



**HAL**  
open science

# The association of algebraic thinking with working memory, control of processing, and speed of processing

Maria Chimoni, Demetra Pitta-Pantazi, Constantinos Christou

## ► To cite this version:

Maria Chimoni, Demetra Pitta-Pantazi, Constantinos Christou. The association of algebraic thinking with working memory, control of processing, and speed of processing. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04417556

**HAL Id: hal-04417556**

**<https://hal.science/hal-04417556>**

Submitted on 25 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# The association of algebraic thinking with working memory, control of processing, and speed of processing

Maria Chimoni, Demetra Pitta-Pantazi and Constantinos Christou

University of Cyprus, Department of Education, Cyprus; [chimoni.maria@ucy.ac.cy](mailto:chimoni.maria@ucy.ac.cy)

*This study aims to provide evidence of the cognitive effort associated with algebraic activity in elementary and lower middle school grades. The participants of the study were 591 students from Grades 4 to 7. A written test with algebraic tasks was used to assess the students' algebraic thinking abilities. Computer-based tests were used to assess students' abilities in working memory, control of processing, and speed of processing. The results of linear regression analysis indicated that working memory significantly predicted the students' algebraic thinking abilities in all grades, while the effect of control of processing and speed of processing was not important. These findings suggest that algebraic activity is demanding of working memory resources, thus offering a new perspective of current understandings of algebra learning in elementary and lower middle school grades.*

*Keywords: Algebraic thinking, working memory, speed of processing, control of processing.*

## Introduction

Algebraic thinking is defined as “a process in which students generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways” (Blanton & Kaput, 2005, p. 99). The ability to formulate generalizations is rooted in the ability to notice “the same and the different” which emerges and evolves as individuals interact with the world, recognize patterns, model classes of objects, and transfer meaning from familiar situations to unfamiliar situations (Radford, 2008, p.38). In this sense, algebraic thinking is exhibited as students become aware of generalizations and express them using various semiotic signs, including verbal expressions, gestures, unconventional symbols, and conventional alphanumeric symbols (Radford, 2008). Furthermore, an essential characteristic of algebraic thinking is “analyticity” that is exhibited as students handle indeterminate quantities in a deductive manner (Radford, 2018).

Algebraic thinking abilities are expected to emerge as early as the elementary school grades and become more formal as students enter the secondary school (Cai & Knuth, 2005). However, relatively little is known about the cognitive effort imposed in algebraic thinking at this age range (Cirino et al., 2019). Studies from the field of educational psychology and cognitive neuroscience had mainly been conducted with middle and higher secondary school students. These findings suggested that algebraic thinking is predicted by cognitive abilities, including working memory, control of processing, and speed of processing (Cirino et al., 2019; Lee et al., 2010; Tolar et al., 2009; Waisman et al., 2014). Therefore, teachers and researchers should be sensitized to the fact that algebraic achievement requires a great deal of cognitive effort (Kieran, 2018). The main aim of the current study is to provide evidence of the cognitive effort associated with algebraic activity of elementary and lower middle school students (grades 4 to 7), investigating the relationship of algebraic thinking abilities with working memory, control of processing, and speed of processing.

## **Theoretical framework**

### **Main characteristics of algebraic thinking**

Radford (2022) suggested that the main characteristic of algebraic thinking is “analyticity” which becomes evident when students deal with algebraic tasks that involve “indeterminacy” and “denotation”. “Indeterminacy” means that the task contains indeterminate quantities (unknown numbers, variables, or parameters). “Denotation” means that these indeterminate quantities must be named or symbolized, either by conventional alphanumeric symbols or in other ways. In these kinds of tasks, students are required to manipulate the indeterminate quantities as if they were known numbers and operate on them (add, subtract, multiply, and divide them). This behaviour reflects a deductive way of work. For example, when students try to solve the equation  $2x + 2 = 10 + x$ , “analyticity” is evident as they operate with the indeterminate quantity to arrive to a solution.

Recently, Kieran (2022) agreed that “analytic-thinking” is a core dimension of algebraic thinking. However, algebraic thinking also encompasses a “structural-thinking” dimension which enables students to look through mathematical objects and decompose or recompose them in various structural ways (Kieran, 2022). For example, when students compute the sum  $47 + 25$ , algebraic thinking facilitates the use of the compensation strategy of addition, that is, adjusting the addends in a convenient way ( $47 + 25 = 47 + 3 + 25 - 3 = 50 + 22$ ). This demonstrates understanding of a general structure based on equality properties, i.e.,  $x + y = (x + a) + (y - a)$  (Britt & Irwin, 2011). To offer a holistic, multi-dimensional view of algebraic thinking, Kieran (2022) also added the dimension of “functional thinking” to capture generalizing related to functional relationships. The process of generalization is considered as a salient thread that runs through all the three dimensions.

