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Grade five students' different experiences of patterns

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The concept of patterns is a fundamental dimension of mathematics as well as an explicit topic for mathematics education. Already at a primary level, patterns are part of syllabus, and later, patterns are also made a part of teaching strategies to explain algebra and functions. In this study we focus on gaining understanding of what experience students has of arithmetic image patterns. Data comes from a lesson design where primary school students work in pairs, thus creating an interview-like situation. The analytic strategy was inspired by phenomenography. The results indicate that there are three levels of experiences, where the lowest level is not functional for expanding into secondary levels of mathematical content such as functions.

Keywords: Functions, patterns, algebra, primary education, Sweden.

Introduction

With algebra, we search for patterns, connections, and relations, aiming to describe features generally (Blanton et al., 2019; Häggström et al., 2019). Thus, an important part of algebra is its explicit element of abstraction. To bridge the alleged gap between the abstractive demands of algebra and the concrete life world of children, image patterns are often used to introduce algebra content in primary school. These lesson activities focus the analysis and generalisation of arithmetic patterns (Blanton et al., 2019), typically in image form, portraying sticks or beads. However, the algebraic, arithmetical, and computational dimensions in image patterns are not clearly delineated (Radford, 2011). Thus, when patterns also is made a mandatory part of curriculum, as is the case in Sweden where this study was carried out (Skolverket, 2022), a substantial number of issues are left for the teachers to resolve.

To prepare lessons, teachers need to have some insight into their students' previous experience and understanding of the intended learning object. Such knowledge will be the point of departure for designing lesson tasks and activities that enables students to discern relevant theoretical dimensions of the content (Larsson, 1986). Therefore, aiming to improve instruction, the issue for this paper is to discuss which different types of experiences students specifically have about patterns and how these experiences, potentially, can be hierarchically structured.

Background and theoretical framework

In western syllabi, algebra has traditionally not been introduced until at secondary levels (Warren & Cooper, 2006; Wettergren et al., 2021). Algebra introduced at secondary levels often relies on the premise that algebra is understood from a basic knowledge of arithmetic (Kieran, 2018). However, educational research from the last decades has argued for a considerably earlier introduction, even at the very beginning of compulsory school (Blanton et al., 2015; Davydov, 2008; Radford, 2018).

As mentioned above, it is not easy to decide if the mathematical foundation for patterns is in algebra or arithmetic. But for practical classroom reasons, at least at compulsory school level, the analytic generalisation dimensions that is inherent in pattern tasks offer rich opportunities for students to work

with mathematically relevant issues of both kinds. One often used argument for letting students work with patterns is based on their similarities with mathematical functions. Therefore, the way to describe this similarity using algebraic symbolism is one method for developing functional thinking (Markworth, 2012; Wilkie & Clarke, 2016). Students' functional thinking develops through a number of cognitive levels (Blanton et al., 2015), however, further knowledge is needed to deeper "understand the mechanisms that promote shifts in children's thinking" (Blanton et al., 2015, p. 546). Reasonably it is a task for education to promote these shifts. One way for education to enable students to develop a capability in algebraic modelling as a part of reasoning is to present the mathematical content in a way that has variables representing quantities or numbers as its point of departure, and gradually replacing these variables with numbers (Davydov, 2008). This implies shifting from a numerical and computational to a generalised and descriptive view on mathematics. Understanding the regularity of patterns rests upon perceiving the pattern's spatial structure, that is, the position of components, but also the pattern's numerical structure. Ignoring the spatial structure and reducing the pattern to a numerical sequence allows the analysis to focus on differences between figures in the pattern. However, this potentially limits the student's possibilities to generalise the pattern into an algebraic form (Markworth, 2012). If a learning activity focusing patterns is to be a basis for developing algebraic thinking it is pivotal that mathematics instruction attempts to develop a spatial/numerical/variable triad. Otherwise, there is a risk that students stop examining structures between variables and instead turn their focus towards numerical connections (Markworth, 2012). A different challenge for teachers is to acknowledge—during lessons—when students are making algebraically sound arguments also in verbal or other form that may deviate from mathematically correct symbolism (Radford, 2011), and to invite them to elaborate those arguments.

