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► **To cite this version:**

Olga Battaïa, Alexandre Dolgui, Nikolai Guschinsky. An exact method for machining lines design with equipment selection and line balancing. *International Journal of Production Research*, 2024, Special issue in memory of Dr. Jean-Marie Proth, 62 (1-2), pp.71-91. 10.1080/00207543.2023.2289182 . hal-04418105v1

HAL Id: hal-04418105

<https://hal.science/hal-04418105v1>

Submitted on 25 Jan 2024 (v1), last revised 7 Feb 2024 (v2)

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An exact optimization method for balancing machining lines minimizing their cost

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Abstract: We study a line balancing problem for a machining line equipped with multi-positional machines, rotary tables and vertical and horizontal machining modules. For this challenging problem in production research, we develop an innovative mathematical model and use a mixed-integer program to solve it. The model is intensively tested on numerous real-life industrial cases as well as generated randomly instances. The obtained results are carefully analysed in order to provide the decision makers with sound managerial insights.

Keywords: Assembly line balancing, CAPP, Combinatorial optimisation, Decision support systems, Flow lines, Mixed Integer Linear Programming, Machining systems design, Machining processes.

1. INTRODUCTION

In the field of production research, line balancing problems have initially been introduced in assembly environment. One of the first description of such a problem was presented for manual assembly lines by Salveson (1955). First studied in its simple version, Simple Assembly Line Balancing problem has been addressed in numerous statements and for different applications. This led to the development of various approximate and exact solution methods. A well-know and renowned from industrial and academic points of views algorithm COMSOAL (a computer Method of Sequencing Operations for Assembly Lines), developed for Chrysler Corporation, was published in IJPR by Arcus (1966). This topic has been a priority of the journal for many time, see for example (Ghosh and Gagnon 1989, Gadinov and Wilhelm 2000).

A number of reviews presenting the state of the art over the time on existing optimisation methods have been successively appeared in literature (Baybars 1986, Erel and Sarin 1998, Rekiek et al. 2002, Scholl and Becker 2006, Dolgui and Proth, 2010). With the development of optimization tools, it became

possible to address complex assembly line balancing problems taking into account real-life attributes of different line layouts, parallel workplaces and workers etc. (Boysen et al. 2007, 2008, Battaia and Dolgui 2013, Dolgui and Proth 2013).

At the same time, the line balancing problems have been introduced for other manufacturing environments including disassembly (Gungor and Gupta 2001) and machining lines (Dolgui et al. 1999). Some studies unified the models used for assembly and disassembly line balancing (Guo et al. 2021, Mete et al. 2018, McGovern and Gupta 2015).

Even if different design problems have been discussed for machining systems previously (Szadkowski 1971, Szadkowski 1997, Spicer et al. 2002), one of the first line balancing problems in machining environment was formulated for transfer lines (Dolgui et al. 1999) and it was labelled as the Transfer Line Balancing Problem (TLBP). Different versions of this problem have been presented, and exact and approximate methods have been developed to solve it (Belmokhtar et al. 2006, Finel et al. 2008, Guschinskaya et al. 2008, 2011, Guschinskaya and Dolgui 2009, Dolgui et al. 2005, 2006a,b,c). Further, the problem of rebalancing transfer lines has been formulated (Makssoud et al. 2014) dealing with the problem of reusability of the machining equipment designed for mass production.

Consequent development in the field of balancing machining lines is related to the technological evolution of the manufacturing sector: from traditional mass production machines (Hitomi 1996), via computer numerical control (CNC) machines or machining centres (Essafi et al., 2010a,b, 2012) to reconfigurable manufacturing systems (Battaia et al. 2017, 2020, 2021; Borisovsky et al. 2013, 2014) passing through the use of customizable modular machines (Dolgui et al. 2008, 2009, Battaia et al. 2016) and transfer lines composed of such machines (Dolgui et al. 2008, 2012, Battaia et al. 2012, 2014a; Borisovsky et al. 2012a,b, Delorme et al. 2012). Different configurations of machining lines with sequential and parallel execution of operations have been discussed in literature (Guschinskaya et al., 2008; Dolgui and Ihnatsenka 2009a,b; Dolgui et al. 2014, Battaia et al. 2014b) as well as different production modes such as mixed-model machining (Battaia et al. 2016), family part production (Battaia et al. 2017) and batch production (Battaia et al. 2020). A general approach of optimisation by decomposition for both optimal process design (combinatorial design of pieces of equipment via solving a line balancing problem) and selection of cutting conditions, e.g. feed rates, for designed pieces of equipment has been proposed in (Dolgui et al. 2020).

A recent article by Boysen et al. (2021) analyses the development of the field of assembly line balancing during last 15 years. The review paper of Battaia and Dolgui (2022) compares this development with the evolution of line balancing problems in disassembly and machining environments. It should be noted that the line balancing problem for robotic lines is mostly considered for assembly and disassembly environments and rarely for machining (Chutima 2022, Daneshmand et al. 2023).

The series of research studies on line balancing for machining lines was started by our team in our INTAS projects for which Dr Jean-Marie Proth was the scientific coordinator: INTAS-96-0820 (Discrete optimization problems in scheduling and computer-aided design, 1997 – 2000); and INTAS-00-0217 (Scheduling and assignment models under uncertainty and real-time constraints with application to manufacturing, communication, computer-aided design and transportation, 2001 – 2004). This work is in continuation of that and later projects.

In machining lines, a part is completed by a series of machining operations like drilling, milling, reaming etc. There exist much less precedence relations between such operations than for an assembly/disassembly process. However, there may exist a lot of exclusion/inclusion constraints for deciding which operations must be performed together, at the same workstation, because of the required tolerance or, on the contrary, cannot be executed at the same workstation because of the impossible fixturing. As the literature review shows, different configurations of machining lines have been studied in the literature providing efficient optimisation tools for the decision makers. The contribution of the current paper is to introduce a new formulation for flow lines equipped with rotary machines and to develop an original and efficient for industrial use MIP models. In Section 2, we present the general statement of the problem. In Section 3, we propose an original MIP model to solve the problem. A case study is presented in Section 4. Experimental results are reported in Section 5. Concluding remarks are given in Section 6.

2. PROBLEM STATEMENT

In this paper, we consider the following optimal design problem for a flow line composed of n rotary transfer machines with turrets (see Fig.1). Each machine is multi-positional, i.e. it may have up to m working positions (1, 2, ..., m), one of these positions is exclusively used for loading and unloading operations. All other working positions are used for machining parts due to the installation of one or several machining modules that are activated sequentially (turrets) or simultaneously (spindle heads) on the parts being machined. Both turrets and spindle heads can be horizontal or vertical. The design of machines restricts the use of the vertical spindle head to one per machine but actionable on all working positions. Only one horizontal spindle head or turret can be installed at each working position.

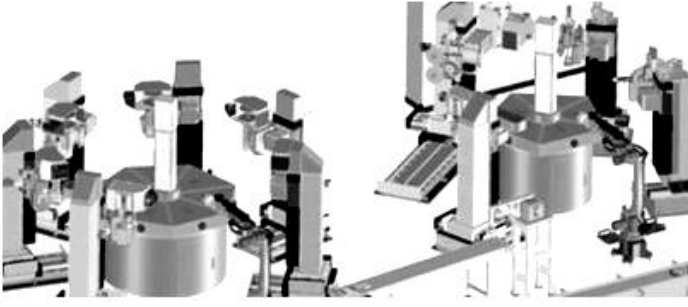


Figure 1. A flow line with rotary machines

The considered line balancing problem consists in the choice of an orientation of the part for each machine, the partitioning of the given set of operations into machines and machining modules, and the choice of cutting modes for each spindle head and turret.

Let \mathbf{N} be the set of machining operations needed for machining of elements of the part, located on η sides and N_s , $s=1, 2, \dots, \eta$, is a subset of operations for machining the elements situated on the s^{th} side of the part. Each side is characterized by tool accessibility directions. The part can be located at a machine in different manners \mathbf{H} that can be represented by a matrix of dimension $\mu \times \eta$, where h_{rs} , $r=1, \dots, \mu$, $s=1, \dots, \eta$, is equal to j , $j=0,1,2$, i.e. 1 if the elements of N_s can be processed by vertical machining modules, 2 if the elements of N_s can be processed by horizontal machining modules and $j=0$ means that processing is prohibited) when the orientation r of the part is selected.

All operations $p \in \mathbf{N}$ are defined by the following parameters:

- the length $\lambda(p)$ of the working stroke for operation $p \in \mathbf{N}$,
- side $s(p)$ of the part to be processed by operation p ,
- the range $[\gamma_1(p), \gamma_2(p)]$ of feasible values of feed rate,
- the set $H(p)$ of feasible orientations of the part (indexes $r \in \{1, 2, \dots, \mu\}$ of rows of matrix \mathbf{H} such that $h_{rs} = 1$ for operation $p \in N_s$).

The designers seek to find optimal values for the following **decision variables**:

- the number n of machines;
- the numbers m_i of working positions at machine i , $i=1, \dots, n$;
- the orientation H_i of the part at machine i , $i=1, \dots, n$;
- the number b_{ikj} of machining modules of type j , $j=1, 2$, that are installed at position k , $k=1, \dots, m_i$, of machine i , $i=1, \dots, n$;
- the set N_{ikjl} from \mathbf{N} , assigned to machining module l of type j at position k of machine i , $i=1, \dots, n$, $k=1, \dots, m_i$, $j=1, 2$, $l=1, \dots, b_{ikj}$;
- the feed rate $\Gamma_{ikjl} \in [\max\{\gamma_1(u) | u \in N_{ikjl}\}, \min\{\gamma_2(u) | u \in N_{ikjl}\}]$ for execution of operations from N_{ikjl} , $i=1, \dots, n$, $k=1, \dots, m_i$, $j=1, 2$, $l=1, \dots, b_{ikj}$.