### **Algebraic thinking and cognitive functions**

Cognitive functions related to processing efficiency and executive control are dominant in problem solving (Tourva et al., 2016). Their role involves ensuring that “currently important information is to be properly attended to, represented, integrated, and understood so that optimum decisions and actions are made, given the current goals” (Demetriou et al., 2020, p.79). Working memory is responsible for maintaining and using information during the execution of problem solving for short periods of time, control of processing warrants that behaviour is goal-directed, and speed of processing reflects the fluency of the brain to register and process information (Tourva et al., 2016).

Empirical studies have shown that the contribution of these cognitive functions to mathematics achievement is crucial and remains stable across grades. Working memory was found to be the most significant cognitive function in later grades (Geary et al., 2017). Students with severe mathematical difficulties were also found to face deficits in working memory (Peng & Fuchs, 2014), control of processing (Chu et al., 2016), and speed of processing (Cirino et al., 2013). Similarly, working memory was found to influence college students’ algebraic achievement (Tolar et al., 2009). Previous research with younger students of Grades 2–3 demonstrated that control of processing and speed of processing contributed to their pre-algebraic knowledge only indirectly via their abilities in arithmetic calculations and word problems (Fuchs et al., 2012).

Overall, relatively little is known about the cognitive functions that are important to algebraic achievement (Cirino et al., 2019). Furthermore, the cognitive effort required from students in elementary and lower middle school grades is underexamined, even though this age range is considered crucial for understanding the formal algebra which will follow in secondary school.

## The current study

The main aim of the current study is to examine the contribution of working memory, control of processing and speed of processing to the algebraic thinking abilities of students in elementary and lower middle school grades. The research question of the study is as follows:

*Is there a predictive relationship between working memory, control of processing, and speed of processing with students' performance in various algebraic tasks?*

## Method



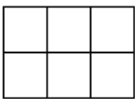
### Participants

The sample of the current study consisted of 591 students who were selected by convenience from ten schools that responded positively to our call to participate in the study: 167 fourth graders (mean age 9 years), 146 fifth graders (mean age 10 years), 147 sixth graders (mean age 11 years), and 131 seventh graders (mean age 12 years). Students in Grades 4 and 5 had experience with the concepts of equivalence and properties of operations. Students in Grade 6 had been introduced to the use of letters for expressing unknowns in equations. Students in Grade 7 were introduced to equation solving.

### Instruments

To assess students' algebraic thinking abilities, a test was developed that addressed the multi-dimensional nature of algebraic thinking. Table 1 presents examples of the tasks.

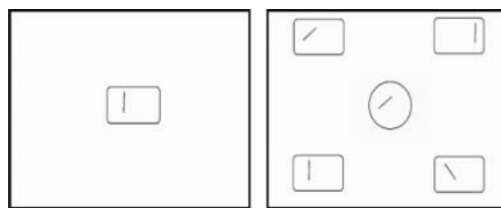
**Table 1: Examples of tasks in the algebraic thinking test**

Example of task	Thinking processes
<p>Nikiforos calculated the sum <math>80 + 50</math> using the following method:</p> $80 + 50 = 80 + 20 + 30 = 100 + 30 = 130$ <p>(a) Explain Nikiforos' method.</p> <p>(b) Use Nikiforos' method to calculate the sum <math>70 + 50</math>.</p>	<p>Look through mathematical objects. Decompose and recombine mathematical objects in convenient structural ways.</p>
<div style="text-align: center;">    </div> <p style="text-align: center;">Figure 1    Figure 2    Figure 3</p> <p>How many squares will Figure 16 have? Show your work.</p>	<p>Generalize within functional situations.</p>
<p>If <math>\star + \star = 4</math>, then</p> $\star + \star + 6 = ?$	<p>Manipulate indeterminate quantities as if they were known numbers and operate on them.</p>

The test involved 11 tasks which captured basic algebraic concepts and procedures: use of general properties of numbers and operations, equation solving, identification of the  $n^{\text{th}}$  term in numerical and figural patterns, interpretation of linear graphs, and analysis of covariational relationships. Students were allowed 40 minutes to take the test. Students were asked to show their strategy for each task in a clear written format.