The inspiration for this paper springs from our joint classroom experience that patterns are mathematically worth while for students to investigate, but at the same time these lessons sometimes deflate into a game of image making. If we believe that a teacher's knowledge about their students' prior experience of a learning objective is vital (Larsson, 1986), a lack of such knowledge will be problematic. One way of understanding students prior experience is through the theory of phenomenography, as described by, for example, Marton (e.g., 2015). In phenomenography the result is in the form of categories describing—often in colloquial terms—the qualitatively different ways a group of people, i.e., a school-class, experience a certain phenomenon as. In the tradition of phenomenography, the word "experience" implies that the understanding is embodied but implicit, and often even difficult for the person to describe. A person's experience of a phenomenon is the sum of aspects within this phenomenon a person has discerned. It comes from how they have been introduced to a certain phenomenon, and under what conditions they have been able to interact with and interpret it. A phenomenographic result forms what Marton (2015) describes as a hierarchical outcome space. The teacher's lesson planning benefits from such phenomenographic knowledge; it helps the teacher to decide what aspects of the learning object that the students need to discern to reach a more qualified understanding. However, the implicit nature of experience makes these aspects difficult to find.

Thus, aiming to scaffold educators' designing and staging lessons, the research question that we aim to answer in this paper is: which qualitatively different ways that Grade 5-students experience patterns can be discerned?

Method

The general interest of this study was to investigate students' experience of patterns in a broad sense of the concept. The research followed an iterative learning study model by designing a research lesson consisting of up to six tasks, see Figure 1. To achieve a conceptual variation in the lesson the task's content were chosen to combine the skills "analyse" and "create", with the methods "describe" and "connect", thus creating the design framework shown in Figure 1. Aiming to design tasks that stimulate whole-class or group discussion, all tasks were set in a playful format (van Oers, 2009), e.g., task 1: "Describe the pattern over the phone to a person that can't see it.", or task 6: "Some students in another class connected this pattern to that expression – how could they have reasoned to come to that conclusion?". Due to space constraints only tasks nr 1 and 3 are presented more thoroughly below.

DESCRIBE	Interpret the meaning and correctness of a pattern description. <i>Task nr: 6</i>	Formulate salient properties in a pattern. <i>Task nr: 1</i>
CONNECT	Identify correct/incorrect correlations between expression and pattern representations. <i>Task nr: 2, 3, 5</i>	Construct a representation, e.g., an image pattern from a description in algebraic or word form. <i>Task nr: 4</i>
	ANALYSE	CREATE

Figure 1: Model of task differentiation and design

Data was gathered from lessons where students worked with tasks in pairs. In addition, each research lesson was accompanied by pre- and post-tests in the form of video-tapes of students working in pairs with tasks. These pre- and post-tests resembled interviews where the interviewer asked the respondents to solve three tasks together. The interviewer encouraged the respondents to use their combined knowledge to solve the task to a point of consensual satisfaction, and instructed the respondents that she would intervene only when explicitly called upon. The interview-questions were thus materialised in the task's triad of text, image and verbal instruction from the interviewer. We call this method, materialised interviewing.

In total 16 interviews, in addition with written materials that the students produced during the research lessons have been analysed. The research lessons as well as the interviews were video-taped and subsequently transcribed.

The tasks were designed in accordance with tasks on patterns regularly found in textbooks and national tests. The design also aimed at encouraging reasoning about algebraic expressions and their connection with patterns. During the research lessons and the materialised interviews the students worked with exploring and describing a given pattern or linking a pattern to different expressions.

The two research cycles were carried out in two different groups in grade 5 (age 11) in a school where two of the authors work as teachers. Grade 5 was chosen for the study because students in Swedish schools traditionally are not introduced to algebra in any formalised form until grade 6. Following research-ethical considerations (Vetenskapsrådet, 2017) all students had written parental consent and were, prior to the interviews and the research lessons, informed that they participated voluntarily. The teachers/researchers that taught the lessons were not involved in the students' ordinary schooling.

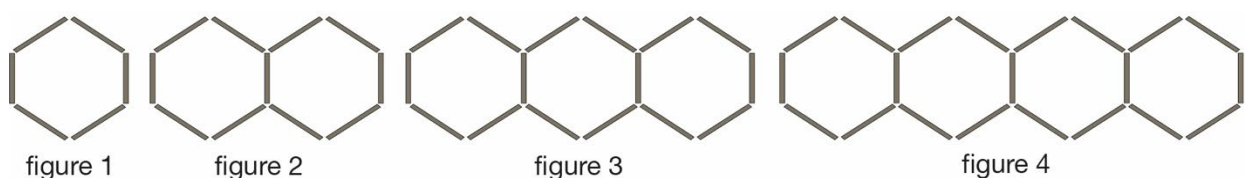
Prior to analysis, lessons and interviews were transcribed and the participants were given pseudonyms. Then, sections where students described or commented on issues connected to the phenomenon “patterns” were marked. At this stage no considerations were made concerning the correctness of these utterances. The analysis was inspired by phenomenography (Larsson, 1986; Marton, 2015), here aiming to understand students’ ways of experiencing arithmetic patterns as a mathematical representation. The focus was on finding similarities and differences in how students discussed mathematically relevant dimensions in the tasks, for example, signs of functional or algebraic thinking. This way the research team, in an iterative way of working, found themes that grouped the utterances together in qualitatively different categories. For example, when two students talk to each other and one of them says, “... should it describe one figure, or should it describe all the figures?”, indicates looking at the pattern as consisting of individual images. This can be compared with a way of understanding patterns as never-ending, as in the utterance by a student, “...these are only four figures, but you could make it a lot longer”, which represents a different category.

