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# Mathematical praxeologies in the Chinese and Singaporean secondary school textbooks: The case of probability

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*This paper presents a so-called praxeological analysis of probability content in secondary school textbooks, based on the notion of praxeology from the Anthropological Theory of the Didactic. The first chapter on probability in one Chinese and one Singaporean mathematics textbook is selected to analyse separately the tasks and corresponding techniques exposed in examples or proposed as exercises, and the theoretical parts round in the exposition of the main text. It can be found that the two textbooks contain three shared types of tasks and demonstrate similar techniques to solve the same tasks. The difference is that PEP covers more types of tasks, while DM provides more examples and exercises. Also, both textbooks indeed discuss mathematical theory, not just explaining how to solve tasks, but PEP has more formal theoretical description based on set theoretical notation.*

*Keywords: Textbooks, praxeology, probability.*

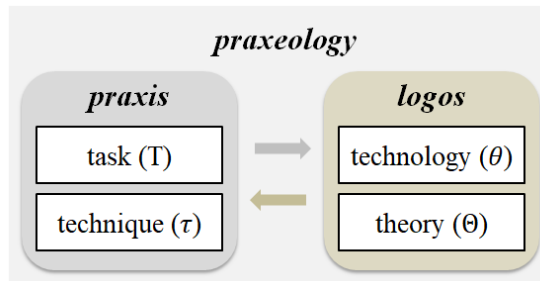
## Introduction

There is no doubt that probability has been an indispensable part of real life and has wide applications in medicine, genetic theory, game strategy, the insurance industry and many other areas. Probability has become fundamental to understanding the events and random phenomena that permeate everyday life (Gal, 2005). Thus all countries in the world regard probability as one of the core modules in mathematics curriculum, such as secondary school mathematics curriculum in China and Singapore, and attach importance to the teaching and learning of probability. As the carrier of specific course content, textbooks are a main resource in mathematics education, which make a significant contribution to the way teachers teach, influencing students' learning of a particular concept (e.g., Stein et al., 2007). So it is important to investigate curricular materials and the potential of such resources to impact students' opportunities to learn probability (Jones & Tarr, 2007).

As a major component in the textbooks, tasks of probability have received the attention of many researchers. Related studies mainly focused on the discussion of the design characteristics of tasks in the textbooks, such as representational forms, cognitive demand levels and contextual features of probability tasks (e.g., Yang & Sinatra, 2019). Such a content analysis focuses only on specific tasks themselves, but does not describe the specific characteristics in theory and practice of the probability content itself. Fittingly, the Anthropological Theory of the Didactic (hereafter "ATD") is trying to questioning what mathematics is about. It can allow us to map, model, and analyze the mathematical choices and didactic choices of the author (dos Santos Verbisck & Bittar, 2021). Thus the present study was set up to apply ATD to textbook analysis so as to explore what resources textbooks provide and how they prepare for teaching and learning probability.

## The notion of praxeology from ATD

ATD as proposed by Chevallard (1999) uses the construct of praxeology as its most basic unit of analysis, which hypothesizes that any activity related to the production, diffusion, or acquisition of knowledge should be interpreted as an ordinary human activity. A praxeology is a model of some specific amalgam of human practice and theory. Figure 1 shows praxis and logos which form the praxeology. Praxis, i.e. the practical block, consists of a task (T) and technique ( $\tau$ ) used to achieve or solve the task. T refers to the goal to be achieved by carrying out one or more action, while  $\tau$  refers to the steps necessary to carry out the task. Tasks presented by textbooks pose a variety of goals which are required to use different techniques to achieve. Each technique entails different actions to select and use mathematical information (Cavalcante & Huang, 2022). Logos, i.e. the theoretical block, is made of a technology ( $\theta$ ) to justify the technique as a valid way of performing T and a wider discourse existing in a coherent body of knowledge to explaining the technology, called theory  $\Theta$ .



**Figure 1: Praxis and logos of the praxeology**

Altogether, a praxeology consists of a quadruple  $(T, \tau, \theta, \Theta)$ , which models an amalgam of human practice and knowledge. An example of such modeling of how the Chinese textbook treats the topic of sample space and random event is presented as follows. A broad type of task can be to determine the sample space  $\Omega$  of a random experiment. Listing or counting all possible outcomes of this random experiment can be done using a variety of techniques ( $\tau$ ). The notions of sample point and sample space belong to the theoretical discourse, including their definition: “Each possible outcome of a random experiment is called a sample point, and the collection of all sample points is called the sample space of experiment (translated from the Chinese textbook)”. Such theoretical notions may also be important in explanations of more specific techniques to solve tasks of type T.