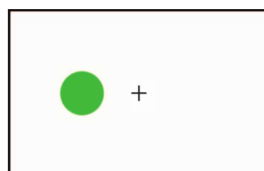
The task in the first row required the students to look through the arithmetical expression  $80 + 50$  and explain the way one of the addends is decomposed, addressing the associative property of addition, i.e.,  $x + y + z = x + (y + z) = (x + y) + z$ . The task in the second row required the students to identify a general rule to determine a specific far term in a figural pattern, i.e.,  $y = 2x$ , where  $y$  is the number of squares and  $x$  the number of the term. The task in the third row required the students to deduce the value of the second expression based on the value of the first. Students' responses were considered as correct only when there was a clear indication of manipulating the first expression as a whole object and replacing it in the second expression with the value of 4. Students' responses that showed replacement of each star symbol with the value of 2 were considered as wrong. Based on Cronbach's alpha, the internal consistency for this test was satisfactory. Cronbach's alpha was  $\alpha=0.75$  for Grade 4,  $\alpha=0.75$  for Grade 5,  $\alpha=0.68$  for Grade 6, and  $\alpha=0.66$  for Grade 7.

To assess students' working memory, control of processing, and speed of processing, computer-based tests were used (Demetriou et al., 2008). The first test (Figure 1) measured the students' working memory by evaluating their ability to remember a figure that appears on the computer screen and to identify it in the next screen within a set of comparable figures.



**Figure 1: Example of tasks included in the working memory test**

The second test (Figure 2) evaluated speed and control of processing. The students were required to focus on the form of a stimuli presented (green circle or blue square) and press the right or left keyboard arrow according to the form of the shape (left arrow for the green circle and right arrow for the blue square). For half of the items in this test, the green circle appeared in the same direction as the keyboard button that had to be pressed (left side of the screen, left arrow). These items measured speed of processing. For the other half of the items, the green circle appeared on the right side of the screen and the student had to press the left arrow (right side of the screen, left arrow). These items measured control of processing.



**Figure 2: Example of tasks included in the speed and control of processing test**

## Scoring

Each task in the algebraic thinking test was graded with 1 mark. For the tasks that had sub-questions, partial credit was given, considering the maximum sum of the marks to be equal to 1. Considering that students had adequate time to complete the test, tasks with no response were graded with 0 marks.

The computer-based tests calculated the total time needed for completing the tasks and the number of correct responses. The final score was students' reaction time. This was calculated by dividing the total time needed for completing the tasks by the number of correct responses. Three different scores were calculated, one for each cognitive function.

## Analysis

To analyse the data collected, we conducted Linear Regression Analysis, using the SPSS statistical package. The dependent variable was students' performance in the algebraic thinking test. The independent variables were their performance in the working memory, control of processing, and speed of processing tests. For each age-group, three regression models were investigated to define the contribution of each cognitive function in separation to students' performance in the algebraic thinking test. This way of analysis allowed us to make assumptions about the dependent variable based on only one independent variable, and to make comparisons across the four grades.

## Results

Table 2 presents the way the students' overall performance in the algebraic thinking test is explained by their performance in the working memory test. The values for *B* present the unstandardized coefficients for each regression equation, *SE* the standard errors associated with the coefficients, and Beta ( $\beta$ ) the standardized coefficients. These results show that working memory is a significant predictor of students' performance in the algebraic thinking test across Grades 4–7. Working memory explains 27% of the variance in the dependent variable in Grade 4, 79% in Grade 5, 70.3% in Grade 6, and 15.4% in Grade 7.

**Table 2: Working memory and algebraic thinking in Grades 4–7**

	Grade 4			Grade 5			Grade 6			Grade 7		
	B	SE	$\beta$	B	SE	$\beta$	B	SE	$\beta$	B	SE	$\beta$
Working Memory	.496	.069	.520*	.957	.047	.890*	.963	.058	.838*	.406	.099	.392*
	R <sup>2</sup> =.270, * <i>p</i> <.05			R <sup>2</sup> =.791, * <i>p</i> <.05			R <sup>2</sup> =.703, * <i>p</i> <.05			R <sup>2</sup> =.154, * <i>p</i> <.05		

Table 3 presents the way the students' overall performance in the algebraic thinking test is explained by their performance in the control of processing test. According to the results, control of processing seems to significantly predict students' performance in the algebraic thinking test in Grades 4, 6, and 7. However, the values of R Square are very low ( $\leq 5\%$ ), that is, a small percentage of the variance in the dependent variable could be explained by control of processing. In Grade 5, the correlation between the two variables is not significant, indicating that speed of processing cannot be considered as a significant predictor of the students' performance in the algebraic thinking test.

