a cognitive approach is necessary to uncover the underlying characteristics of various mathematical processes (Duval, 2006). Access and manipulation of mathematical objects are connected to the method of representing them, leading to Duval's (1993) famous paradox of lack of direct access. According to Duval, direct access to mathematical objects is not possible and they must be represented through signs and semiotic representations. He viewed the visualisation process as the creation of these representations, not just the visualisation of the objects. He categorised representations into those created intentionally through semiotic systems and those produced automatically through organic or physical means. The system in which a representation is created is crucial as it affects the processing of mathematical thinking, referred to as the "register of representation." A semiotic representation shows relationships between object elements, and often multiple registers of representation must be utilised. Duval identified four levels of apprehension for the learning of the geometry (Duval, 1995; 1999):

- *perceptual apprehension* occurs when a person identifies shapes and sub-shapes;
- *sequential apprehension*, involved in constructing or describing a geometrical figure based on mathematical rules;
- *discursive apprehension*, focused on recognizing and explaining mathematical properties through speech acts;
- *operative apprehension*, which involves how an individual interacts with the figure, such as through mereologic, optic, or place ways.

Nowadays it is widespread in literature that it is necessary to handle various semiotic representations to achieve the gradual and conscious cognitive construction of the object. Among the possible representations of a mathematical object, graphic representation is very relevant in the processes of learning mathematics and in 1994 Duval points out that graphical representations are semiotic representations in the same way as algebraic writing or language. Vinner (1987) highlighted how visualisation, as a means of representing mathematical objects, is central to learning processes concerning one of the main topics in advanced mathematical learning, functions and its graphs. The difficulty of students handling the different function representations has been extensively studied in the literature (Sfard, 2008; Nachlieli & Tabach, 2012).

Tirosh and Stavy (1999b) defined “Same amount of A-same amount of B” (more briefly called “Same A - Same B”) and “More A – More B” as intuitive rules. These rules are used by students when they have to make a comparison between two measures of two figures, such as area and volume or distance and length. In their study, it can be observed that younger students have more problems with length and distance, while older students have more problems with area and volume. Tirosh and Stavy concluded that the origin of the “Same A - Same B” rule or “More A – More B” rule may be an intuitive rule but also an overgeneralization from the previous successful experiences (Tirosh & Stavy, 1999a; 1999b). In their work they presented an example of this intuitive rule about the conservation of area and volume. Specifically, they presented to the students two figures with the same rectangle rotated by  $90^\circ$  and asked them if the two shapes have the same area. After this question, they asked the students if the cylinders built by the rectangles have the same volume. The results showed that there is a strict correlation between conservation of area and conservation of volume, this highlighted the use of the intuitive rule “Same A - Same B”.

Our work aimed to investigate the ability of high school students to handle the geometric representations and to construct new figures following the indications. Specifically, we think that the request to draw a square with twice the area of the starting square, might trigger an intuitive rule, such as Same A - Same B or More A – More B. Our research question is the following: *What processes do students enact when handling geoboard representations of geometric figures? Are there intuitive rules, such as Same A-Same B, that guide their resolution process?*

## **Methodology**

The experiment was conducted in high school of Rome and involved two classes consisting of 50 students. The students ranged in age from 15-17 years old. The reason for this choice was due to the students’ knowledge of geometry; in fact, in the middle school and in the early years of the high school, they dealt with various geometry topics, such as the Pythagoras’s theorem. Students worked individually or in pairs using personal tablet (we collected about 35 protocols). In this situation the students were unassessed: this choice is motivated by the desire to leave students free to solve the task as they see fit. Students were told that they would not be assessed and were encouraged to write down their thoughts in an unpinned way. The task consists of two following questions:

*Draw a square and try to construct another square that has an area twice the area of the starting square and then one that has an area four times as large. Have you succeeded? Explain how you did it.*

*Try to construct a square that has an area equal to 5. Have you succeeded? Explain how you did it.*

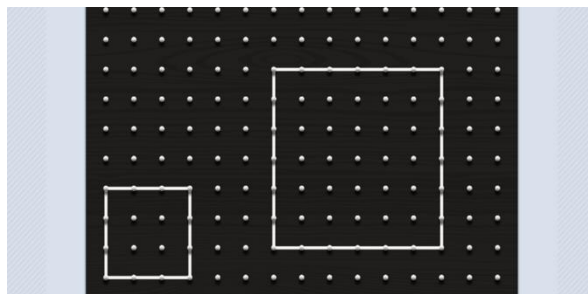
The choice of these questions is justified by the desire to investigate the ability of the students to represent in “non-standard” ways the figures. In fact, to solve the task the students must represent the square without sides parallel to the screen. The experiment required the students to solve the task using Geoboard software. This software is composed of a lattice where it is possible to construct figures using the rubber bands and nails in the software. This software offers the capability to create shapes with a fixed structure that adhere to a lattice grid. We opted to utilize this software to explore diverse methods for constructing squares with areas either double or five times that of the initial squares. Notably, the rigid construction requirement compels students to create squares in

unconventional positions to successfully complete the task. This approach enables us to assess students' proficiency in constructing squares in non-standard orientations, as well as investigate the presence of intuitive principles such as 'More A – More B' and 'Same A – Same B'.