These decision variables can be represented by $P_{ikjl}=(N_{ikjl},\Gamma_{ikjl})$, $P_{ikj}=(P_{ikjl}|l=1,\dots,b_{ikj})$, $P_{ik}=(P_{ikj}|j=1,2)$, $P_i=(P_{ik}|k=1,\dots,m_i)$, $P=((P_i,H_i)|i=1,\dots,n)$,

The feed rate defined the cutting time of each machining module in the following manner:

the execution time $t^b(P_{ikjl})$ of set of operations N_{ikjl} with the feed rate $\Gamma_{ikjl} \in [\underline{\Gamma}_{ikjl}, \bar{\Gamma}_{ikjl}]$ is equal to $t^b(P_{ikjl}) = L(N_{ikjl})/\Gamma_{ikjl} + \tau^a$, where $L(N_{ikjl}) = \max\{\lambda(p)|p \in N_{ikjl}\}$, $\underline{\Gamma}_{ikjl} = \max\{\gamma_1(p)|p \in N_{ikjl}\}$, $\bar{\Gamma}_{ikjl} = \min\{\gamma_2(p)|p \in N_{ikjl}\}$ and τ^a is an additional time for advance and disengagement of tools.

If the turret of type j is installed at position k of machine i , then the execution time of operations from set N_{ikj} is equal to $t^h(P_{ikj}) = \tau^g b_{ikj} + \sum_{l=1}^{b_{ikj}} t^b(P_{ikjl})$, $j=1,2$, where τ^g is an additional time for one rotation of a turret.

If all N_{ikjl} are empty then $t^h(P_{ikj})=0$. If $b_{ikj}=1$ then $t^h(P_{ikj})=t^b(P_{ikj1})$.

If the vertical spindle head is installed on machine i , then $\Gamma_{ik11}=\Gamma_i$ and $\Gamma_i \in [\max\{\underline{\Gamma}_{ikj1} \gamma_1(p)|k=1,\dots,m_i\}, \min\{\bar{\Gamma}_{ikj1}|k=1,\dots,m_i\}]$.

The production time of one working position $t^p(P_{ik})$ is equal to $\tau^r + \max\{t^h(P_{ikj})|j=1,2\}$, where τ^r is an additional time for one rotation of the rotary table.

Finally, the cycle time of machine i , $t(P_i)$ is equal to $\max\{t^p(P_{ik})|k=1,\dots,m_i\}$ and the cycle time of the flow line is defined by its slowest machine, i.e. $t(P) = \max\{t^p(P_i)|i=1,\dots,n\}$.

Taking into account the fact that at the beginning and end of machining of O parts not all the working positions are occupied, the total time $T(P)$ for machining O parts is equal to:

$$T(P) = t(P)(O + \sum_{i=1}^n m_i + n - 1).$$

We assume that the given productivity is provided, if the total time $T(P)$ does not exceed the available time T_0 .

Let C_{sm} , C_{sp} , C_{st} , C_{mm} , C_{sh} , and C_{sw} be the relative costs for one machine, one position, one turret, one machining module of a turret, one spindle box, and one workhead with a single spindle, respectively. Since the vertical spindle head (if it is present) is common for several positions, its size (and therefore the cost) depends on the number of positions to be covered. Let k_{\min}^{ih} and k_{\max}^{ih} be the minimal and the maximal position of the common vertical spindle head at machine i , $i=1,\dots,n$. Then the cost of the vertical spindle head can be estimated as $C_{sh} + (k_{\max}^{ih} - k_{\min}^{ih})C_{vp}$, where C_{vp} is the relative cost for covering one additional position by vertical spindle head. If the vertical spindle turret is installed, its cost can be estimated by $C_{st} + C_{mm}b_{ik1}$. In the similar way, the cost $C(b_{ik2})$ for performing set of operations N_{ik2} by associated b_{ik2} machining modules can be assessed as follows:

$$C(b_{ik2}) = \begin{cases} 0 & \text{if } b_{ik2} = 0, \\ C_{sw} & \text{if } b_{ik2} = 1 \text{ and } |N_{ik21}| = 1, \\ C_{sh} & \text{if } b_{ik2} = 1 \text{ and } |N_{ik21}| > 1, \\ C_{st} + C_{mm}b_{ik2} & \text{if } b_{ik2} > 1. \end{cases}$$

The cost $Q(P)$ of a design solution is calculated as the total cost of all equipment used in the corresponding flow line:

$$Q(P) = C_{sm}n + \sum_{i=1}^n (C_{sp}m_i + C_{sh} \text{sign}(|N_{i1}|)(1 - \sum_{k=1}^{m_i} \text{sign}(|N_{ik12}|))) + \sum_{k=1}^{m_i} \text{sign}(|N_{ik12}|)(C_{st} + C_{mm}b_{ik1}) + C_{vp}(k_{\max}^{ih} - k_{\min}^{ih}) + \sum_{k=1}^{m_i} C(b_{ik2}) + C_{ro} \sum_{i=1}^{n-1} \rho(H_i, H_{i+1}),$$

where $N_{i1} = \bigcup_{k=1}^{m_i} \bigcup_{l=1}^{b_{ik1}} N_{ik1l}$, $\text{sign}(a) = 1$, if $a > 0$, and $\text{sign}(a) = 0$, if $a \leq 0$, $\rho(H_i, H_{i+1}) = 1$, if $H_i \neq H_{i+1}$, and $\rho(H_i, H_{i+1}) = 0$, otherwise.

The design decision P should satisfy the following constraints:

– precedence constraints which define possible sequences of operations, they are specified by a directed graph $G^{OR}=(N, D^{OR})$ where an arc $(p, p') \in D^{OR}$ if and only if the operation p has to be executed before the operation p' . It should be noted that if such operations p and p' belong to different sides of the part then they cannot be executed at the same position without violating the precedence constraint.

– inclusion constraints which oblige to perform some pairs of operations from N at the same machine, by the same turret, by the same spindle head or even by the same spindle, they are defined by undirected graphs $G^{SW}=(N, E^{SW})$, $G^{SP}=(N, E^{SP})$, $G^{ST}=(N, E^{ST})$, and $G^{SM}=(N, E^{SM})$, where the edge $(p, p') \in E^{SM}$ ($(p, p') \in E^{ST}$, $(p, p') \in E^{SP}$, $(p, p') \in E^{SW}$) if and only if the operations p and q must be executed in the same machining module (turret, position, machine).

– exclusion constraints which prohibit the mutual assignment of some pairs of operations from N to the same machine, to the same turret, or to the same spindle head, they are defined by undirected graphs $G^{DM}=(N, E^{DM})$, $G^{DT}=(N, E^{DT})$, $G^{DP}=(N, E^{DP})$ and $G^{DW}=(N, E^{DW})$, where the edge $(p, p') \in E^{DM}$ ($(p, p') \in E^{DT}$, $(p, p') \in E^{DP}$, $(p, p') \in E^{DW}$) if and only if the operations p and p' cannot be executed in the same machining module (turret, position, machine).

- constraints on the maximal number n_0 of machines, on the maximal number m_0 of working positions at the same machine and on the maximal number b_0 of machining modules in a turret;

- constraints on feasible orientations of the part for execution of operations;

- the impossibility to perform operations from N_s at one machine by machining modules of different

types;

– productivity constraints to provide the required output.

In the following section, we propose an original mathematical formulation of this problem.

3. MIP FORMULATION

We can build set \mathbf{N}' based on graph G^{SM} . Let $G_i^{SM} = (N_i^{SM}, E_i^{SM})$, $i=1, \dots, n^{SM}$, be connectivity components of G^{SM} including isolated vertices. Only one vertex (operation) \wp_i is chosen from each set N_i^{SS} , let $\chi(p) = \wp_i$ for all $p \in N_i^{SS}$ and included into \mathbf{N}' .

Let us introduce the following notation:

- X_{pq} decision variable which is equal to 1 if operation p from \mathbf{N}' is assigned to block $q=2(i-1)m_0 + b_0 + 2(k-1)b_0 + (j-1)b_0 + l$, i.e. l -th machining module of spindle head or turret type j at the k -th position of the i -th machine
- S_i decision variable which is equal to $1/T_i$ if a vertical spindle head is installed at the i -th machine
- Y_{ikj}^s auxiliary variable which is equal to 1 if at least one operation from N_s is assigned to spindle head or turret of type j at the k -th position of the i -th machine
- Y_{ik2}^w auxiliary variable which is equal to 1 if a horizontal work head is installed at the k -th position of the i -th machine
- Y_{ik2}^h auxiliary variable which is equal to 1 if a horizontal spindle head is installed at the k -th position of the i -th machine
- Y_{ikjl} auxiliary variable which is equal to 1 if the l -th machining module of spindle head or turret type j is installed at the k -th position of the i -th machine
- $Y_{i1\min}$ auxiliary variable which is equal to k if k is the minimal position covered by vertical spindle head or turret of the i -th machine
- $Y_{i1\max}$ auxiliary variable which is equal to k if k is the maximal position covered by vertical spindle head or turret of the i -th machine
- Y_{i1h} auxiliary variable which is equal to 1 if the vertical spindle head or turret is installed at the i -th machine
- Y_{i1t} auxiliary variable which is equal to 1 if the vertical turret is installed at the i -th machine
- X_{pik11} auxiliary variable which is equal to $X_{pq} Y_{i1h}$, $q=2(i-1)m_0 + b_0 + 2(k-1)b_0 + 1$
- Z_i auxiliary variable which is equal to 1 if at least one operation is assigned to the i -th machine

- Z_{ik} auxiliary variable which is equal to 1 if at least one operation is assigned to the k -th position of the i -th machine
- h_{ri} auxiliary variable which is equal to 1 if elements of the part are machined with the r -th orientation at the i -th machine
- ρ_{ii+1} auxiliary variable which is equal to 1 if the part is reoriented between machines i and $i+1$, $i=1, \dots, n_0-1$
- F_{ikjl} an auxiliary variable for determining the time of execution of operations from \mathbf{N} in the l -th machining module of spindle head or turret type j at the k -th position of the i -th machine
- F_{ik} an auxiliary variable for determining the time of execution of operations from \mathbf{N} at the k -th position of the i -th machine
- T an auxiliary variable for determining the time of execution of all the operations from \mathbf{N} at the flow line
- T_i auxiliary variable which is equal to TZ_i
- T_{ik} auxiliary variable which is equal to TZ_{ik}

We calculate in advance parameters $t_{pp'} = \max(\lambda(p), \lambda(p'))/\min(\gamma_2(p), \gamma_2(p')) + \tau^a$. They represent the minimal time necessary for execution of operations p and p' in the same machining module. It is assumed that $(p, p') \in E^{DM}$ if $\min(\gamma_2(p), \gamma_2(p')) < \max(\gamma_1(p), \gamma_1(p'))$.

