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# Relationships among mathematics beliefs, fluid intelligence, cognitive reflection and mathematics achievement in secondary school students

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*This study examines the interrelationships between non-verbal abstract reasoning (fluid intelligence), cognitive reflection, students' mathematics-related beliefs and mathematics achievement in second-year students at secondary school (n = 121). In addition, path analysis is used to test the predictive and mediational role played by the students' mathematics-related beliefs on mathematics achievement. Results confirm the association between fluid intelligence, cognitive reflection, and mathematics achievement, and reveal that the mathematics-related belief system offers an independent contribution to the prediction of mathematics achievement while also mediating the effect of fluid intelligence on said achievement. Results suggest the mediating role of students' mathematics-related beliefs and corroborate previous findings suggesting a circle of influence between reasoning abilities and beliefs on academic achievement.*

*Keywords: Metacognition, beliefs, mathematics achievement, secondary schools.*

## Introduction

Mathematical competence is a main component of school curriculum and frequently determines not only academic success but also the students' future career opportunities and employability possibilities (OECD, 2019). Different factors contribute in an interrelated manner to predict individual differences in mathematics performance. These include domain-specific knowledge, heuristics methods, positive mathematics-related beliefs, meta-knowledge, and self-regulatory abilities (Schoenfeld, 1992; De Corte et al., 2010).

Several studies have shown that adequate student beliefs regarding themselves as mathematics learners are necessary for mathematical learning, influencing mathematics achievement (De Corte et al., 2010; Gómez-Chacón et al., 2014; Goldin et al., 2016). Student belief systems embrace the implicit or explicit subjective conceptions that they hold to be true about (1) mathematics education (e.g., beliefs about mathematical learning and problem solving), (2) about themselves as mathematicians (e.g., intrinsic/extrinsic goal orientation, self-efficacy), and (3) about the mathematics class context (e.g., the role and functioning of their teacher). As the authors argued, these types of beliefs interact with each other and with the students' prior knowledge and determine their mathematical learning and problem-solving activities. This study examines the interrelationships between fluid intelligence, cognitive reflection and students' mathematics-related beliefs and mathematics achievement in second-year students at secondary school.

Several studies have shown that higher-order thinking skills have a significant influence on learning in school; they have pointed to the influence of intelligence (g-factor) on academic achievement in various subjects, specifically in the school subjects of mathematics-science (Roth et al., 2015). Fluid intelligence (Gf) is understood as the capacity for abstract reasoning and establishing new

relationships between multiple mental representations (Cattell, 1987). This variable is of interest for our study due the influence on abstract and spatial reasoning (Primi et al., 2010). Studies revealed that a high level of intelligence in 11 to 14-year-old students is associated with higher initial scores on numerical reasoning, abstract reasoning, verbal reasoning, and spatial reasoning (Primi et al., 2010). Assuming that Gf is an influential factor in mathematics learning and achievement, we explore the kind of relationship that exists between cognitive ability and beliefs. In this study, we use a classic measure and index of Gf based on visual-spatial reasoning abilities: the Raven's Advanced Progressive Matrices Test (Raven et al., 1995).

Likewise, dual thought process theories have explored the relationship between intuitive and analytical thinking in mathematics (Barrouillet, 2011; Frederick, 2005; Gómez-Chacón et al., 2014). The reasoning and decision-making aspects faced by individuals in a mathematical problem-solving environment require the interaction between both and the cognitive reflection, understood as a self-regulatory skill. The contributions of this paper relate to two main points. Firstly, we examine the relationships and relative contributions of the fluid intelligence - non-verbal abstract reasoning- and cognitive reflection as underlying mathematics reasoning processes. Secondly, assuming the relevance of individual beliefs in learning, we investigate the mediation of students' mathematics-related beliefs between those cognitive abilities and mathematics achievement in second-year secondary school students. Few studies have yet to simultaneously considered these aspects as variables that support individual differences in a specific main area of the school curriculum such as mathematics. Therefore, further research is necessary to address this gap. Participation of an adolescent sample seemed particularly relevant since reasoning skills are increasingly important during this developmental period (Barrouillet, 2011), in which learning activities become more complex.