## Method and research questions

In TIMSS, textbook analysis focused on investigating the content profiles of textbooks (Schmidt et al., 1997). With the deepening of textbook research, the content of textbook analysis became more and more diversified and specific, including mathematical practice and praxeological reference model at task level (e.g., Wijayanti & Winsløw, 2017). Based on the method as in Wijayanti and Winsløw (2017) further described, we firstly analyse all examples in the main text to identify the techniques the textbooks present students with. Techniques are explicit in the demonstrated solutions of examples. Then the exercises are solved and analyzed in terms of the types of tasks found in examples. During this process, if there appear new types of tasks, these types will be added to describe and to classify all exercises precisely and objectively. For each type, we count the number of tasks. If a worked example or exercise contains multiple questions, we count each question as a task. After that, the exposition in

the main text is analyzed to reveal how textbooks discuss mathematics at the theory level, including what the specific mathematical knowledge is and how it develops and is presented.

Also international comparative studies of textbooks are now also increasingly, so as to understand and learn from other countries. A cross-national comparison of mathematics textbooks can provide insight into what and how mathematical content is presented and taught in different educational systems (Han et al., 2011). In this study, secondary school textbooks from China and Singapore (hereafter “PEP” and “DM”) were selected to make a comparison on how probability is dealt with in the textbooks. As both countries have national authorization of textbooks, these two textbooks can be considered to be in accordance with the national mathematics curriculum in each country. And these two textbooks can be considered as representative by considering their popularity and wide use. The details of textbooks selected for this study are displayed in Table 1.

**Table 1: Details of mathematics textbooks from China and Singapore**

Country	The chapter in textbook	Abbreviation
China	Chapter 10 (pp. 225-264), published by People’s Education Press in 2019	PEP
Singapore	Chapter 12 (pp. 349-399), published by Oxford University Press and Star Publishing Pte Ltd in 2020	DM

The first chapter of probability in two textbooks (see Table 1 above) was chosen for analysis, rather than all the content of probability appearing in secondary school. Thus the study addressed the following two research questions, i.e., (1) What are the mathematical praxeologies developed in the first chapter of probability in PEP and DM? (2) What are the similarities and differences between these praxeologies in the two textbooks?

## Results

### The practice block in Chinese and Singaporean mathematics textbooks

Types of tasks related to probability in the analyzed chapter can be classified into three clusters: identify sample space and random event, calculate probability, and estimate probability by using frequency. Also, the chapter presents a few exceptional tasks that do not belong to these eight main types; they are typically more theoretical. As shown in table 2, the two textbooks contain three shared types of tasks, i.e.  $T_1$ ,  $T_2$  and  $T_6$ . Compared with DM, PEP additionally provides types of tasks related to “relationship between events”, “mutual independence between events” and “statistical probability” in the worked example and exercises. And it should be pointed out that, although type of tasks related to statistical probability do not appear in DM, it contains a subsection entitled “CLASS ACTIVITY 3” for having students to compare and discuss the experimental and theoretical values of probability through computer simulation.

The two textbooks demonstrate similar technique to solve the first two types of tasks ( $T_1$ ,  $T_2$ ), both requiring students to represent all possible outcomes of the random experiment to determine the sample space. And the technique of  $T_2$  is based on that of the previous type of tasks. Also, there are two specific ways to perform  $\tau_1$  related to  $T_1$ , that is, listing all possible outcomes directly or using a

sample space diagram, the latter is suitable for the random experiment involving two processes. DM clearly divides random events into single events and simple combined events, and thus specifically mentions the term of sample space diagram in the main text to help students to represent possible outcomes of simple combined events; while PEP only presents how to apply the sample space diagram in the related worked examples.

In addition, we further counted the total number of examples and exercises belonging to each type of tasks in each textbook. It can be found that the total number of tasks provided in DM is more than that in PEP, where the former has 208 tasks and the latter has 161 tasks. While PEP covers more types of tasks, DM provides a greater number of examples and exercises. Besides, except for  $T_2$ , the number of tasks belonging to the other two types of tasks in DM is more than that in PEP, especially for  $T_6$ . Statistics show that the proportion of  $T_6$  in PEP and DM reaches 43.48% and 82.69% respectively, thus calculating the probability of a random event ( $T_6$ ) is the dominant type of tasks in both textbooks, in which students are required to find the probability of a certain event in the context of a given random experiment.

It is worth noting that the techniques for  $T_6$  can be mainly classified into two categories: one is direct calculation based on the classical combinatorial definition of probability ( $\tau_{61}$ ), which requires students to count the number of possible outcomes contained in the random experiment and that in the corresponding event; the other is indirect calculation based on applying the basic properties of probability ( $\tau_{62}$ ), requiring students to firstly identify the relationship between the event asked to calculate the probability with other events. The relationship between different events that students need to identify in PEP include mutually exclusive, complementary or mutually independent; while that in DM only includes the first two.