Result

The result indicates that students’ different experiences of patterns can be organised in three categories that form a range of increasing complexity, a hierarchical outcome space. In this section we present the categories, accompanied with a short transcript from the data.

Category 1. Students experience patterns as images

The students discern a pattern as repetitive where the images are recurrent. Within this category students discern a repetitive dimension restricted to the images visible. With Figure 2 as an example this would mean that the pattern is 1, 2, 3 and 4 hexagons and the following section is again, 1, 2, 3 and 4 hexagons. Or the students experience patterns as the one-to-one connection between one of the figures in the pattern and an expression, for example, student A’s question to student B, “... should it describe all the figures?”, as in Excerpt 1.



Teacher’s written and verbal instruction: “Wich expression, or expressions, can describe the pattern?
 $S = 5 \cdot n + 1$ $S = 5 \cdot n + 6$ $S = 6 \cdot n$ $S = 6 + 6 + 6$ $S = 6 \cdot n + 5$ ”

Figure 2: Task number 3 reproduced from lesson video

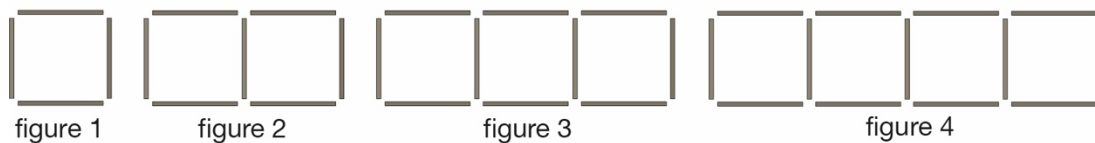
Excerpt 1

- Student A: Here we have five times something. Plus six. So if we have, for example five times two. Plus six. Then it will be sixteen. So it can be figure 3 then.
- Student B: Aha. If it's five times, <thinks>, five times one.
- Student A: Then there will only be five.
- Student B: Plus six and then it can be figure eleven. [points at the second figure in the pattern]. Wait, should it describe one figure, or should it describe all the figures?
- Student A: Ehm, which expressions describe the pattern it says. Ok, there can be two, there can be more? So then I guess that, <stops>
- Student B: We shall describe the whole pattern, or?
- Student A: Something like this [points at one expression] can describe many (figures) in the pattern, we don’t know if everyone can do all, but we can try and see this one.

Discussing patterns this way indicates that the students neither discerns the continuing quality of patterns, nor its connection to algebraic symbolism. The student experience that geometrical characteristics can reveal relationships between components, or that numerical structures within a certain element is central.

Category 2. Students experience patterns as a row of systematically larger images

Within this category the students experience patterns as a connection of figures, infinitely growing in size following a structure that is additive for each new figure. From this follows implicitly that a pattern contains figures that are not shown in the image.



Teacher's instruction: "Describe the pattern to a person that can't see it."

Figure 3: Task number 1 reproduced from lesson video

Excerpt 2

Student C: [Talks about task number 1 (see Figure 3).] There is a ceiling, a floor, and a wall, and a wall. And then it becomes a square. And then, in the second figure, you do almost the same thing, but you add three matchsticks (...) just a floor, yes <thinks> no, a wall. And a ceiling <stops>

Student D: Yes, because you already have one, a wall there that <stops>

Student C: Aha, and then there will be two squares <thinks> and then it continues like that until you get four, and then it continues like that, well, for as long as you like.

Student D: Well, there are, these are only four figures, but you could make it a lot longer.

Typical for this second category, is that students experience a quality of infinity in patterns, and that the pattern grows – both in number of figures and the figures themselves. Here this is seen when the students compare the number of components in different figures and searches for a constant difference between consecutive figures, aiming to describe figures that are not visible in the pattern.