## **Method**

### **Objective**

The main objective of this study is to examine the relationships between fluid intelligence (Gf), cognitive reflection (CRT, Cognitive Reflection Test) and mathematics-related beliefs with mathematics achievement in second-year secondary school students. Based on a past review, we expect to find positive correlations between Gf, cognitive reflection, and students' mathematics-related beliefs. We also expect the three measures to be significant positive correlates of mathematics achievement (Hypothesis 1). Specifically, we predict a tendency to underestimate CRT problem difficulty, such that the correlation between erroneous intuitive responses and the rating of CRT difficulty should be positive (Hypothesis 2). We also expect that students' mathematics-related belief system mediates the relationship between reasoning abilities and mathematics achievement (Hypothesis 3).

### **Participants**

Second-year secondary school students (n = 121, 13-14 years old) were recruited from a private school in an average socioeconomic level urban area of Madrid (Spain). Of this initial sample, 10 subjects were removed from the initial data set due to their failure to complete all the tasks. This resulted in 111 students (55 males and 66 females) from five different class groups. None of them

had repeated their grade, and all the participants had an academic level that was in accordance with the curricular standards for second-year studies.

## **Instruments**

### **Fluid intelligence**

Raven's Advanced Progressive Matrices (RPMT; Raven et al., 1995), was used as a test measuring non-verbal abstract reasoning. It is considered a classic measure and an index of fluid intelligence. The RPMT consists of sixty visual analogy problems. To solve each problem of the test, the participant is required to identify the relevant features of an array of visual abstract figures and shapes, plus an empty box, discover the rules governing the presentation of the diverse figural elements, use the rules to determine the missing element in the box and then choose the correct element to be selected from amongst several alternative responses arranged below the matrix. The dependent variable was the number of correctly solved items. Cronbach's alpha was .75.

### **Cognitive reflection**

A Spanish version of the Cognitive Reflection Test (CRT; Frederick, 2005) was used to measure cognitive reflection ability in problem solving. In addition to the three problems used in the initial test, we also included two additional issues proposed in the study of Gómez-Chacón et al. (2014). The CRT assesses an individual's willingness and aptitude to be reflective when faced with finding solutions to text-based mathematical reasoning problems. To solve the five problems, no time limit was set and no alternatives were provided to the participants to choose in an open-ended response format. Afterwards, participants were also asked to evaluate the percentage of their classmates that would give a correct solution to each problem (i.e., ratings of difficulty of the problems). The solutions to each problem and the estimated percentages had to be written down in a booklet. Three types of measures were provided for each problem: correct response, intuitive response, and a rating of the problem's difficulty. The total score in CRT was calculated as the number of correct answers. Reliability of Cronbach's alpha range between values of 0.60 and 0.73 (see, Campitelli & Gerrans, 2014). For this sample, Cronbach's alpha was 0.57.

### **Mathematics-related beliefs**

The CreeMat questionnaire (CreeMat questionnaire) is a Spanish version designed to assess belief systems about mathematics (Gómez-Chacón et al., 2014). The theoretical approach underlying the questionnaire is based on an integrative understanding of belief systems and on Mathematics-Related Beliefs Questionnaire (MRBQ) as a belief evaluation instrument (see Op't Eynde et al., 2002; De Corte et al., 2010). This framework identifies distinct relevant belief categories and defines students' belief systems as implicitly or explicitly subjective conceptions that they considered to be true (1) about mathematics education, (2) about themselves as learners of mathematics, and (3) about the mathematics classroom context. These beliefs, in close interaction with each other and with the students' prior knowledge, determine their mathematical learning and problem-solving activities in the class. The CreeMat questionnaire has been satisfactorily used in the research, not only in Spain but also in other countries (see Gómez-Chacón et. al., 2014; Mello-Román & Gómez-Chacón, 2022).