**Table 3: Control of processing and algebraic thinking in Grades 4–7**

	Grade 4			Grade 5			Grade 6			Grade 7		
	B	SE	$\beta$	B	SE	$\beta$	B	SE	$\beta$	B	SE	$\beta$
Control of processing	.190	.066	.225*	.103	.059	.136	.122	.059	.159*	.131	.047	.213*
	R <sup>2</sup> =.050, * <i>p</i> <.05			R <sup>2</sup> =.018			R <sup>2</sup> =.025, * <i>p</i> <.05			R <sup>2</sup> =.045, * <i>p</i> <.05		

Table 4 presents the way the students' overall performance in the algebraic thinking test is explained by their performance in the speed of processing test. According to the results, speed of processing significantly predicts students' performance in the algebraic thinking test in Grades 4 and 6. However, the values of R Square are low (<5%). Therefore, speed of processing seems not to be an important predictor of the students' algebraic thinking in these grades. Moreover, speed of processing does not seem to influence students' performance in Grades 5 and 6.

**Table 4: Speed of processing and algebraic thinking in Grades 4–7**

	Grade 4			Grade 5			Grade 6			Grade 7		
	B	SE	$\beta$	B	SE	$\beta$	B	SE	$\beta$	B	SE	$\beta$
Speed of processing	.172	.066	.204*	.064	.055	.092	.106	.054	.150	.119	.044	.207*
	R <sup>2</sup> =.042, * <i>p</i> <.05			R <sup>2</sup> =.009			R <sup>2</sup> =.023			R <sup>2</sup> =.043, * <i>p</i> <.05		

## Discussion

The current study investigated the association of algebraic thinking with working memory, speed of processing, and control of processing in students of the elementary and lower middle school grades. The results showed that the significance of working memory for exhibiting algebraic thinking remains stable across grades 4–7. This finding agrees with previous longitudinal studies that indicated the important role of working memory to mathematics achievement (Geary et al., 2017). The influence of control of processing and speed of processing was not found to be significant in any of the four grades, indicating that these cognitive functions do not predict students' algebraic thinking abilities.

Overall, the findings of this study suggest that working memory shapes students' effort to learn algebra at this age range. Working memory plays a crucial role in maintaining and using information during the execution of problem solving for short periods of time (Tourva et al., 2016). In this sense, algebraic thinking is related to the ability to select information out of a greater set, maintain this information, and use it effectively. For example, when computing a sum like  $80 + 50$  (Table 1) students need to retain that 50 can be decomposed into  $20 + 30$  which transforms the arithmetical expression into  $80 + 20 + 30$ . Then by adding first  $80 + 20$ , a convenient number is created which allows a quick computation of the final sum. "Structural-thinking", that is, looking through mathematical objects and noticing structure is an important aspect, but this structure needs to be retained for successfully solving the task. Without sufficient working memory, the information would be lost before it could be combined into a coherent, complete thought (Cirino et al., 2019). Similarly, in a pattern task, like the one presented in Table 1, students need to retain information in respect to the relationship involved as this is represented by a set of specific terms and check whether this

relationship also applies to all subsequent terms (Pittalis et al., 2020). Therefore, “functional thinking” and the process of generalization seems to be facilitated by working memory resources. In equation solving, working memory may also assist retaining information from one step to the next one. As Radford (2018) suggested, this procedure involves transformations of expressions until a final answer in respect to the values of unknowns is deduced.

In sum, the finding that algebraic thinking requires a great deal of cognitive effort related to working memory is important in respect to the requirements of algebra learning in elementary and lower middle school grades. Furthermore, this finding complies with previous studies that uncovered the intense mental activity of older students when dealing with algebraic tasks, indicating that algebra is not merely about the execution of automatised techniques for manipulating letter-symbolic expressions (Waisman et al., 2014). A pedagogical implication would be adapting teaching and learning processes in a way that working memory resources are supported, i.e., by teaching students to record in a written format important information during algebra problem solving.

Future studies may investigate further the role of working memory, control of processing, and speed of processing by conducting regression analyses that examine simultaneously the contribution of each of the three cognitive functions to algebraic achievement and not in separation as in the current study. Possible indirect effects of control of processing and speed of processing should also be examined. In addition, the contribution of the interaction of these cognitive functions to algebraic achievement can be investigated, to gain a clearer picture of their role in dealing with algebraic tasks.