For each operation $p \in \mathbf{N}$, we calculate a set $B(p)$ of block indices from $\{1, 2, \dots, 2n_0m_0b_0\}$, a set $K(p)$ of position indices from $\{1, 2, \dots, n_0m_0\}$ and a set $M(p)$ of machine indices from $\{1, 2, \dots, n_0\}$, where operation $p \in \mathbf{N}$ can be potentially assigned.

Let $I(i)=[2(i-1)m_0b_0+1, 2im_0b_0]$, $I(i, k)=[2(i-1)m_0b_0+2(k-1)b_0+1, 2(i-1)m_0b_0+2kb_0]$, $I(i, k, j)=[2(i-1)m_0b_0+2(k-1)b_0+(j-1)b_0+1, 2(i-1)m_0b_0+2(k-1)b_0+(j-1)b_0+2kb_0]$, and $I(i, k, j, l)=[2(i-1)m_0b_0+2(k-1)b_0+(j-1)b_0+l, 2(i-1)m_0b_0+2(k-1)b_0+(j-1)b_0+l_0]$, respectively.

3.1 Cost calculation

The objective function is as follows:

$$\begin{aligned} \text{Min} \quad & \sum_{i=1}^{n_0} (C_{sm} Z_i + C_{sp} \sum_{k=1}^{m_0} Z_{ik} + \sum_{k=1}^{m_0} (C_{sh} Y_{ik2}^h + C_{sw} Y_{ik2}^w)) + (C_{st} + 2C_{mm}) \sum_{k=1}^{m_0} \sum_{j=1}^2 Y_{ikj2} + \\ & C_{mm} \sum_{k=1}^{m_0} \sum_{j=1}^2 \sum_{l=3}^{b_0} Y_{ikjl} + C_{sh} (Y_{i1t} + Y_{i1h} - \sum_{k=1}^{m_0} Y_{ik12}) + C_{vp} (Y_{i1\max} - Y_{i1\min}) + C_{ro} \sum_{i=1}^{n_0-1} \rho_{ii+1} \end{aligned} \quad (1)$$

3.2 Assignment constraints

Let $M_1 = \max\{1/\gamma_1(p) \mid p \in \mathbf{N}\}$, $M_2 = \max\{1/\gamma_2(p) \mid p \in \mathbf{N}\}$, $M(i,k,2) = |\{p \in \mathbf{N} \mid I(i,k,2) \cap B(p) \neq \emptyset\}|$.

Equations (2) provide assignment of each operation from \mathbf{N}' exactly to one machining module.

$$\sum_{q \in B(p)} X_{pq} = 1; p \in \mathbf{N}' \quad (2)$$

Expressions (3) guarantee that each predecessor p of operation p' is executed before operation p' .

$$\sum_{\substack{q \in \bigcup_{i=1}^{i-1} \bigcup_{k'=1}^{m_0} \bigcup_{j'=1}^2 I(i',k',j') \cap B(p)}} qX_{\chi(p)q} + \sum_{q \in I(i,k,j) \cap B(p)} qX_{\chi(p)q} \leq \sum_{q \in I(i,k,j) \cap B(p')} (q-1)X_{\chi(p')q},$$

$$(p,p') \in D^{OR}; p, p' \in \mathbf{N}; i \in M(p'), k \in K(p'); j=1,2 \quad (3)$$

Expressions (4) – (6) are used to model inclusion constraints for machines, positions and turrets, respectively.

$$\sum_{q \in I(i) \cap B(p)} X_{\chi(p)q} = \sum_{q' \in I(i) \cap B(p')} X_{\chi(p')q'}; (p,p') \in E^{SW}; p, p' \in \mathbf{N}; i \in I(p) \cap I(p') \quad (4)$$

$$\sum_{q \in I(i,k) \cap B(p)} X_{\chi(p)q} = \sum_{q' \in I(i,k) \cap B(p')} X_{\chi(p')q'}; (p,p') \in E^{SP}; p, p' \in \mathbf{N}; k \in K(p) \cap K(p') \quad (5)$$

$$\sum_{q \in I(i,k,j) \cap B(p)} X_{\chi(p)q} = \sum_{q' \in I(i,k,j) \cap B(p')} X_{\chi(p')q'}; (p,p') \in E^{ST}; p, p' \in \mathbf{N}; k \in K(p) \cap K(p'); j=1,2 \quad (6)$$

Expressions (7) – (10) provide exclusion constraints for machines, positions, turrets, and machining modules, respectively.

$$\sum_{q \in I(i) \cap B(p)} X_{\chi(p)q} + \sum_{q' \in I(i) \cap B(p')} X_{\chi(p')q'} \leq 1, (p,p') \in E^{DW}; p, p' \in \mathbf{N}; i \in I(p) \cap I(p') \quad (7)$$

$$\sum_{q \in I(i,k) \cap B(p)} X_{\chi(p)q} + \sum_{q' \in I(i,k) \cap B(p')} X_{\chi(p')q'} \leq 1, (p,p') \in E^{DP}; p, p' \in \mathbf{N}; k \in K(p) \cap K(p') \quad (8)$$

$$\sum_{q \in I(i,k,j) \cap B(p)} X_{\chi(p)q} + \sum_{q' \in I(i,k,j) \cap B(p')} X_{\chi(p')q'} + Y_{kj2} \leq 2, (p,p') \in E^{DT}; p, p' \in \mathbf{N}; k \in K(p) \cap K(p'); j=1,2 \quad (9)$$

$$X_{\chi(p)q} + X_{\chi(p')q} \leq 1; (p,p') \in E^{DB}; p, p' \in \mathbf{N}; q \in B(p) \cap B(p') \quad (10)$$

Equations (11) prohibit assignment of operations from N_s to machining modules of type j if there is no feasible orientation of the part for such an execution.

$$X_{\chi(p)q} = 0; p \in N_s; s=1, \dots, \eta; i \in I(p); k \in K(p); \{h_{rs}=j | r=1, \dots, \mu\} = \emptyset; q \in I(i, k, j) \cap B(p) \quad (11)$$

$$\sum_{\substack{q \in B(p) \cap \\ k \in K(p)}} \bigcup_{I(i, k, j)} X_{\chi(p)q} = \sum_{\substack{q' \in B(p') \cap \\ k \in K(p')}} \bigcup_{I(i, k, j)} X_{\chi(p')q'};$$

Equations (12) guarantee assignment of operations from N_s to the same type of spindle head or turret at the same machine.

$$p, p' \in N_s; i \in I(p) \cap I(p'); j=1, 2; s=1, \dots, \eta \quad (12)$$

Expressions (13) – (14) determine the existence of the l -th machining module of type j at the k -th position of the i -th machine.

$$Y_{ikjl} \leq \sum_{p \in \mathbf{N}, q \in I(i, k, j, l) \cap B(p)} X_{\chi(p)q}; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (13)$$

$$\sum_{p \in \mathbf{N}, q \in I(i, k, j, l) \cap B(p)} X_{\chi(p)q} \leq |N| Y_{ikjl}; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (14)$$

Equations (15) do not allow empty machining modules.

$$Y_{ikjl-1} \geq Y_{ikjl}; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2; l=2, \dots, b_0 \quad (15)$$

Equations (16) prohibit a vertical turret and horizontal machining module at the same position.

$$Y_{ik12} + Y_{ik21} \leq 1; i=1, \dots, n_0; k=1, \dots, m_0 \quad (16)$$

Expressions (17) – (20) determine the existence vertical spindle head or turret at the i -th machine.

$$Y_{i1t} + Y_{i1h} \leq \sum_{k=1}^{m_0} Y_{ik11}; i=1, \dots, n_0 \quad (17)$$

$$\sum_{k=1}^{m_0} Y_{ik11} \leq m_0(Y_{i1t} + Y_{i1h}); i=1, \dots, n_0 \quad (18)$$

$$Y_{i1t} + Y_{i1h} \leq 1; i=1, \dots, n_0 \quad (19)$$

$$Y_{i1t} = \sum_{k=1}^{m_0} Y_{ik12}; i=1, \dots, n_0 \quad (20)$$

Constraints (21) – (25) provide that the feed rate for vertical spindle head at the i -th machine will be chosen from $[\max\{\underline{\Gamma}_{ikj1} \gamma_1(p) | k=1, \dots, m_i\}, \min\{\bar{\Gamma}_{ikj1} | k=1, \dots, m_i\}]$. In fact, $X_{pik11}=1$ if and only if $Y_{i1h} = 1$ and $X_{pq}=1$ for $q=2(i-1)m_0 + 2(k-1)b_0 + 1$ due to (21) – (23). In that case $S_i \leq 1/\gamma_1(p)$ and $S_i \geq 1/\gamma_2(p)$ for $p \in N_{i1}$.