Moreover, according to the specific probabilistic laws that are applied, the second class of techniques  $\tau_{62}$  for  $T_6$  can be further classified in four categories: (1) it is possible to obtain the probability of the event opposite to the event whose probability is to be calculated, then the corresponding technique is to substitute  $P(\bar{E})=1-P(E)$  for calculation, sometimes  $P(E)$  needs to be calculated firstly; (2) the event for which the probability is to be calculated can be identified as two mutually exclusive events, then the corresponding technique is based on the rule  $P(A \cup B)=P(A)+P(B)$ , sometimes students need to calculate  $P(A)$  and  $P(B)$  firstly; (3) the technique of this type is a combination of (1) and (2), consisting of three key steps, one is to identify  $E=A \cup B$ , the second is to substitute  $P(A \cup B)=P(A)+P(B)$  for calculating  $P(E)$ , and the third is to use  $P(\bar{E})=1-P(E)$  for the calculation of the final result; (4) the technique of this type is a combination of (1) and  $\tau_5$ , consisting of three key steps, one is to identify  $E=A \cap B$ , the second is to substitute  $P(A \cap B)=P(A)P(B)$  for calculating  $P(E)$ , and the third is to use  $P(\bar{E})=1-P(E)$  for the calculation of the final result. It should be noted that the type of tasks solved by this technique only appear in PEP.

**Table 2: Types of tasks and corresponding techniques in two textbooks<sup>1</sup>**

Cluster	Types of tasks	Techniques	PEP	DM
identify sample space and random event	T <sub>1</sub> : Determine the sample space of a random experiment.	$\tau_1$ : Represent all possible outcomes of this random experiment.	$\surd(20)$	$\surd(24)$
	T <sub>2</sub> : Determine the elements of an event from a random experiment.	$\tau_2$ : Represent all possible outcomes of the random experiment, and then identify which possible outcomes belong to the given event.	$\surd(20)$	$\surd(12)$
	T <sub>3</sub> : Identify the relationship between different random events from a random experiment.	$\tau_3$ : Represent these random events in the form of set, and then translate the given information into the relationship between these sets.	$\surd(23)$	×
calculate probability	T <sub>4</sub> : Determine whether two events are mutually independent in a random experiment.	$\tau_4$ : Calculate $P(A) \cdot P(B)$ , $P(A \cap B)$ , and then compare.	$\surd(6)$	×
	T <sub>5</sub> : Given $P(A)$ , $P(B)$ , calculate $P(A \cap B)$ , where $A$ and $B$ are mutually independent.	$\tau_5$ : Calculate $P(A) \cdot P(B)$ .	$\surd(5)$	×
	T <sub>6</sub> : Given a random experiment, calculate the probability of a random event occurring.	$\tau_{61}$ : Calculate $P = k/n$ , where $k$ and $n$ refer to the number of outcomes belong to this event and total number of possible outcomes in this random experiment. $\tau_{62}$ : By identifying the relationship between different events, apply basic theoretical properties of a probability function to calculate the probability of the given event.	$\surd(70)$	$\surd(172)$
estimate probability by using frequency	T <sub>7</sub> : Given a data set, calculate the frequency of an event.	$\tau_7$ : Calculate $f = m/n$ , where $m$ and $n$ refer to the number of times the event occurs and the total number of trials in the experiment.	$\surd(11)$	×
	T <sub>8</sub> : Given a random experiment, design simulated trials to estimate the probability of a certain random event occurring.	$\tau_8$ : Generate random numbers with various instrumented techniques.	$\surd(6)$	×

**The theoretical block in Chinese and Singaporean mathematics textbooks**

At the level of theory, on the one hand, there is a corresponding technology in the textbooks explaining why the technique can be used to solve the task for each type of tasks. Technological elements in both textbooks can be divided into two categories: one is about terms and definitions in probability theory, such as sample space, the probability of an event, mutually exclusive events, complementary events, stability of frequency, etc.; the other is about some properties of probability and their theoretical results, such as  $P(\bar{E}) = 1 - P(E)$ ,  $P(A \cup B) = P(A) + P(B)$  where events  $A$  and  $B$  are

<sup>1</sup> “ $\surd$ ” indicates that the type of tasks appears in the textbook; while “ $\times$ ” indicates that the type of tasks doesn’t appear in the textbook. The number in parentheses refers to the number of tasks included in each type.

mutually exclusive. Also, different from DM, PEP specifically introduces the term of sample point to represent the possible outcomes of a random experiment; and present the definition of mutually independent events and relevant properties due to the existence of types of tasks related to them.