Students are tasked with constructing the figure within the software and providing an explanation of the reasoning behind their solution. We gathered both screenshots of students' Geoboard creations and their written explanations, conducting an analysis that specifically emphasizes their ability to represent and substantiate their choices. Drawing from the insights of Duval's research, we applied the levels of apprehension as a framework to assess the development of geometric understanding.

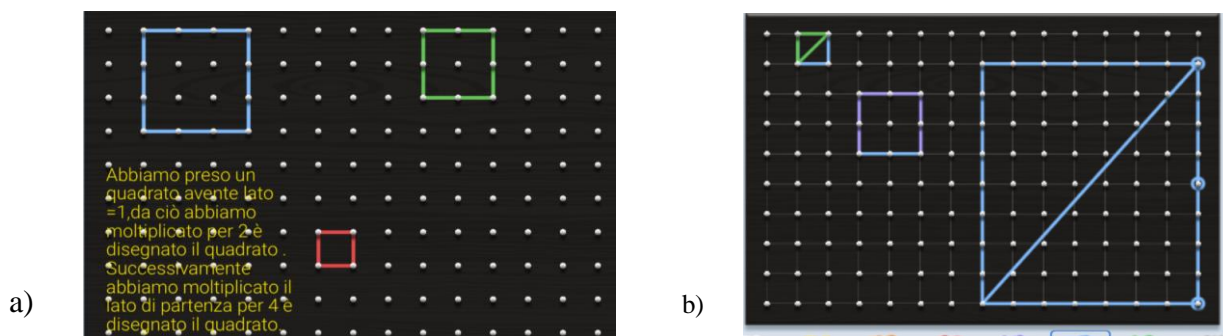
### Analysis and discussion

In the following figures, we showed some examples of the students that answered incorrectly to first question, highlighting the intuitive rules involved during the solving process.



**Figure 2: Implementation of “More A – More B rule**

Protocol in Figure 2 shows that students multiply the side of the starting square by two. We analysed this action related to “More A - More B”, because they answered the request “*to construct another square that has an area twice the area of the starting square*”. They were not able to handle the mathematical rules and applied an intuitive rule to solve the task, without checking the result obtained. These students did not provide explanations of their answers, but also a graphical representation.



**Figure 3: Examples of inconsistency between explanations and procedure implemented**

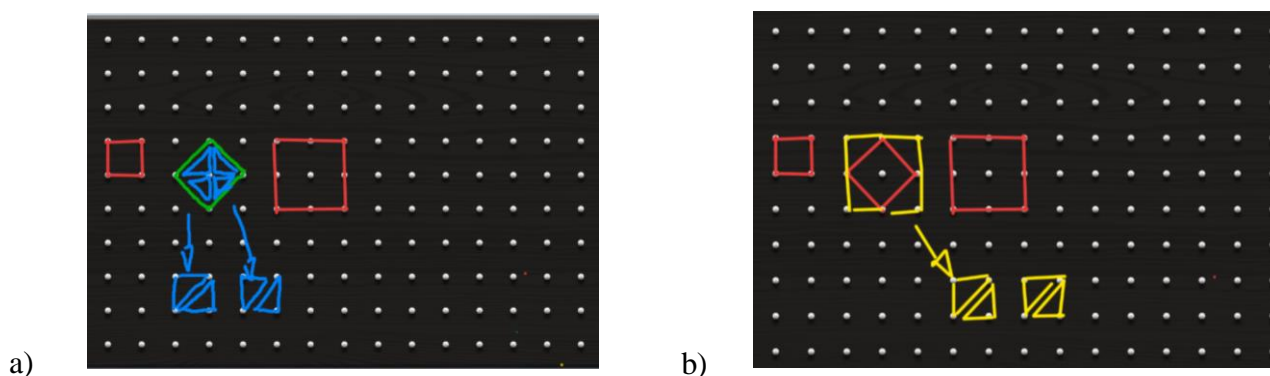
In Figure 3, we can see that the students applied intuitive rules, and, in this case, they explained their motivations. In Figure 3 a), the students described the procedure applied, but it was not consistent with the results shown. In fact, they said:

We took a square with a side equal to 1, afterwards we multiplied it by two and we drew the square. Next, we multiplied the starting side by four and we drew the square.