$$X_{pik11} \leq X_{pq}; i=1, \dots, n_0; k=1, \dots, m_0; q=2(i-1)m_0 b_0+2(k-1)b_0+1 \quad (21)$$

$$X_{pik11} \leq Y_{ih}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (22)$$

$$X_{pik11} \geq Y_{ih} + X_{pq} - 1; i=1, \dots, n_0; k=1, \dots, m_0; q=2(i-1)m_0 b_0+2(k-1)b_0+1 \quad (23)$$

$$S_i + M_1 X_{pik11} \leq M_1 + 1/\gamma_1(p); p \in \mathbf{N}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (24)$$

$$S_i - M_2 X_{pik11} \geq 1/\gamma_2(p) - M_2; p \in \mathbf{N}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (25)$$

Expressions (26) – (27) determine the existence of the k -th position at the i -th machine.

$$Z_{ik} \leq Y_{ik11} + Y_{ik21}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (26)$$

$$Y_{ik11} + Y_{ik21} \leq 2Z_{ik}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (27)$$

Equations (28) do not allow empty positions.

$$Z_{ik-1} \geq Z_{ik}; i=1, \dots, n_0; k=2, \dots, m_0 \quad (28)$$

$$(m_0 - k + 1)Y_{ik11} + Y_{1\min} \leq m_0 + 1; i=1, \dots, n_0; k=1, \dots, m_0 \quad (29)$$

Expressions (29) – (32) determine $Y_{1\min}$ and $Y_{i1\max}$ for the i -th machine. If there is no vertical machining at the i -th machine ($Y_{i1t} = 0, Y_{ih} = 0, Y_{ik11} = 0, k=1, \dots, m_0$), then $Y_{1\min} = 0$ and $Y_{i1\max} = 0$ due to (31) – (32).

Otherwise $Y_{i1\max} = k_{\max}^{ih}$ and $Y_{i1\min} = k_{\min}^{ih}$ due to (29) – (32) and minimizing (1).

$$Y_{i1\max} \geq kY_{ik11}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (30)$$

$$Y_{i1\max} \leq m_0(Y_{i1t} + Y_{ih}); i=1, \dots, n_0 \quad (31)$$

$$Y_{i1\min} \leq m_0(Y_{i1t} + Y_{ih}); i=1, \dots, n_0 \quad (32)$$

If $Y_{ik22}=1$ ($Y_{ik21}=1$) or $Y_{ik21}=0$ ($Y_{ik22}=0$) then $Y_{ik2}^w=0$ and $Y_{ik2}^h=0$ due to (33). If $Y_{ik21}=1$ and $Y_{ik21}=0$, then

$Y_{ik2}^w + Y_{ik2}^h = 1$ due to (33). If in this case $\sum_{p \in \mathbf{N}, q \in I(i,k,2) \cap B(p)} X_{\chi(p)q} > 1$, then (33) is valid only for $Y_{ik2}^w = 0$

since $M(i,k,2) > 1$.

$$Y_{ik2}^w + Y_{ik2}^h + Y_{ik22} - Y_{ik21} = 0, i=1, \dots, n_0; k=1, \dots, m_0 \quad (33)$$

$$M(i,k,2)Y_{ik2}^w + Y_{ik22} + \sum_{p \in \mathbf{N}, q \in I(i,k,2) \cap B(p)} X_{\chi(p)q} \leq M(i,k,2) + 1, i=1, \dots, n_0; k=1, \dots, m_0 \quad (34)$$

Constraints (35) – (37) allow execution by vertical machining modules of operations not more of one side.

$$Y_{ikj}^s \leq \sum_{p \in N_s, q \in I(i,k,j) \cap B(p)} X_{\chi(p)q}; s=1, \dots, \eta; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2 \quad (35)$$

$$\sum_{p \in N_s, q \in I(i,k,j) \cap B(p)} X_{\chi(p)q} \leq |N_s| Y_{ikj}^s; s=1, \dots, \eta; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2 \quad (36)$$

$$\sum_{s=1}^{\eta} Y_{ik1}^s \leq 1; i=1, \dots, n_0; k=1, \dots, m_0 \quad (37)$$

Expressions (38) – (39) determine the existence of the i -th machine.

$$Z_i \leq \sum_{k=1}^{m_0} Z_{ik}; i=1, \dots, n_0 \quad (38)$$

$$\sum_{k=1}^{m_0} Z_{ik} \leq m_0 Z_i; i=1, \dots, n_0 \quad (39)$$

Equations (40) do not allow empty machines.

$$Z_{i-1} \geq Z_i; i=2, \dots, n_0 \quad (40)$$

Constraints (41) – (42) provide the choice of a single feasible orientation of the part for each machine.

$$h_{ri} + Y_{ikj}^s \leq 1; r=1, \dots, \mu, i=1, \dots, n_0; k=1, \dots, m_0; s=1, \dots, \eta; j=1, 2; j \neq h_{rs} \quad (41)$$

$$\sum_{r=1}^{\mu} h_{ri} = 1; i=1, \dots, n_0 \quad (42)$$

Let $h_{ri} = 1$ and $h_{r_{i+1}i+1} = 1$. Then $\rho_{ii+1} = 1$ if $r_i \neq r_{i+1}$ due to (43) and $\rho_{ii+1} = 0$ otherwise since (1) is minimized.

$$\mu \rho_{ii+1} \geq \sum_{r=1}^{\mu} r h_{ri} - \sum_{r=1}^{\mu} r h_{r_{i+1}i+1}; \mu \rho_{ii+1} \geq \sum_{r=1}^{\mu} r h_{r_{i+1}i+1} - \sum_{r=1}^{\mu} r h_{ri}; i=1, \dots, n_0-1 \quad (43)$$

3.3 Time calculation

Expressions (44) – (45), (46) and (47) are used for estimation of execution time of operations from \mathbf{N} by the l -th machining module, vertical spindle head and at the k -th position of the i -th machine, respectively.

$$F_{ikjl} \geq t_{pp} X_{\chi(p)q}; p \in \mathbf{N}; j=1, 2; i=1, \dots, n_0; k=1, \dots, m_0; l=1, \dots, b_0; q \in I(i,k,j,l) \cap B(p) \quad (44)$$

$$F_{ikjl} \geq t_{pp'} (X_{\chi(p)q} + X_{\chi(p')q} - 1); p, p' \in \mathbf{N}; j=1, 2; \\ i=1, \dots, n_0; k=1, \dots, m_0; l=1, \dots, b_0; q \in I(i,k,j,l) \cap B(p) \cap B(p') \quad (45)$$

$$F_{ik11} \geq \lambda(p)/S_i + (\lambda(p)/\gamma_1(p) + \tau^a) X_{pik11} - \lambda(p)/\gamma_1(p); p \in \mathbf{N}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (46)$$

$$F_{ik} \geq \sum_{l=1}^{b_0} F_{ikjl} + 2\tau^s Y_{ikj2} + \tau^s \sum_{l=3}^{b_0} Y_{ikjl} + b_0 \tau^s (Y_{ikj} - 1); i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2 \quad (47)$$

Expressions (48) are used for estimation of $T(P)$.

$$T \geq F_{ik} + \tau^r; i=1, \dots, n_0; k=1, \dots, m_0 \quad (48)$$

Expressions (49) – (50) and (51) – (52) determine T_{ik} , $i=1, \dots, n_0$, $k=1, \dots, m_0$, and T_i , $i=1, \dots, n_0$, respectively.

$$T_{ik} \leq T_0/O Z_{ik}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (49)$$

$$T - T_{ik} \leq T_0/O (1 - Z_{ik}); i=1, \dots, n_0; k=1, \dots, m_0 \quad (50)$$

$$T_i \leq T_0/O Z_i; i=1, \dots, n_0 \quad (51)$$

$$T - T_i \leq T_0/O (1 - Z_i); i=1, \dots, n_0 \quad (52)$$

Constraint (53) guarantees that the total time for machining O parts does not exceed T_0 .

$$(O - 1)T + \sum_{i=1}^{n_0} (T_i + \sum_{k=1}^{m_0} T_{ik}) \leq T_0 \quad (53)$$

3.4 Bound constraints

$$X_{pq} \in \{0, 1\}; p \in \mathbf{N}'; q \in B(p) \quad (54)$$

$$Y_{ikj}^s \in \{0, 1\}; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2; s=1, \dots, \eta \quad (55)$$

$$Y_{ikjl} \in \{0, 1\}; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (56)$$

$$Y_{i1\min}, Y_{i1\max} \in \{0, 1, \dots, m_0\}; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (57)$$

$$Y_{i1t} \in \{0, 1\}; i=1, \dots, n_0 \quad (58)$$

$$Y_{i1h} \in \{0, 1\}; i=1, \dots, n_0 \quad (59)$$

$$X_{pik11} \in \{0, 1\}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (60)$$

$$Z_{ik} \in \{0, 1\}; i=1, \dots, n_0; k=1, \dots, m_0 \quad (61)$$

$$S_{i1} \in [\min\{1/\gamma_2(p) | p \in \mathbf{N}\}, \max\{1/\gamma_1(p) | p \in \mathbf{N}\}]; i=1, \dots, n_0 \quad (62)$$

$$F_{ikjl} \in [0, \bar{t} - \tau^r]; i=1, \dots, n_0; k=1, \dots, m_0; j=1, 2; l=1, \dots, b_0 \quad (63)$$

$$F_{ik} \in [0, \bar{t} - \tau^r]; i=1, \dots, n_0; k=1, \dots, m_0 \quad (64)$$

$$F_i \in [\underline{t}, \bar{t}]; i=1, \dots, n_0 \quad (65)$$

$$T \in [O\underline{t}, T_0] \quad (66)$$

$$T_{ik} \in [0, T_0]; i=1, \dots, n_0; k=1, \dots, m_0 \quad (67)$$

$$T_i \in [0, T_0]; i=1, \dots, n_0 \quad (68)$$

where $\underline{t} = \max\{\lambda(p)/\gamma_2(p) + \tau^a + \tau^r | p \in \mathbf{N}\}$; $\bar{t} = T_0 / O$.

