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► To cite this version:

Juncal Goñi-Cervera, Irene Polo-Blanco, Hélia Oliveira, Ana Cláudia Henriques. Strategies used by a student with autism when solving a pictorial pattern task under an instructional approach. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04417673

HAL Id: hal-04417673

<https://hal.science/hal-04417673>

Submitted on 25 Jan 2024

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Strategies used by a student with autism when solving a pictorial pattern task under an instructional approach

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Student with autism in inclusive schools often require additional support for mathematical learning. We present an exploratory case study, based on an adapted instructional experience with a 9-year-old-student with autism, with the aim of characterising the strategies he uses when solving pictorial pattern tasks. We followed previous adaptation of instructional methodologies for autistic students, as combining the use of manipulatives and visual support. The results show that the student advanced from bald answers towards operating strategies by relying on the use of tables and guided by the instructor. However, he showed no evidence of functional strategies and kept using the recursive relationship, which seems to be promoted by the use of the table. We conclude by providing clues as to how to help autistic students develop functional thinking.

Keywords: Autism, pictorial pattern task, functional thinking, adaptation of methodologies.

Introduction

Algebra is one of the subjects studied in secondary school with which many students struggle. In an effort to mitigate these difficulties, it has been suggested to start working with functional tasks from early ages, as they provide an appropriate context for developing generalisation and algebraic thinking (Kieran et al., 2016). Working with geometric patterns from early ages, may provide an appropriate context for developing generalisation and algebraic thinking, and particularly to understand functional relationships (Kieran et al., 2016). Some studies on functional thinking have focused on studying the strategies and representations shown by early-age-neurotypical students (e.g., Blanton & Kaput, 2004; Pinto & Cañadas, 2021) when solving generalisation tasks that involve functional relationships. For example, Pinto and Cañadas (2021) identified in 48 children aged 8 to 11 a variety of strategies and the use of different representation systems, predominantly numerical representation or natural language. Their findings also indicated that while only half of the youngest participants showed evidence of functional relationships in their responses, the majority of 10-11-year-old students did.

Autism spectrum disorder (ASD) is a neurobiological developmental disorder that manifests during the first years of life and lasts throughout the life cycle. Its main symptoms are: deficits in communication and social interactions and restrictive and repetitive patterns of behaviour, interests or activities (APA, 2013). Autistic students may have difficulties in planning, in understanding other people's mental states (also known as theory of mind), and they may have a limited attention span (Ozonoff & Schetter, 2007). However, they may also possess a unique visual thinking ability that is often associated with the disorder, which allows them to think and reason through images and visual systems, as explained by Grandin (1995). Visual cues from representations can therefore guide students with ASD in carrying out academic tasks. This particular way of thinking can be

advantageous for certain tasks, and educators should recognize and take advantage of these strengths and other aptitudes to enhance their learning experience.

Often, students with autism with and without intellectual disability require support for mathematical learning (Chen et al., 2019). In the particular case of generalisation, the difficulties with abstraction prevalent in individuals with ASD could influence their generalisation ability. In addition, the inflexibility characteristic of some people with ASD (Minsheiw et al., 2002) could lead them to remain in incorrect strategies and not move towards more sophisticated ones that would allow them to generalise. Recent studies have been interested in studying algebraic thinking in students with some type of educational needs, specifically, in students with autism spectrum disorder (Goñi-Cervera et al., 2022a; Goñi-Cervera et al., 2022b). In this line, Goñi-Cervera et al. (2022b) explored the strategies followed by 17 autistic and 17 neurotypical students (5 to 9-year-old-students) when solving a questionnaire with different questions about specific cases and the general case of a drawing pattern. A similar study extended the sample to 26 students (6 to 11-year-old-students) with autism (Goñi-Cervera et al., 2022a). Both studies observed that the most frequent strategy was the use of incorrect operations, represented verbally (e. g. “if there are 14 at four tables and four at one, 14 plus 4 is 18”) and numerically (e.g. writing “ $6 + 5 = 11$ ”), and the second most frequent was modeling with drawing and counting. This modeling strategy was the one that led to the most correct answer in both groups of students, especially for obtaining consecutive cases. This work also focused on describing the type of generalisation achieved by the students, concluding that only three students generalised, starting with a correct modeling strategy in the consecutive terms and continuing towards operations in the non-consecutive ones to finally generalise.