From the figure, we can observe that the largest square has a side three times bigger than the starting square and not four times. This highlights that there is an inconsistency with the motivation and the procedure applied and there was a lack of verification at the end of the solving process.

In figure 3 b), it is possible to see another rule applied, in this case the student justified the procedure writing “Starting to 1” and he wrote down the formula of area of square. Next, he wrote down some calculations, specifically he multiplied the side of the starting square by two and after the result by four. In this way he obtained three squares, one with side 1, one with side 2 and one with side 8. In the figure, he did not draw three squares, but he drew two squares and a rectangle, showing the lack of the checking process. These difficulties could be related to the difficulty to manage different representation, because they have difficulty to link what they represent in the figure and what they perceive from the figure (Duval, 1995). From these protocols we can point out the presence of the intuitive rule “More A – More B”, they constructed the squares with the side multiplied by two or by four, thinking that there is a conservation in area as well. It is evident that the students rely totally on this rule and not implement a verification process that would allow them to recognize errors of representation and that the area of the shapes was not that requested by the task.

In the following figures we showed two examples of students that gave the correct answer to first question, and it is possible to see two different strategies to solve it.

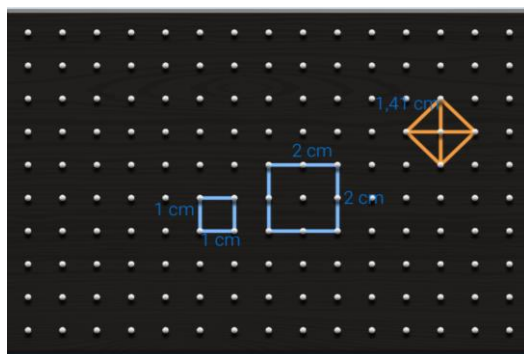


**Figure 4: Examples of students that gave the correct answer to task 1**

In Figure 4 a) the student divided the starting square in two triangles, and he composed a new square combining the parts to create a square with an area twice the area of the starting square. The biggest square was constructed in the same way, as shown in the figure 4 b). In this protocol we can observe the presence of a handling of visual representations; in fact, the student was able to decompose the starting figure and combine the parts to construct a new figure. This student provided a written justification, but he also used the graphical representation to better explain the procedure. His explanation is the following:

The square with twice the area was constructed by dividing the starting square into two triangles, then I took two squares equal to the starting one, divided each into two triangles, and formed the largest square with the four triangles (image).

The use of the visual mediator is highlighted in the figure, there were the indicators to make the process clear, and in the written justification, when he recalled the image.



**Figure 5: Use of the Pythagoras' theorem to solve the task**

In Figure 5, it is possible to see the implementation of another strategy to solve the question. The student chose as a starting square a square with side 1, he identified that the second square should have a side equal to square root of two, probably using the Pythagoras' theorem.

These solutions of the question are not linked to an intuitive rule, but to a visualisation process that allowed them to see shapes and sub shapes and to construct geometric figures, according to the first levels of Duval (1999). Moreover, in these protocols there was a use of the aimed not only to represent the problematic situation, but to show the strategy chosen and to allow others to understand the process applied.

No students answered the second question of the task correctly, some students stated:

Is not possible because there is no perfect square equal to 5.

Other students declared that they were not able to construct it.

These data pointed out that there is a difficulty stepping outside the ordinary patterns and to see figures in a different way. To solve this question, students should apply the Pythagoras' theorem or to use Pythagorean triplets to find the new side of the square. None of the students were able to use one of these tools to answer the question.

## **Conclusions**

The presence of a constrained environment in the task proposed allowed us to explore and to analyse the development of the visualisation process in the students and the management of the mathematical objects. This environment allowed the students to explore the construction of the geometrical shapes and the exploration of the concept of area not only because of a calculation but as a mathematical concept. Regarding our research question we can observe that it is possible to highlight two different approaches to this task: who used an intuitive rule and who decided to decompose the starting figure.



The first approach did not make it possible to achieve the goal, in fact we identified a presence of an intuitive rule, linked to the intuitive rule individuated by Tirosh and Stavy (1999). The existence of an intuitive rule is correlated with the lack of the validation process because the students had full confidence in their choice, and they did not apply a verification process, and this is highlighted by the inconsistency from the screenshot and the explanation given. The second approach showed a different management of the figures, they used the graphical representation to solve the task. This strategy made it possible to visualise the application of the first levels of Duval (1999), in fact the students were able to manage the representations and to visualise the area as a mathematical concept and not as a procedure or a formula to apply.

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