Consistent with this theoretical framework, which recognises belief systems and the dynamics of interaction between beliefs, the CreeMat questionnaire assesses four dimensions in the development of beliefs. For the purposes of this study, we took 10 items that cover these four dimensions: 1) affective and behavioural engagement in mathematical learning (EngagBe), e.g., “I work hard in mathematics”; 2) confidence and beliefs regarding one’s competence in mathematics (Self-efficacyBe), e.g., “I learn mathematics quickly”; 3) domain-specific mathematical beliefs (MathBe), e.g., “Mathematics allows us to better understand the world we live in”; and 4) beliefs about mathematical problem-solving (ProbSolBe), e.g., “Mathematics classes should not place much importance on problem solving”. Participants used a 5-point Likert scale from “Strongly disagree” (1) to “Strongly agree” (5) indicating the degree to which they agreed with the statement. In this study, Cronbach’s alpha coefficients were 0.73 for the overall scale (10 items), 0.58 for the EngagBe (4 items), 0.72 for the Self-efficacyBe (2 items), 0.61 for the MathBe (2 items) and 0.5 for the ProbSolBe (2 items) dimensions.

### **Mathematics achievement**

According to Roth et al. (2015), school grades are a good measure of academic performance, as they include information on school performance over a broad period and are based on different sources. In this study the school grades in Mathematics at the end of the school term, a numerical score on a scale of 0 to 10 points, were used as performance criteria in Mathematics. This evaluation focuses on theoretical, procedural, and attitudinal mathematical content within the discipline and curriculum and is assigned by the mathematics teacher responsible for the course. There have been control mechanisms in place to guarantee a fair grade independent of the teacher. Both exams and correction rubric for exams have been unified; randomized exams were double corrected, and differences were not to be higher than 0.5 points; and the number of tests was high, minimizing the impact of punctual deviations of individuals.

## **Results**

### **Descriptive and correlational analyses**

Descriptive statistics, mean (M) and standard deviation (SD) for the key study measures are presented in Table 1, as well as bivariate Pearson correlation coefficients between the variables. As seen in Table 1, the pattern of interrelationships in the total sample shows that all measures except the Cognitive Reflection Superficial performance (CRT\_Superficial) are significantly and positively correlated with each other in the low to moderate range (from  $r = .16$  to  $r = .51$ ). More specifically, the strength of the association between Gf and the students’ mathematics-related beliefs (CreeMat, Mathematics beliefs\_Total) is moderate and positive, and both constructs are positively and moderately related to mathematics achievement. The four belief system dimensions are positively and mildly correlated with Gf, but are moderately correlated with mathematics achievement (ranging from  $r = .20$  to  $r = .51$ ). Of special relevance is the association between students’ beliefs about their competence in mathematics (i.e., Self-efficacyBe) and Gf, as well as between self-efficacy beliefs and mathematics achievement ( $r = .21$  and  $.51$ , respectively). As for cognitive reflection measures, a pattern of low positive correlations between CRT\_correct performance and Gf, self-efficacy beliefs and mathematics achievement ( $r$  ranging from  $.16$  to  $.26$ ) is found. In contrast, the CRT\_superficial

score does not reach a significant association with Gf, student's mathematics-related beliefs (mathematics beliefs\_total, EngagBe, Self-efficacyBe, MathBe, ProbSolBe) and mathematics achievement measures. Students estimated that the percentage of their classmates who would be able to give correct solutions to CRT problems was high ( $M = 67.5\%$ ;  $SD = 16$ ). However, the CRT was rather difficult for participants, since there are more wrong intuitive or superficial responses (46.62 %) than correct ones (11.78 %),  $t = 6.68$ ,  $p < .001$ , Cohen's  $d = 0.63$ . The rating of the problem's difficulty (i.e., estimated difficulty) negatively associates with CRT\_Total correct but positively with CRT\_Superficial answers.