3.5 Calculations of $B(p)$ and $K(p)$

Each row of matrix \mathbf{H} defines in one-to-one manner the orientation H and the corresponding partition of \mathbf{N} to \mathbf{N}_1 and \mathbf{N}_2 . For such a partition, we can calculate $B(p, H)$ and $K(p, H)$ using algorithms (Dolgui *et al.* 2009, Dolgui *et al.* 2006). Then, $B(p)$ and $K(p)$ are defined as $B(p) = \bigcup_{H \in \mathbf{H}} B(p, H)$ and

$K(p) = \bigcup_{H \in \mathbf{H}} K(p, H)$ respectively.

In the next section, we illustrate how the proposed model can be used in practice in order to decrease the solution time and to improve the quality of the final solution.

4. INDUSTRIAL CASE STUDY

The approach proposed in this paper will be illustrated on the following real-life industrial example. A machining line to produce the part presented in Figure 2 has to be designed.

There are 21 elements (features $H1-H21$) to be machined located on two sides of the part. Set \mathbf{N} included 64 operations. Their parameters and predecessors are presented in Table 1. Exclusion constraints for operations concerning machines, positions, turrets, and machining modules are given in Table 2 to 5, respectively. Inclusion constraints are given by families $E^{SW} = \{(1,4,8,9,11), (21,24,25,28), (29,31,32,34,35,37,38,40), (41,43,50,51,56,58), (59,61,62,64), (44,46,47,49)\}$, $E^{SP} = \emptyset$, $E^{ST} = \{(1,9,29,32,35,38), (2,6,10,13,16,19,30,33,36,39)\}$, $E^{SM} = \{(1,5), (3,7), (9,12,15,18), (11,14,17,20), (21,25), (59,62)\}$. Other parameters are: $O=140$, $m_0=5$, $\tau^a = \tau^g = \tau^r = 0.1$ min, $T_0=384$ min, $C_{sm}=30$, $C_{sp}=10$, $C_{st}=5$, $C_{mm}=2$, $C_{sh}=4$, $C_{sw}=2$, $C_{ro}=0.5$, $\mathbf{H} = \{(0, -1), (2, -1), (-1, 0), (-1, 2)\}$.

The model (1)-(68) was used for designing a flow line for machining the given part. The obtained optimal solution shown in Table 6 consists of 3 machines. The first machine includes 5 horizontal working positions

(the work head at the first position, the spindle heads at the second and third positions, the turret at the fourth position, and the spindle head at the fifth position). The second machine utilizes 4 horizontal working positions with turrets. The third machine is equipped with 5 horizontal working positions (with turrets installed at the first, second and third positions, and a spindle head at the fourth position as well as a work head at the fifth position). The part is reoriented after machining at the first machine. The total time $T(P)$ is equal to 2.393 min and the cost of the line is equal to 327.5.

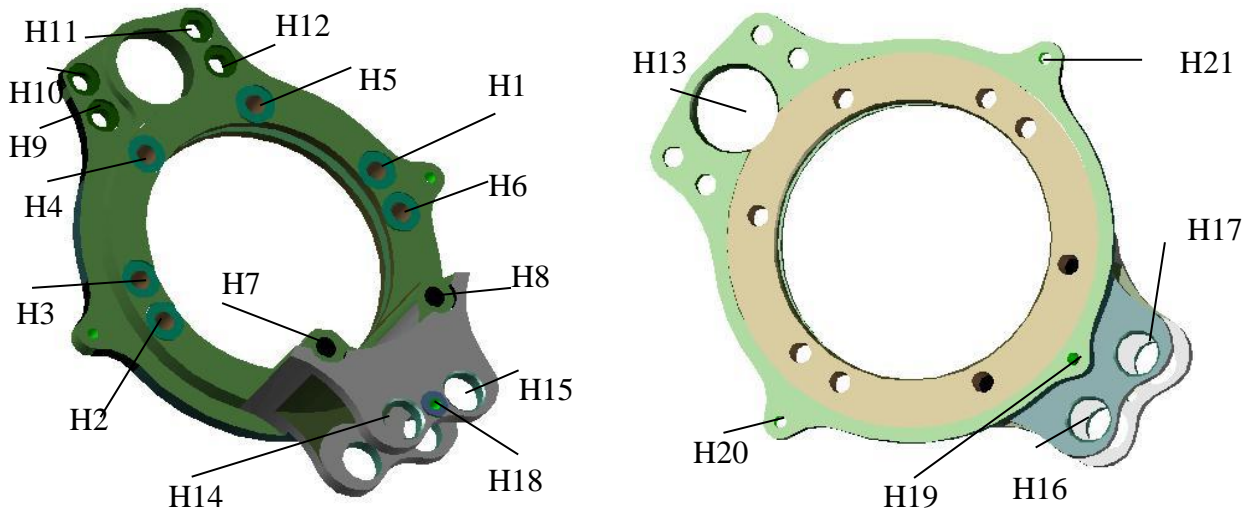


Figure 2. The part to be machined.

Table 1. Parameters of operations and their predecessors

Operation p	Hole	Side s	$\lambda(p)$ mm	$\gamma_1(p)$ mm/min	$\gamma_2(p)$ mm/min	Predecessors
1	H1	1	45	16.2	99.5	44 47
2	H1	1	7	17.5	58.5	1 5
3	H1	1	42	21.1	129.3	2 6
4	H1	1	42	23.8	143.7	3 7
5	H2	1	45	16.2	99.5	44 47
6	H2	1	7	17.5	58.5	1 5
7	H2	1	42	21.1	129.3	2 6
8	H2	1	42	23.8	143.7	3 7
9	H3	1	45	16.2	99.5	44 47
10	H3	1	7	17.5	58.5	9 12 15 18
11	H3	1	42	21.1	129.3	10 13 16 19
12	H4	1	45	16.2	99.5	44 47
13	H4	1	7	17.5	58.5	9 12 15 18
14	H4	1	42	21.1	129.3	10 13 16 19
15	H5	1	45	16.2	99.5	44 47
16	H5	1	7	17.5	58.5	9 12 15 18

17	H5	1	42	21.1	129.3	10 13 16 19
18	H6	1	45	16.2	99.5	44 47
19	H6	1	7	17.5	58.5	9 12 15 18
20	H6	1	42	21.1	129.3	10 13 16 19
21	H7	1	58	16.2	99.5	44 47
22	H7	1	41	16.2	99.5	21 25
23	H7	1	82	16.2	99.5	22
24	H7	1	8	17.4	74.9	23
25	H8	1	58	16.2	99.5	44 47
26	H8	1	41	16.2	99.5	21 25
27	H8	1	82	16.2	99.5	26
28	H8	1	8	17.4	74.9	27
29	H9	1	24	16.2	99.5	44 47
30	H9	1	7	20.2	86.4	29
31	H9	1	21	21.1	129.3	30
32	H10	1	24	16.2	99.5	44 47
33	H10	1	7	20.2	86.4	32
34	H10	1	21	21.1	129.3	33
35	H11	1	24	16.2	99.5	44 47
36	H11	1	7	20.2	86.4	35
37	H11	1	21	21.1	129.3	36
38	H12	1	24	16.2	99.5	44 47
39	H12	1	7	20.2	86.4	38
40	H12	1	21	21.1	129.3	39
41	H13	2	34	85.4	341.8	-
42	H13	2	33	8.4	49.3	41
43	H13	2	29	11.4	69	42
44	H14	1	42	13.3	89.5	-
45	H14	1	21	14.6	94.4	44 47
46	H14	1	33	17.2	116.4	45
47	H15	1	42	13.3	89.5	-
48	H15	1	21	14.6	94.4	44 47
49	H15	1	33	17.2	116.4	48
50	H16	2	18	14.6	94.4	41
51	H17	2	18	14.6	94.4	41
52	H18	1	30	17.7	106.1	44 47
53	H18	1	7	26.3	112.3	52
54	H18	1	13	20.1	123.2	53
55	H18	1	29	239.3	478.5	4 8 11 14 17 20 24 28 31 34 37 40 46 49 54
56	H19	2	39	17.7	106.1	41
57	H19	2	13	20.1	123.2	56
58	H19	2	28	239.3	478.5	43 50 51 57 60 63
59	H20	2	23	17.7	106.1	41
60	H20	2	13	20.1	123.2	59 62
61	H20	2	24	239.3	478.5	43 50 51 57 60 63
62	H21	2	23	17.7	106.1	41
63	H21	2	13	20.1	123.2	59 62
64	H21	2	24	239.3	478.5	43 50 51 57 60 63

Table 2. Incompatibility of operations in one machine

Operations	Incompatible operations
21 22 23 26 27	1 2 3 16 19
29 30 39	21 22 23 26 27
41 42	1 2 3 16 19 21 22 23 26 27 30 39
45 48	1 2 3 16 19 30 39 41
50 51	1 2 3 16 19 21 22 23 26 27 30 39 44 45 48 49
52 53 54	1 2 3 16 19 30 39 41 42 50 5
55	21 22 23 26 27 41 42 44 45 48 49 50 51 52 53 54
56 57 58 59 60 63	1 2 3