Category 3. Students experience patterns as a rule that enables connection between images

The student experiences a pattern as connectable to a general rule in a way that one expression fits all figures in the pattern. These rules can be represented without using algebraic symbols or connections to an explicit algebraic expression, see Excerpt 3:

Excerpt 3

Student E: [Talks about Task number 1.] That's matchsticks, it increases with three each time.

Student G: But now it is like this, three, it is figure 1, times three plus one.

Student E: If you think like that, that kind of calculation <thinks> if you take three times one plus one, then we have the first figure which is four.

Student G: Yes.

Student E: Figure 2, it is like three times two, it is six plus one.

Student G: And then it continues like that.

In contrast, Excerpt 4 illustrates a different task (see Figure 2), one that contains explicit algebraic expressions. This invites the students to connect algebraic expressions to patterns and to describe the general relations.

Excerpt 4

- Student H: [Talks about Task number 3 (see Figure 2).] This is eleven. If you take five times two [points at the second figure in the pattern].
- Student J: Or three. It works even if it's three here [points at n in the expression] instead of the n .
- Student H: Five times three, it's fifteen <thinks> but it's sixteen because, if you take for example five times three, then it's fifteen [points at the n and the 5 in the expression] and then plus one, it's sixteen, and if we look at figure 3 <J interrupts>
- Student J: Then it will be sixteen.
- Student H: Sixteen matchsticks
- Student J: Then it works for all here [points at all figures in the pattern].

Despite being differently framed, the two examples illustrates the students' ambition to connect the pattern to a formula or describe it in general terms. This can be done following stringent algebraic symbolism, or in more colloquial ways. Rather, the two examples illustrate a difference in which affordances lie within a certain task, but regardless of whether the task contains explicit algebraic expressions or symbolism, this third category expands the understanding of students experience of patterns as something that can be described generally, beyond the specifics of the matchsticks, marbles or other elements shown in the image: As this last example illustrates:

Excerpt 5

Student C: Five times n means that it increases with five every time. Look, how many times you want, like plus four, then plus four means it must start with four more than what it grows.

Summary and discussion

Aiming to describe the qualitatively different ways students experience patterns, the result of this study is the categories. We believe that teaching methods (see e.g., Cooper & Warren, 2011) will be difficult to make successful without this detailed knowledge (Larsson, 1986; Marton, 2015).

Relating to the research question, the analysis indicates a hierarchical structure in the experiencing of patterns, ranging from singular images via systematically larger images to rules. The experience of patterns as separate images and seeing the pattern as the geometrical shape of the elements in one single figure, in agreement with Ekdahl (2013) and Fred & Björklund Boistrup (2017), implies that Category 1 can be seen as non-functional and non-algebraic.

The second category of experience, "a row of systematically larger images", is especially interesting because it could mean that the student is in a kind of transit (Wilkie & Clarke, 2016); from seeing the systematic growth in category 2 to identifying the general rule in category 3. The fact that student C is in both categories lends strength to this argument. Even though category three connects to functional thinking (Blanton & Kaput, 2011; Stephens et al., 2017), continued research is needed to understand which aspects that are critical for the students to discern to develop a more qualified understanding of patterns. In addition, the excerpt from students E and G, in category 3, implies that students in Grade 5 can grasp a rather qualified way to operate on patterns, by ways where they treat the indeterminate quantities in an analytic way (Radford, 2011). Further, this was not seen until the post-tests, implying that some element in the lesson enabled them to discern this knowledge. Hence, even though this study is not quantitative, the fact that at least one pair of students expressed this competence points towards the idea that all students potentially could be made to master it (Davydov, 2008). One implication of the present study is that since the point of departure for some students is the non-mathematical Category 1-experience, lessons need to secure the possibility for students to experience patterns in ways consistent with Category 2 (Pittalis et al., 2020). However, continued

research is needed to pinpoint more exactly how a learning situation can be designed and staged for this to happen, thus how lessons can examine algebraic relations in ways that invite algebraic thinking and not only algebraic symbolism, as pointed out by Radford (2011).

Lastly, the framework used for choosing lesson tasks (Figure 1) has developed during the analytic iterations of the research lessons, therefore the framework can be seen as a part of the results of this study. The theory of variation (Marton, 2015) points towards the necessity of contrasting variation in successful instruction. However, designing teaching tasks with such variation and contrast is not trivial to accomplish in practice. Therefore—and admitting the tentative dimension of the task framework-results—we believe that teaching practice could benefit from continued research focusing this lesson design tool.

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