The aforementioned studies with students with ASD focus on the evaluation of spontaneous strategies in solving tasks in a functional context. The results of these studies have served as a basis for the design of methodologies adapted to help students in developing generalisation strategies. Specifically, the present work reports an instructional experience using one of these adapted direct instruction methodologies with a 9-year-old student with ASD to help him develop strategies that promote the generalisation within pictorial pattern tasks. In this work we intend to describe the performance of a student with autism while solving a task that includes a pictorial pattern, in terms of the strategies he develops by supporting him through the instructional sequence. The first aim is to support the process, the second aim is to see the strategies that can emerge with this support.

Method

We present an exploratory study to describe the strategies evidenced by the student during an instructional process assuming a descriptive qualitative approach. This work is part of a project aimed at describing the mathematical abilities of students with ASD and their relationship with cognitive variables (Polo-Blanco et al., in press). For the instruction presented below, we selected a student among the participants. At the beginning of the study, Philip was 9 years and 5 months old, obtained a mathematical age measured by TEMA-3 (Ginsburg & Baroody, 2007) of 6 years and 10 months, and an IQ of 88 measured by WISC-V (Wechsler, 2015). He was enrolled in 4th grade in an ordinary school but followed a math curriculum corresponding to 2nd and 3rd grade. He received 6 hours per week of specialist support at school. He had already participated in a teaching experiment aimed at

learning to solve arithmetic word problems with the same instructor, with good behavioural and learning outcome. He had not been exposed to generalisation in pictorial growing patterns before.

Procedure

Prior to instruction, the participant solved five pre-tests with pictorial pattern tasks involving for each test, one of the functions, $f(x) = x + 1$, $f(x) = x + 2$, $f(x) = x + 3$, and $f(x) = 2x$, without support material or instructor's help. After the pre-test sessions, the instruction that consisted of four sessions, each one aimed at working with one of the above functions, began. The present study details his performance during the second pre-test and the second instructional session, both involving the function $f(x) = x + 2$, with different pictorial patterns (Figure 1).

Direct instruction was conducted to aid student's understanding of the task and to promote solving strategies. The instructional session was divided into: (1) model phase, in which the instructor modelled the situation with dot stickers under the observation of the student for steps 1, 2 and 3; (2) guidance phase, in which the student solved the proposed task using dot stickers or multilink, guideline sheet, table and with the necessary feedback from the instructor (Figure 1, left) and (3) evaluation phase, in which the student solved a new task without the instructor's help, which involved the same function but with a different pictorial pattern (Figure 1, right). At the beginning of the instructional session the student was given a worksheet with the pictorial pattern and written questions (see Figure 1). Then, at the beginning of the evaluation phase, another similar worksheet was provided. The following task was proposed at different stages of the instruction:

Look at this figure that grows and grows with each step:

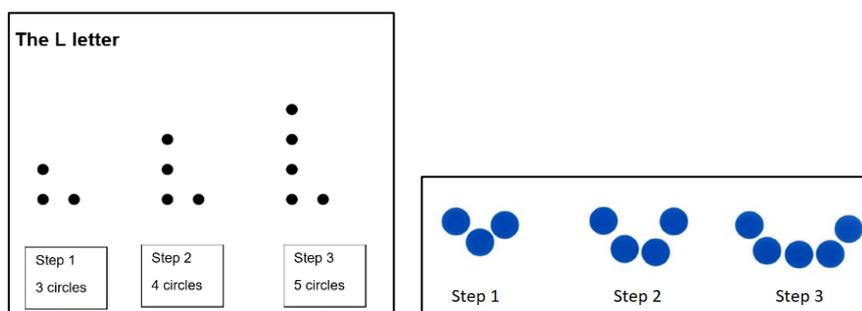


Figure 1: Graphical representation of a pictorial pattern task involving $f(x)=x+2$, for the model and guidance phases (left) and evaluation (right)

Then, the student was presented with the following written questions: (Step 4) *Build the figure in step 4. How many circles did you need?* (Step 5); *Build the figure in step 5. How many circles did you need?* (Step 10); *How many circles do we need to build the figure in step 10? How did you know?* (Step 25); *How many circles do we need to build the figure in step 25? How did you know?* (Step 12); *How many circles do we need to build the figure in step 12? How did you know?* (Any step); *How many circles do we need to build the figure in any step? How did you know?*