Table 3. Incompatibility of operations in one position

Operations	Incompatible operations
7	4
8	3
11 31 34 37 40 45 46 48 49 54	4 8
55	1 2 3 8 11 19 24 25 28 44 46 47 48 52 53
58	50 51 56
59 60 62 63	58
61	42 43 50 51 56 57 59
64	42 43 50 51 56 57 62

Table 4. Incompatibility of operations in one turret

Operations	Incompatible operations
22 23	21
24 25	22 23
26 27	21 24 25
28 44 46 47 49	22 23 26 27
50 51	41 42 43
52 53	22 23 26 27
56	41 42
57	41 42 43
59	41
62	41 59 60
63	59 60
64	61

Table 5. Incompatibility of operations in one machining module

Operations	Incompatible operations
9 10	5 6 7
11	7
18 19	1 2 3
20	3
29 30	12 13
32	13 29 30

33	12 13 29 30
34	31
35	16
36	15 16
38 39	15 16 35 36
40	37
44	21 24
45	21 22 23 24
47	25 28 44 45
48	25 26 27 28 44 45 46
49	45 46
51	50
52 53	44 45 47 48
54	44 45 46 47 48 49
56 57	50 51

Table 6. Obtained solution

Set N_{ijkl}	Operations	$L(N_{ijkl})$	Γ_{ijkl}	$t^b(P_{ijkl})$	$t^p(P_{ik})$
N_{1121}	41	24.1	242.1	0.199	0.299
N_{1221}	42 51 59 62	17.3	25.5	0.777	0.877
N_{1321}	43 50 60 63	15.5	36.9	0.52	0.62
N_{1421}	56	22.8	61.9	0.468	
N_{1422}	57	7.5	70.3	0.207	0.975
N_{1521}	58 61 64	28	287.1	0.217	0.317
N_{2121}	44	21.8	46.4	0.569	
N_{2122}	47	21.8	46.4	0.569	
N_{2123}	21 25	33.1	56.7	0.683	2.221
N_{2221}	26 45	22.2	51.1	0.534	
N_{2222}	22 48	22.2	51.1	0.534	
N_{2223}	23 27	46.7	56.7	0.924	2.393
N_{2321}	46	17.1	60.4	0.384	
N_{2322}	49	17.1	60.4	0.384	1.067
N_{2421}	24 28 52	26.8	61.9	0.534	
N_{2422}	53	4	63.8	0.162	
N_{2423}	54	7.5	70.3	0.207	1.303
N_{3121}	9 12 15 18 32 35	25.6	56.7	0.552	
N_{3122}	1 5 29 38	25.6	56.7	0.552	1.405
N_{3221}	2 6 13 16	4.1	34	0.22	
N_{3222}	10 19 30 39	4.1	34	0.22	
N_{3223}	33 36	4	49.1	0.181	1.02
N_{3321}	3 7 31 37	24	73.8	0.425	
N_{3322}	11 14 17 20 34 40	24	73.8	0.425	1.15
N_{3421}	4 8	19.6	67.1	0.392	0.492
N_{3521}	55	29	287.1	0.221	0.321

In Table 7, we present the size of the corresponding MIP model and the computation results obtained with the academic version of Gurobi 9.5.1 for $n_0=3, 4, 5$ with available solution time of 1 hour. In this table, MIP corresponds to model (1) – (68) without providing an initial solution and MIP_h corresponds to the case when an initial heuristic solution with the objective value of 344.5 was given to Gurobi solver.

Table 7. Computation results

n_0	Number of variables	Number of constraints	Cost		Bound		Solution time, sec	
			MIP	MIP _h	MIP	MIP _h	MIP	MIP _h
3	7794	240718	327.5	327.5	327.5	327.5	229.6	231.4
4	10391	320972	327.5	327.5	214.5	222	3600	3600
5	12989	401227	-	327.5	206.5	207	3600	3600

The obtained results show that the problem can be solved easily for $n_0=3$, but the size of the model increases significantly for $n_0=4$ and 5. However, with an initial heuristic solution even far from the optimum, a better solution can be found by the solver. Therefore, if an ad hoc solution exists, it can be fed to the solver to check if a better solution can be found due to the optimisation.

5. EXPERIMENTAL STUDY

The purpose of these additional tests is to evaluate the effectiveness of the proposed approach. We compare the results to industrial problems and random generated tests. The characteristics of 20 industrial problems are presented in Table 8. In this table, $|\mathbf{N}|$ is the number of operations, OS^{OR} is the order strength of precedence constraints, OS^{DM} , OS^{DT} , OS^{DP} , OS^{SM} , OS^{ST} , OS^{SP} , and OS^{SW} are the densities of graphs G^{OR} , G^{DM} , G^{DT} , G^{DP} , G^{SM} , G^{ST} , G^{SP} , and G^{SW} respectively, and NO is the number of rows of matrix \mathbf{H} . Experiments were carried out on Lenovo notebook (1.80 Ghz, 16 Gb RAM) with the academic version of Gurobi 9.5.1. We solve the problems using depth first searching shortest path (DF), decomposition approach (DA), as well as MIP , MIP_h , and MIP_b with maximal available time of 1 hour. DF and DA are adopted from (Battaïa et al. 2014) to take into account the possible reorientation of the part. MH represents a heuristic which provides initial solutions for Gurobi. MH differs from DF in solving subproblems. The first feasible solution is taken for MH . MIP_b differs from MIP_h in providing the initial solution. The MH solution is used for MIP_h and the best of DF and DA solutions is used for MIP_b . Results obtained with the default

values of Gurobi' parameters are presented in Table 9.

Table 8 Parameters of industrial problems

Problem	N	OS^{OR}	OS^{DM}	OS^{DT}	OS^{DP}	OS^{DW}	OS^{SM}	OS^{ST}	OS^{SP}	OS^{SW}	NO	n_0	m_0
1	24	0.163	0.638	0.029	0.000	0.000	0.025	0.000	0.000	0.000	4	2	3
2	24	0.210	0.717	0.000	0.000	0.000	0.033	0.000	0.000	0.000	4	2	3
3	24	0.163	0.000	0.312	0.098	0.467	0.025	0.000	0.000	0.000	4	3	4
4	24	0.210	0.022	0.507	0.123	0.217	0.033	0.000	0.000	0.000	4	3	4
5	32	0.133	0.782	0.000	0.000	0.000	0.022	0.000	0.000	0.000	8	3	3
6	32	0.133	0.040	0.508	0.085	0.234	0.022	0.000	0.000	0.000	8	4	3
7	34	0.139	0.756	0.012	0.000	0.000	0.023	0.000	0.000	0.000	10	3	3
8	34	0.118	0.756	0.009	0.000	0.000	0.011	0.000	0.000	0.000	10	3	4
9	34	0.139	0.007	0.581	0.078	0.230	0.023	0.000	0.000	0.000	10	4	4
10	34	0.118	0.007	0.581	0.078	0.230	0.011	0.000	0.000	0.000	10	5	4
11	59	0.152	0.385	0.065	0.000	0.000	0.020	0.000	0.000	0.000	4	3	4
12	59	0.139	0.067	0.036	0.060	0.506	0.009	0.000	0.000	0.000	4	4	4
13	61	0.164	0.374	0.072	0.000	0.000	0.016	0.000	0.000	0.000	4	3	4
14	61	0.154	0.084	0.048	0.060	0.486	0.009	0.000	0.000	0.000	4	4	4
15	64	0.159	0.414	0.080	0.000	0.000	0.018	0.000	0.000	0.000	4	4	4
16	64	0.148	0.081	0.046	0.058	0.516	0.008	0.000	0.000	0.000	4	4	4
17	68	0.173	0.431	0.093	0.000	0.000	0.017	0.000	0.000	0.001	4	3	4
18	68	0.163	0.101	0.409	0.249	0.000	0.007	0.000	0.000	0.000	4	6	4
19	68	0.163	0.100	0.119	0.072	0.468	0.007	0.000	0.000	0.000	4	5	4
20	68	0.163	0.104	0.269	0.067	0.320	0.007	0.000	0.000	0.000	4	4	4

Table 9 Experimental results

Test	Cost						Bound			Solution time, sec					
	DF	DA	MH	MIP	MIP_h	MIP_b	MIP	MIP_h	MIP_b	DF	DA	MH	MIP	MIP_h	MIP_b
1	147.5	147.5	147.5	136.5	136.5	136.5	136.5	136.5	136.5	0.7	1.1	0.04	1.58	1.45	1.47
2	156.5	152.5	156.5	151.5	151.5	151.5	151.5	151.5	151.5	0.5	1	0.02	9.16	8.01	9.37
3	193	244.5	193	184	184	184	184	184	184	0.8	0.8	0.07	2.2	2.2	2.14
4	220	220	221.5	209.5	209.5	209.5	209.5	209.5	209.5	0.4	1	0.02	11.9	10.11	12.11
5	216	208	222	177.5	177.5	177.5	177.5	177.5	177.5	7.3	2.4	0.12	67.2	99.0	75.7
6	277.5	273.5	282	252.5	252.5	252.5	252.5	252.5	252.5	7.4	2.3	0.12	191.9	168.4	158.9
7	209	205	213	190	190	190	190	190	190	1.3	2.3	0.06	147.9	236.4	201.9
8	209	172.5	207	172.5	172.5	172.5	172.5	172.5	172.5	1.3	2.8	0.07	540.6	441.3	468.6
9	317	279.5	300	225	225	225	225	225	225	1.1	2.6	0.06	169.5	201.0	180.0
10	313	270.5	302	225	225	225	225	225	225	1.2	2.3	0.06	313.6	313.9	315.8
11	238	209.5	240	201.5	201.5	201.5	201.5	201.5	201.5	11.7	24.9	2.78	802.4	952.9	739.8
12	302.5	382	303.5	262.5	262.5	262.5	262.5	262.5	262.5	7.4	7.7	1.73	1296	1408	570.5
13	267	265	279	247.5	247.5	247.5	223	222.5	216.5	32.5	35.2	10.17	3600	3600	3600.
14	316.5	405.5	328.5	295.5	295.5	295.5	295.5	295.5	295.5	10	7.9	2.94	1944	2527	1322.
15	308.5	296	329.5	282	282	282	219.5	205.5	170	34.5	28.8	9.14	3600	3600	3600
16	330.5	403	342.5	302.5	302.5	304	281.5	276.5	285.5	10.1	8	2.95	3600	3600	3600