**Table 1: Means (M), Standard Deviations (SD) and Pearson Correlation Coefficients (unilateral, with  $*p < .05$ ;  $**p < .01$ )**

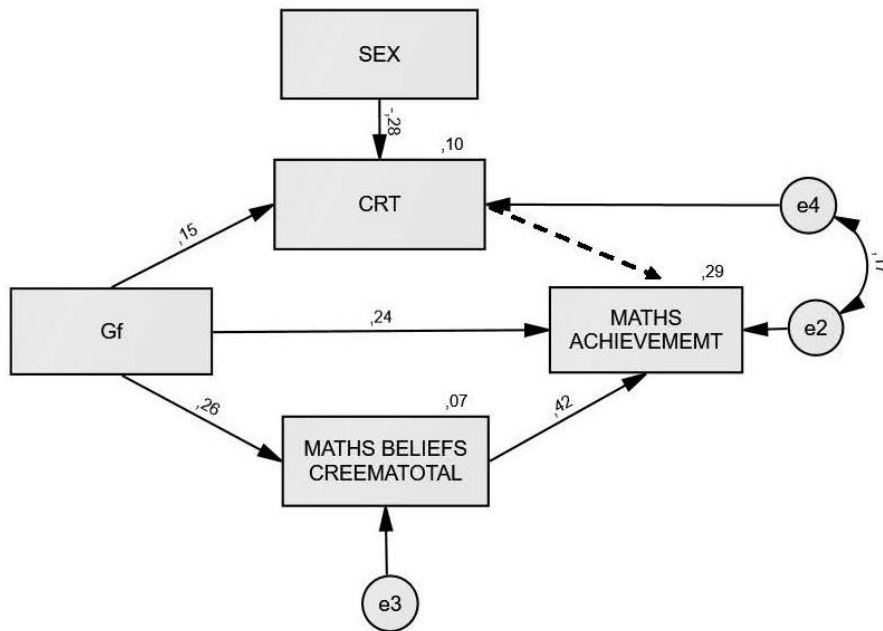
	1	2	3	4	5	6	7
<b>1. FLUID INTELLIGENCE (Gf)</b>	1	.26**	.15	.21*	.18*	.18*	.35**
<b>2. MATHEMATICS BELIEFS Total</b>		1	.80**	.67**	.68**	.56**	.48**
<b>3. EngagBe</b>			1	.30**	.43**	.26**	.34**
<b>4. Self-efficacyBe</b>				1	.35**	.25**	.51**
<b>5. MathBe</b>					1	.14	.20*
<b>6. ProbSolBe</b>						1	.27**
<b>7. MATHEMATICS ACHIEVEMENT</b>							1
<i>n</i> = 111							
<i>M</i>	47.41	33.84	12.16	6.72	7.75	7.19	6.85
<i>SD</i>	5.54	6.25	3.04	2.10	1.93	1.91	1.8
<b>Min. – Max.</b>	31-57	16-47	4-19	1-10	1-10	0-10	3-10

Even though it was not a main objective of this research, we examine sex-based differences in measures. No significant differences between male and female second-year secondary school students are found for Gf, mathematics-related beliefs, and mathematics achievement ( $p > .05$  in all cases). However, boys ( $M = 17.82$ ,  $SD = 22$ ) performed significantly better than girls ( $M = 7.7$ ,  $SD = 14.8$ ) in CRT\_Total\_Correct,  $t(1,109) = 2.9$ ,  $p = .005$ , Cohen's  $d = 18.1$ . For the total sample, CRT\_Total\_Correct score ( $M = .60$ ) is lower than CRT\_Superficial ( $M = 1.70$ ,  $t(110) = 7.73$ ,  $p < .001$ , Cohen's  $d = 1.50$ ) and CRT incorrect scores ( $M = .90$ ;  $t(110) = 2.16$ ,  $p = .03$ , Cohen's  $d = 1.49$ ), whereas the mean number of superficial answers is higher than incorrect answers ( $t(110) = 6.36$ ,  $p < .001$ , Cohen's  $d = 1.32$ ).