On the other hand, some of the theory explaining the technologies is presented in the analyzed chapter of two textbooks. Both textbooks discuss theoretical probability and statistical probability, where the former is the likelihood of an event happening based on all the possible outcomes, and the latter depends on the actual results reflected in data (from observations or experiments). For example, for the definition of the probability of an event  $E$  in a random experiment, DM states that the premise of this definition is the random experiment has a finite number of equally likely outcomes; while PEP presents the notion of classical models of probability before giving the definition of the probability of an event, to help students understand the classical definition of probability.

It is worth noting that both textbooks explain why the classical properties of probability behind some of types of tasks should hold. Both textbooks follow a “special to general” approach to help students to find the properties of probability, that is, to obtain general theoretical results from the calculation of the probability of specific events. The difference is that DM is based on how to calculate the probability of an event to explain why the formula holds true; while PEP uses set theoretical notation to design deductive proofs. Taking the formula for complementary events as an example, DM provides the following proof,

Suppose that there are  $n$  outcomes in a sample space and that  $m$  of the outcomes is favourable to the event  $A$ . Then the number of outcomes not favourable to the event  $A$  is  $n - m$ , thus

$$P(\text{not } A) = \frac{n-m}{n} = \frac{n}{n} - \frac{m}{n} = 1 - \frac{m}{n} = 1 - P(A).$$

While PEP uses set theory to explain the meaning of two complementary events, and then gives the following proof,

Because events  $A$  and  $\bar{A}$  are complementary, thus  $A \cap \bar{A} = \emptyset$ ,  $A \cup \bar{A} = \Omega$ . So

$$1 = P(\Omega) = P(A \cup \bar{A}) = P(A) + P(\bar{A}) \Rightarrow P(\bar{A}) = 1 - P(A).$$

Also, it can be found that some results from the probability theory are merely named and outlined informally in two textbooks. For example, DM gives such a narrative in the main text, “when the number of trials in an experiment is large, the experimental probability of an event approximates its theoretical probability”; while PEP states that the frequency of an event will gradually stabilize at the value of the theoretical probability of it occurring as the number of trials increases, and mentions the term of the law of large numbers. Both textbooks do not specifically explain why the theoretical probability can be approximated by statistical probability as the number of trials increases, but only give this conclusion.

## Discussion and conclusion

In this study, we examine the tasks and theoretical knowledge in the first chapter of probability in PEP and DM, from which it can be inferred that what learning opportunities the two textbooks provide for students to learn probability. On the one hand, it can be found content coverage of the first

chapter of probability in PEP is wider than that of DM according to the types of tasks and the theoretical block contained in the two textbooks. This may be related to the diversity of the teaching sequences of secondary school curriculum content in different countries. Also, it can be seen that content emphasis in the two textbooks is the probability calculation of events, which is the main but not the only type of tasks. A notable finding is that DM covers fewer types of tasks, but it provides students with more opportunities to repeatedly use mathematical knowledge to solve problems, especially for the calculation of probability. Tasks could support the teaching and learning of problem solving (Jäder et al., 2019). Such a design also gives more choices for teachers to select tasks which are in different contexts to better cultivate students' ability of mathematical problem solving.

On the other hand, regarding how students learn probability, the results show that students not only need to know what knowledge is, including some concepts, properties and formulas in probability theory, but also need to master how knowledge is obtained and how to apply knowledge. Probability learning plays an important role in improving students' logical reasoning competency, and it should be avoided that using mathematical formulas to teaching probability; otherwise the development of students' conceptions of probability will be essentially based only on the use of formulas (Zhang & Cheng, 2017; Yang & Sianturi, 2019). Both textbooks not only present how to obtain the formulas of probability properties by induction, but also guide students to prove why the formulas hold true through deduction. Such treatment permeates the idea of mathematical reasoning, which can avoid that the alienation of probability teaching into the calculation teaching of just applying formulas.

As Chaput et al. (2011) state, "the classical approach only leads to reduce the understanding of probability to the counting of elementary events and to the overuse of combinatorics; while the frequentist approach, because of its empirical nature, only generates didactic issues like the confusion between the observation of reality and theoretical knowledge. " The textbooks do not present an overly narrow view of probability, and both textbooks introduce theoretical probability and experimental probability, and clarify the difference. Of course, it is still somewhat limited at the theory level, mainly because some of the theory such as the law of large numbers are merely named and outlined informally, for which the reason might be related to the authors' consideration of the learning characteristics and knowledge acceptance ability of students.

To sum up, this paper specifically discusses which types of tasks are provided in the first chapter of probability in Chinese and Singaporean secondary school textbooks and the knowledge processing related to them. The follow-up research could model and study didactic praxeologies in relation to the mathematical praxeologies found in the textbook, that is, explore how teachers and students deal with these praxeologies in practice.

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