17	290.5	294.5	296.5	283.5	283.5	283.5	225	212.5	233	28.6	30.9	25.58	3600	3600	3600
18	424.5	456.5	489.5	-	463.5	348	166	165	168.5	34.7	16.4	12.81	3600	3600	3600
19	411	474.5	427	-	371.5	374	236.5	212.5	201.5	8.7	5	1.85	3600	3600	3600
20	358.5	424.5	371	-	329	334	232	244	230	10.6	12.8	2.43	3600	3600	3600

In Table 10, we present computation results for three series of 50 randomly generated problems. If all problems have not been solved by *MIP*, we do not calculate the minimal, maximal and average gaps as well as the minimal, maximal and average improvements of *DF* and *DA*. The improvement is calculated as $(C_1/C_2 - 1) * 100$, where C_1 is equal to the cost of *DF* or *DA* and C_2 is equal to the cost of *MIP*, *MIP_h*, or *MIP_b*, respectively. The negative improvement means that the solution found by *MIP_h* is worse than the solutions of *DF* or *DA*.

Table 10 Experimental results for random generated problems

Number of features to be machined	10	15	20
Minimal number of operations	27	40	55
Maximal number of operations	47	65	83
Average number of operations	34	52	69
Number of not solved problems by <i>MIP</i>	0	1	8
Number of not solved problems by <i>MIP_h</i>	0	0	0
Number of not solved problems by <i>MIP_b</i>	0	0	0
Number of optimally solved problems by <i>MIP</i>	49	21	11
Number of optimally solved problems by <i>MIP_h</i>	49	20	11
Number of optimally solved problems by <i>MIP_b</i>	49	35	21
Minimal time solution of <i>DF</i> , sec	1.4	3.9	9.3
Maximal time solution of <i>DF</i> , sec	13.9	122.8	347.9
Average time solution of <i>DF</i> , sec	5.39	24.71	66.61
Minimal time solution of <i>DA</i> , sec	0.2	0.8	2.5
Maximal time solution of <i>DA</i> , sec	20.4	61.3	44.9
Average time solution of <i>DA</i> , sec	5.15	16.82	24.31
Minimal time solution of <i>MIP</i> , sec	0.19	18.89	10.39
Maximal time solution of <i>MIP</i> , sec	3600	3600	3602
Average time solution of <i>MIP</i> , sec	218.2	2552	3202
Minimal time solution of <i>MIP_h</i> , sec	0.17	20.86	32.99
Maximal time solution of <i>MIP_h</i> , sec	3600	3600	3600
Average time solution of <i>MIP_h</i> , sec	176.9	2601	3131
Minimal time solution of <i>MIP_b</i> , sec	0.17	20.2	9.75
Maximal time solution of <i>MIP_b</i> , sec	3600	3600	3600
Average time solution of <i>MIP_b</i> , sec	139.7	1757	2671
Minimal time to reach best value of <i>MIP</i> , sec	0.19	4	10
Maximal time to reach best value of <i>MIP</i> , sec	807	3600	3600
Average time to reach best value of <i>MIP</i> , sec	94.44	1530	2331
Minimal time to reach best value of <i>MIP_h</i> , sec	0.17	2	6
Maximal time to reach best value of <i>MIP_h</i> , sec	398	3172	3487
Average time to reach best value of <i>MIP_h</i> , sec	65.9	1186	1714
Minimal time to reach best value of <i>MIP_b</i> , sec	0.17	2	4

Maximal time to reach best value of MIP_b , sec	769	2978	3600
Average time to reach best value of MIP_b , sec	54.26	602.9	1102
Minimal gap of MIP , %	0	-	-
Maximal gap of MIP , %	2.86	-	-
Average gap of MIP , %	0.06	-	-
Minimal gap of MIP_h , %	0	0	0
Maximal gap of MIP_h , %	3.64	46.54	53.07
Average gap of MIP_h , %	0.07	11.5	13.19
Minimal gap of MIP_b , %	0	0	0
Maximal gap of MIP_b , %	2.60	42.52	45.25
Average gap of MIP_b , %	0.05	6.02	6.63
Minimal improvement of DF by MIP , %	0	-	-
Maximal improvement of DF by MIP , %	20.37	-	-
Average improvement of DF by MIP , %	6.14	-	-
Minimal improvement of DF by MIP_h , %	0	0	-28.32
Maximal improvement of DF by MIP_h , %	20.37	37.86	45.41
Average improvement of DF by MIP_h , %	6.14	9.19	12.51
Minimal improvement of DF by MIP_b , %	0	0	0
Maximal improvement of DF by MIP_b , %	20.37	37.86	52.78
Average improvement of DF by MIP_b , %	6.14	9.65	15.33
Minimal improvement of DA by MIP , %	0	-	-
Maximal improvement of DA by MIP , %	29.81	-	-
Average improvement of DA by MIP , %	4.03	-	-
Minimal improvement of DA by MIP_h , %	0	-4.09	-23.46
Maximal improvement of DA by MIP_h , %	29.81	21.51	26.4
Average improvement of DA by MIP_h , %	4.03	4.41	7
Minimal improvement of DA by MIP_b , %	0	0	0
Maximal improvement of DA by MIP_b , %	29.81	21.51	26.4
Average improvement of DA by MIP_b , %	4.03	4.84	9.64

The obtained results clearly show that the complexity of the problem increases with the number of features to be machined. It can be also seen in Figures 3 to 5 which show the evolution of the optimality gap over the time for different number of features.