As can be seen in the task, questions were asked for particular consecutive (steps 4 and 5), non-consecutive (steps 10, 25 and 12) and the general term (any step) cases. To solve the task, the student had at his disposal the following materials that the instructor encouraged him to use: (1) worksheet with the pictorial pattern (see Figure 1) and with the written questions, where the student provided

the answers, (2) guideline sheet, where the steps to follow to solve the task appeared accompanied by pictograms, and (3) multiple representations: pictorial (pictorial pattern provided in the task), manipulative (multilink and dot stickers) and tabular (a table with two columns with the headings: "step" (left) and "number of circles" (right), where the student was encouraged to write down the results obtained in the different questions). The steps in the guideline sheet were: (1) build the figure, (2) surround what is repeated, (3) fill in the small number step, (4) fill in the big number step, (5) fill in any step and (6) fill in backward. These steps were accompanied by pictograms showing dot stickers and images referring to the table.

For data analysis, based on strategies defined by Goñi-Cervera et al., (2022a), we differentiate between: (a) NA: non-answering, where the student failed to answer orally or in writing, or said he did not know the answer; (b) BA: bald answering, where an answer was given but no explanation; (c) MM: modeling with manipulatives and counting, if the student used the manipulatives at hand to depict the tasks posed; (d) MD: modeling with drawing and counting, where he depicted the situation with drawing; and (e) O: operating, where he added orally, in writing or on their fingers. In addition, the type of representation provided in the instruction on which the student relied in each of the questions is also considered (Cooper & Warren, 2008), in particular: (a) P: pictorial, when the student relies on the pictorial pattern provided in the text, (b) M: manipulative, when he uses multilink and/or dot stickers, and (c) T: tabular, when he relies on the table.

Results

This section shows the results of the pre-test session and the instructional session in which the function involved was $f(x)=x+2$ (Figure 1). Table 1 reflects the strategies shown spontaneously by Philip, as well as the representations on which he relied to demonstrate the strategy for each step of the figure in the pre-test, instructional guidance phase and instructional evaluation phase. Incorrect answers are marked with an asterisk (*).

Table 1: Strategies, representations and responses by Philip in different phases

		Step 4	Step 5	Step 10	Step 25	Step 12	Any step
Pre-test	Strategy	MD	MD	BA	BA	BA	BA
	Representation	P	P	-	-	-	-
	Answer	4*	5*	10*	25*	12*	'Depends on the number'*
Guidance phase (instruction)	Strategy	MD	MD	O	O	O	O
	Representation	P, T	P, T	T	T	T	T
	Answer	6	7	12	27	14	'Counting by ones'*

Evaluation phase (instruction)	Strategy	O, MM	O, MM	O	O	O	O
	Representation	P, T, M	P, T, M	P, T	P, T	P, T	T
	Answer	6	7	12	27	14	'The next number'*

Pre-test

As can be seen in Table 1, during the pre-test session Philip answered incorrectly to all the questions, providing as a response the same number for which he was asked (“five” for step 5, etc). He employed also drawings, but showed not understanding the pattern by drawing as many circles as the step of the figure (drawing four circles for step 4, for example).

Instruction model phase (steps 1, 2 and 3)

At the beginning of the instruction, the instructor showed the student the pictorial pattern task and the materials available. The instructor began by modeling with dot stickers under the observation of the student the terms provided in the task (steps 1, 2 and 3, see Figure 1). In this way, the instructor relied on the guideline sheet and modelled the steps, making sure that the student understood what the instructor was doing.

Instruction guidance phase

Next, the instructor repeated the above by involving the student. In particular, she invited the student to model the situation with dot stickers for steps 1, 2 and 3 and guided him during the task with different actions such as asking him about the number of circles used, and inviting him to write the answer in the two columns table. If the student showed difficulties, the instructor invited him to count the dot stickers. Next, she asked the student for the circles needed for steps 4 and 5 and he drew on the worksheet the correct number of circles and counted them (counting strategy from modeling with drawing). Philip then wrote down all the answers regarding steps 4 and 5 in the table.

Subsequently, Philip began to solve the non-consecutive terms (step 10 and forward), challenged by the instructor. For this, he abandoned the modeling-with-drawing strategy and relied exclusively on the table. For example, to answer step 10, Philip filled in the intermediate steps between step 5 and step 10, and next, he wrote in the table the number of circles in the right column. For the next steps, he kept adding numbers consecutively in the table in each column (see Figure 2, left). When asked how he got the answers, Phillip answered "by counting one more" (operations strategy).