## References

- Blanton, M. L., & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education*, 36(5), 412–446.
- Britt M.S., & Irwin K.C. (2011). Algebraic thinking with and without algebraic representation: A pathway for learning. In J. Cai & E. Knuth (Eds.) *Early algebraization. Advances in mathematics education* (pp 137–159). Springer-Verlag. [https://doi.org/10.1007/978-3-642-17735-4\\_10](https://doi.org/10.1007/978-3-642-17735-4_10)
- Cai, J., & Knuth, E. (2005). Introduction: The development of students’ algebraic thinking in earlier grades from curricular, instructional and learning perspectives. *ZDM—The International Journal on Mathematics Education*, 37(1), 1–4.
- Chu, F.W., van Marle, K., & Geary, D.C. (2016). Predicting children's reading and mathematics achievement from early quantitative knowledge and domain-general cognitive abilities. *Frontiers in Psychology*, 7(775). <https://doi.org/10.3389/fpsyg.2016.00775>
- Cirino, P.T., Tolar T.D., & Fuchs, L.S. (2019). Longitudinal algebra prediction for early versus later takers. *The Journal of Educational Research*, 112(2), 179–191. <https://doi.org/10.1080/00220671.2018.1486279>
- Demetriou, A., Kazi, S., Makris, N., & Spanoudis, G. (2020). Cognitive ability, cognitive self-awareness, and school performance: From childhood to adolescence. *Intelligence*, 79. <https://doi.org/10.1016/j.intell.2020.101432>
- Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., & Bryant, J. D. (2010). The contributions of numerosity and domain-general abilities to school readiness. *Child Development*, 81(5), 1520–1533. <https://doi.org/10.1111/j.1467-8624.2010.01489.x>



- Geary, D. C., Nicholas, A., Li, Y., & Sun, J. (2017). Developmental change in the influence of domain-general abilities and domain-specific knowledge on mathematics achievement: an eight-year longitudinal study. *Journal of Educational Psychology*, *109*(5), 680–693. <https://doi.org/10.1037/edu0000159>
- Kieran, C. (2018). Algebra teaching and learning. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Springer. [https://doi.org/10.1007/978-3-319-77487-9\\_6-5](https://doi.org/10.1007/978-3-319-77487-9_6-5)
- Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: background, overarching dimensions, and new directions. *ZDM — The International Journal on Mathematics Education*, *54*, 1131–1150. <https://doi.org/10.1007/s11858-022-01435-6>
- Lee, K., Yeong, S.H.M., Ng, S.F., Venkatraman, V., Graham, S., & Chee, M.W.L. (2010). Computing solutions to algebraic problems using a symbolic versus a schematic strategy. *ZDM—The International Journal on Mathematics Education*, *42*, 91–605. <https://doi.org/10.1007/s11858-010-0265-6>
- Peng P., & Fuchs, D. (2014). A meta-analysis of working memory deficits in children with learning difficulties: Is there a difference between verbal domain and numerical domain? *Journal of Learning Disabilities*, *49*(1), 3–20. <https://doi.org/10.1177/0022219414521667>
- Pittalis, M., Pitta-Pantazi, D., & Christou, C. (2020). Young students’ functional thinking modes: The relation between recursive patterning, covariation thinking and correspondence relation. *Journal for Research in Mathematics Education*, *51*(5), 631–674. <https://doi.org/10.5951/jresmetheduc-2020-0164>
- Radford, L. (2008). Iconicity and contraction: A semiotic investigation of forms of algebraic generalizations of patterns in different contexts. *ZDM — The International Journal on Mathematics Education*, *40*(1), 83–96. <https://doi.org/10.1007/s11858-007-0061-0>
- Radford, L. (2018). The emergence of symbolic algebraic thinking in primary school. In C. Kieran (Ed.), *Teaching and learning algebraic thinking with 5- to 12-year-olds: The global evolution of an emerging field of research and practice* (pp. 3–25). Springer. [https://doi.org/10.1007/978-3-319-68351-5\\_1](https://doi.org/10.1007/978-3-319-68351-5_1)
- Radford, L. (2022). Early algebra: Simplifying equations. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of European Research in Mathematics Education (CERME12)* (pp. 588-595). ERME / Free University of Bozen-Bolzano.
- Tolar, T.D., Lederberg, A.R., & Fletcher, J.M. (2009). A structural model of algebraic achievement: computational fluency and spatial visualisation as mediators of the effect of working memory on algebraic achievement. *Educational Psychology*, *29*(2), 239–266. <https://doi.org/10.1080/01443410802708903>
- Tourva, A., Spanoudis, G. & Demetriou, A. (2016). Cognitive correlates of developing intelligence: The contribution of working memory, processing speed and attention, *Intelligence*, *54*, 136–146. <https://doi.org/10.1016/j.intell.2015.12.001>
- Waisman, I., Leikin, M., Shaul, S. & Leikin, R. (2014). Brain activity associated with translation between graphical and symbolic representations of functions in generally gifted and excelling in mathematics adolescents. *International Journal of Science and Mathematics Education*, *12*(3), 669–696. <https://doi.org/10.1007/s10763-014-9513-5>