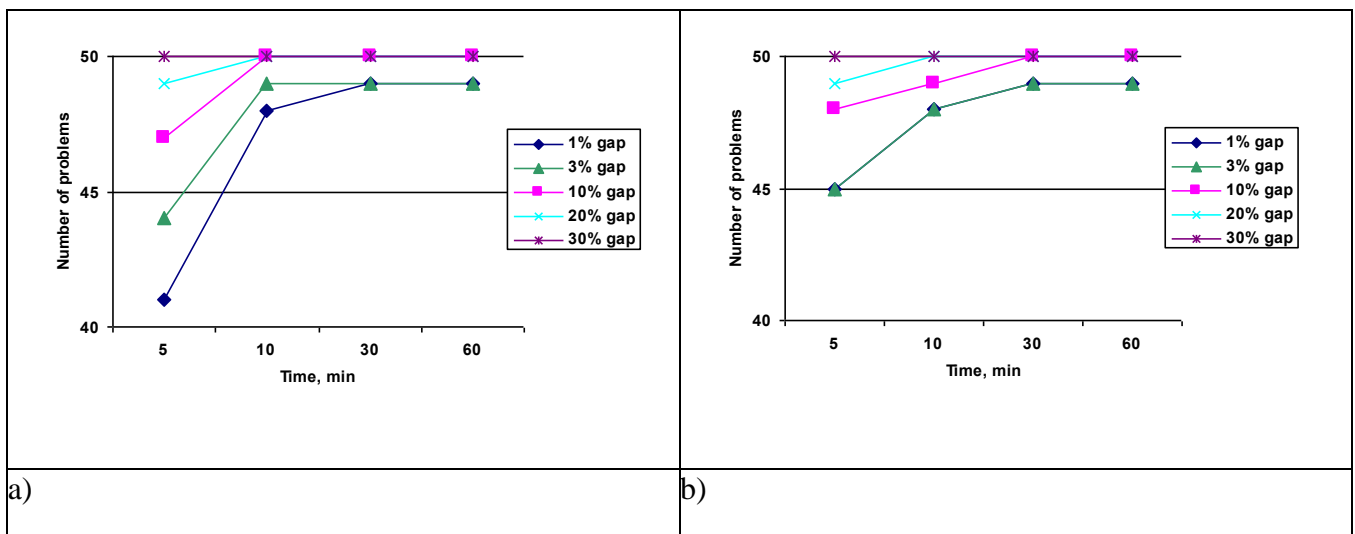


Fig. 2. Diagrams of MIP_h (a) and MIP_b (b) for 10 features

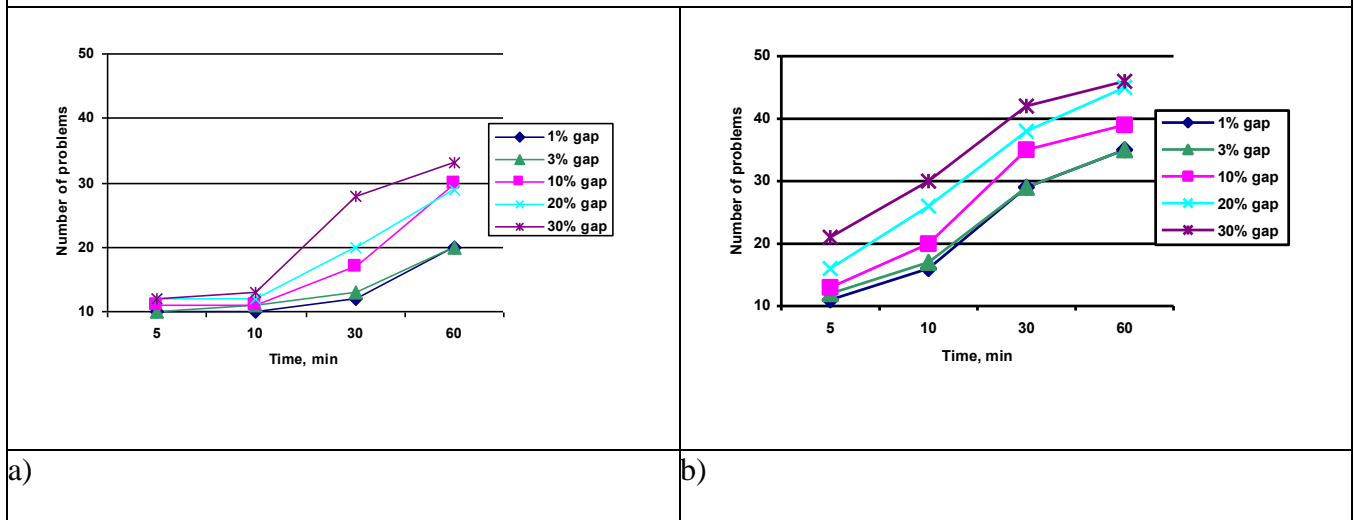


Fig. 3. Diagrams of MIP_h (a) and MIP_b (b) for 15 features

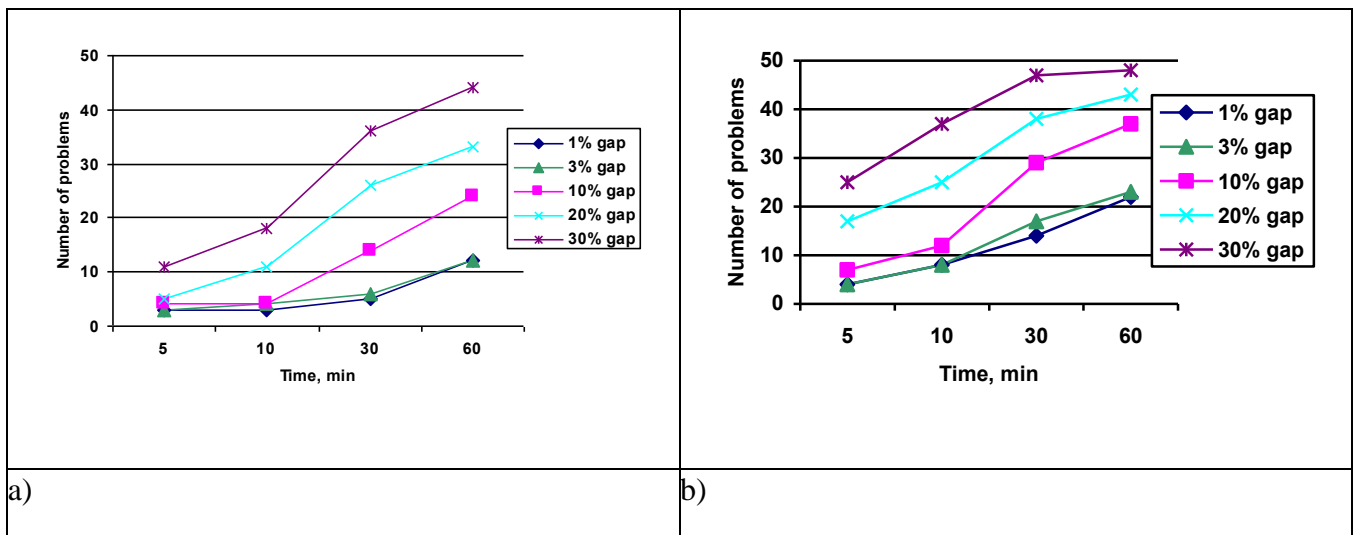


Fig. 4. Diagrams of MIP_h (a) and MIP_b (b) for 20 features

We further explore for MIP_b the impact of different values of Gurobi parameters $MIPFocus$ and $ImproveStartTime$, for the instance of 10 features which have not been solved to optimality. For this instance, for $MIPFocus=1$ (the priority is given to good quality feasible solutions), the obtained gap was 3.6458%. For $MIPFocus=2$ (the priority is given for proving the optimality), the obtained gap was 1.5625%. For $MIPFocus=3$ (the best objective bound is moving very slowly), the obtained gap was 0.5208%. The $ImproveStartTime$ parameter is used in order to modify the behavior of research after the specified time has elapsed shifting from proving optimality to finding better feasible solutions from that point onward. For the tested instance, for $ImproveStartTime=1800$ seconds, the obtained gap was 7.8125%.

In Table 11, we present results for other hard problems (the gap obtained with the default Gurobi parameters was not less than 20%). In this table, *NV* and *NC* are the numbers of variables and constraints of MIP model, *OBJ* is the cost of the obtained solution, *BND* and *GAP* are the lower bound on the optimal cost and the gap between *OBJ* and *BND* reported by Gurobi. Index 3 corresponds to MIPFocus=3. We indicate in bold the best value obtained for the objective function.

Table 11 Experimental results for hard problems

<i>N</i>	<i>NV</i>	<i>NC</i>	<i>OBJ</i>	<i>BND</i>	<i>GAP, %</i>	<i>OBJ</i> ₃	<i>BND</i> ₃	<i>GAP</i> _{3, %}
15 features								
58	5493	127881	208.5	148.5	28.777	214	155	27.5701
58	5253	133112	174	123	29.3103	174	134	22.9885
51	4783	101420	209	145	30.622	209	142	32.0574
65	4327	133404	205	167	18.5366	215	153.5	28.6047
58	3065	77607	160.5	134.5	16.1994	170.5	133	21.9941
49	4523	105540	184	128	30.4348	184	120	34.7826
50	4533	102459	200.5	158.5	20.9476	199.5	167.5	16.0401
57	5163	120693	190.5	109.5	42.5197	190.5	143	24.9344
54	4733	118157	173	116.5	32.659	173	141.5	18.2081
20 features								
72	5793	238208	191	147	23.0366	153	151	1.3072
69	9485	331577	279.5	153	45.2594	242.5	212	12.5773
64	5473	188142	158	131.5	16.7722	158	158	0
71	4951	186586	194.5	166.5	14.3959	196.5	177.5	9.6692
75	6783	259225	206.5	151	26.8765	209.5	182.5	12.8878
64	5873	195975	192.5	147	23.6364	197.5	179.5	9.1139
70	6333	232326	240.5	210.5	12.474	240.5	223	7.2765
77	6483	266564	197	148	24.8731	197	180.5	8.3756
Industrial examples								
61	5009	185722	247.5	206.5	16.5657	247.5	195	21.2121
64	6403	271783	282	223.5	20.7447	290.5	203	30.1205
68	5189	231485	283.5	233	17.8131	283.5	219	22.7513
68	11049	391284	398.5	168.5	57.7164	354	197	44.3503
68	11049	392134	372.5	201.5	45.906	381.5	250	34.4692
68	8839	332441	334	230	31.1377	339	241.5	28.7611

We can see that for 20 instances, the use of default parameters of Gurobi provided the best value of the objective function while the use of different parameters of Gurobi provided the best value for 13 instances. In total, the solution for 5 instances could be improved due to the use of MIPFocus=3 and ImproveStartTime=1800 seconds. If the available computing time is not too limited, the best option in practice is to launch the model twice with the different parameters and to keep the best solution at the end.

6. CONCLUSIONS

In this paper, a very complex optimisation problem for preliminary design of machining lines was considered

dealing with line balancing the lines equipped with multi-positional machines, rotary tables and vertical and horizontal machining modules. Both combinatorial decisions for assignment of operations and pieces of equipment to working positions as well as cutting mode selection for pieces of equipment were taken into account. For such a problem, the number of possible solutions increase extremely quickly and to master this complexity and to increase the efficiency of lines and decrease their cost, advanced decision aid tools are necessary.

For this problem, we developed a novel powerful mathematical model allowing us to take efficiently into account more parameters to consider at the design stage. The model integrates many advanced techniques of mixed integer linear programming and combines profound knowledges on the properties of such lines, problem-oriented design approaches, focused engineering techniques and cutting-edge optimisation skills.

An extensive numerical experiment has been conducted on a large set of real-life instances from industry and randomly generated instances in order to analyse the performances of the model. The obtained results show that even for very difficult and large-scale problems, good quality solutions can be obtained within 1 hour of calculation. The allowed calculation time can be reduced for smaller scale instances. Thus, the model can be effectively used in real life design procedures to reduce the line cost at the preliminary stage of line design when essential structural and functional parameters of designed line are decided.

The overall approach, specific modelling knowledges, developed techniques to increase the efficiency of the model, optimisation insights, etc. presented in this paper could be also useful for scholars working on modelling other complex real life mixed combinatorial problems in production and logistics.

The proposed model is deterministic, in the future research, we are planning to develop robust optimisation approaches for the preliminary design stage as well as to consider the line exploitation phase in order to treat unseen events that can prevent the line from its designed functioning.

The paper is an successful example how production research is function: starting from a real life problem in industry with all its complexity, the problem interests a large community of decision makers, after understanding all the details of engineering approaches and challenges of the problem staying not solved, a serious scientific study is realized based on a solid background in decision aid mathematics, and an applied decision aid tool is developed which can be used in large number of applications in industry and able to produce significant help for decision makers. Such an approach was often used in the research work of Dr. Jean-Marie Proth.

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