Figure 2: Task resolution during the guidance phase (left) and evaluation phase (right)

When asked about the general term, Philip also answered "counting one more" which was coded as an operation strategy. In order for him to understand the general term, the instructor encouraged him to particularize other cases such as 20 or 100. For case 20 he looked up the solution in the table he had previously filled in, and correctly answered "22". When asked about 100, which was not represented in the table, he answered "101" and provided no further explanation.

Instruction evaluation phase

In the evaluation phase, Philip filled every step in the table from the first to the requested one, and after obtaining the solution, he used dot stickers to represent it. For example, to build the figure in step 4, he first filled in the table, correctly obtaining the answer 6, and then took 6 dot stickers and pasted them on the worksheet in the shape of the pattern (Figure 2, right). For the general term, he answered, similarly as in the guidance phase, that he would need "the next number" of dot stickers.

Discussion and conclusions

This work shows how a student with ASD improved his understanding of the task from the pre-test to the end of the instructional session. During the instruction model phase, the modeling strategy was used by the instructor for the first steps, and encouraged in the guidance phase to correct errors and to accompany the student towards a correct answer. The student moved towards the operations strategy (in particular, adding one to the previous case) for further steps, which led to correct answers in all particular cases. He showed understanding of the way the sequence was formed (by drawing the circles of the sequence, for example), but did not transfer this knowledge for generalising the quantitative relation between the order and the terms of the sequence. The instructor's support played a crucial role in enhancing Philip's performance during the training. In particular, inviting him to solve the tasks or suggesting how to do so gave him confidence in his responses. Also, the use of the table was shown to be effective in completing the task in the particular cases but this support did not allow Philip to show evidence of functional relationships, but rather, in all cases, he relied on a "plus one" recursion relationship. In that sense, although the task was designed to differentiate consecutive from non-consecutive cases, there was no such differentiation for the student. Despite not showing to identify the functional relationship, Philip showed progress in his strategies, as he started out evidencing mostly incorrect bald-answer strategies in the pre-test, and through instruction he was able to move towards the operations strategy using the table as support. This is in line with what has been observed by Radford (2008) in the sense that there is a generalisation in the "plus one" recursive relationship, what the author calls "an arithmetic generalisation", but that it is not of algebraic nature.

This work contributes to the field of algebraic thinking for students with autism. Its results are in line with previous studies with students with ASD and functional tasks, in the sense that, although the student identified a regularity, he did not manage to generalise to any step of the sequence (Goñi-Cervera et al., 2022a). Regarding the strategies employed, the results of this work coincide with previous studies with children of similar ages, both typically developing (Blanton & Kaput, 2004) and with ASD (Goñi-Cervera et al., 2022a, 2022b), in which the most frequent strategy was operations. Philip's inflexibility, expressed in this case by refusing to abandon the use of the table, is a behaviour that has been observed in previous studies involving autistic students (Polo-Blanco et al.,

2019). Although the participant in this study failed to generalise, in addition to incorporating some aspects that have been shown to be beneficial in ASD students (pacing the task or using visual supports), other aspects that can foster the identification of generalisation strategies should be further explored. For example, in the context of visual patterns, it is important that children are encouraged to make connections between the pictorial and numerical representations (Pinto & Cañadas, 2021), so that they grasp the relationship between the term and the respective order, before focusing only on the numerical sequence. Given that students with autism often present difficulties in interpreting language, it would be interesting to delve into the role of language when facing tasks involving a functional relationship, and to investigate to what extent difficulties in generalising are related to the understanding of some key words involved in these tasks, such as "any". Furthermore, the use of tabular representation, which has been shown to be effective for the development of functional strategies in students without autism (Blanton & Kaput, 2004), may have made it difficult for the participant of this study to abandon the recursive relationship. This could be due to the fact that representing all the steps in the table always led him to the correct solution, thus provided him with security, in line with what has been observed in other studies on strategies in students with ASD (Polo-Blanco et al., 2019). Moreover, a preference for more rudimentary strategies could be related to the type of rigid thinking characteristic of ASD (Minsheu et al., 2002). In that sense, the students' preference for tabular representation could be considered to help him focus his attention on the relationship between the two columns of the table and thus supporting him to identify the functional relationship.

Acknowledgment

SUBVTC-2022-0004, PID2022-136246NB-I00 and a "Concepción Arenal" grant (Gob. Cantabria)

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