### Model testing

As shown in Figure 1, the hypothetical model is tested on the sample. When the direct effect between CRT and mathematics achievement is removed, the model demonstrates an acceptable fit to our data ( $\chi^2 = 4.16$ ,  $df = 4$ ,  $p = .38$ ;  $CFI = .99$ ,  $NFI = .93$ ,  $RMSEA = .02$ ). Thus, the standardized coefficients of all

paths of the model are significant, except for the effect between Cognitive Reflection Total and mathematics achievement. As expected, the results indicate that Gf predicts mathematics achievement ( $\beta = .24, p = .004$ ) and the mathematics-related beliefs total measure ( $\beta = .26, p = .005$ ), which in turn is positively related to mathematics achievement ( $\beta = .42, p < .001$ ). The results also reveal a significant association between sex and cognitive reflection total correct performance ( $\beta = .28, p = .002$ ). However, we find only a marginally significant positive association between Gf and CRT\_Tot correct performance ( $\beta = .15, p < .09$ ) and an insignificant negative association between CRT\_Tot correct and mathematics achievement ( $\beta = -.17, p = .26$ ).



**Figure 1: Standardized path coefficients among variables**

We observe an indirect relationship between Gf and mathematics achievement mediated by mathematics-related beliefs (Math Beliefs Total). To analyse this, we restrict the paths from Gf to the Mathematics Beliefs Total, and from mathematics beliefs to mathematics achievement to 0 in the indirect model. In this case, the direct relationship between Gf and mathematics achievement ( $\beta = .34, p < .004$ ) decreased when the Math Beliefs Total measure was introduced as a mediator, but remained significant ( $\beta = .24, p = .005$ ). The bootstrapping results reveal that the mediating effect of mathematics-related beliefs has a significant indirect relationship ( $\beta = .11, p = .009$ ; 95% CI: 0.03-0.18). The decrease in direct effect, while remaining statistically significant, suggests a partial mediating effect.

## Discussion and conclusion

We investigated the interrelationships between Gf, cognitive reflection, and mathematics-related beliefs measures with mathematics achievement in second-year secondary school students. The results advance our understanding in two ways. First, new evidence is offered on the interplay between non-verbal visuospatial reasoning abilities —as measured by a fluid intelligence test—, cognitive reflection —as measured by the CRT— and mathematics achievement. Second, evidence is presented on the mediating role of students’ mathematics-related beliefs between Gf and mathematics

achievement. The model proposed to illustrate the plausible mediating role of mathematics-related beliefs is shown in Figure 1 and it has a good fit to the data.

The interrelationships among the key variables that we hypothesised to support individual differences in mathematics achievement, the results confirm a pattern of reliable positive inter-correlations between Gf, cognitive reflection, and students' mathematics-related beliefs with mathematics achievement. Overall, the magnitude of the correlations is low to moderate for this sample. The results show that students with higher Gf have higher mathematics achievement, better cognitive reflection performance and more favourable mathematics-beliefs than those with lower Gf. Accordingly, a close relationship between the diverse kinds of reasoning studied in this paper —fluid intelligence and cognitive reflection— and mathematics achievement has been shown. It is especially remarkable that mathematics beliefs are the most closely related variable to mathematics, followed by a positive association between Gf and mathematics achievement —as measured by teacher-assigned grades—. Therefore, it appears that the students' beliefs measures play a more relevant role in mathematics achievement in this age period than any other variable considered in this study. The role of reasoning processes in mathematics is also confirmed (Gómez-Veiga et al., 2018).

Regarding cognitive reflection, the results corroborate the idea that most of the participants replied with the wrong intuitive responses in the CRT. Another relevant finding is the low-moderate association between Gf and CRT-correct performance, and between mathematics achievement and the CRT\_correct measure. The relationship between Gf and mathematics achievement in secondary school students (in this study, second year) emphasizes the role of students' mathematics-related belief system in the pathway between reasoning abilities and mathematics academic achievement. More specifically, the results of the mediational analysis suggest a partial mediation through mathematics-related beliefs in the relationship between Gf and mathematics achievement (Primi et al., 2010). The mediation may be explained by the strength of the correlations between the variables. This study confirms the relevant role of mathematics-related beliefs in mathematics achievement (Gómez-Chacón et al., 2014; Goldin, et al., 2016) and shows that it is better explained when considering the mediating effect of intelligence on students' mathematics-related beliefs.

Our study has certain limitations that future work should address. To increase the generalizability of our findings, a confirmatory methodological approach is needed. It should include a wider sample of students and additional years of secondary school education, as well as exploring more factors that may influence academic performance and the self-efficacy beliefs.

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