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Exploring the structure of multiplication tasks in Norwegian textbooks within a developmental approach

Nils-Jakob Herleiksplass¹, Viktor Freiman² and Raymond Bjuland¹

¹University of Stavanger, Department of Education and Sport Science, Stavanger, Norway;
nils-jakob.herleiksplass@uis.no

²Université de Moncton, Canada

This paper examines the tasks suggested for Norwegian primary schools working upon a developmental education in mathematics (DEM) approach. 82 tasks from a 2nd grade textbook aiming to introduce multiplication were analysed, revealing a variety of features and structures reflecting the complexity of a key mathematical concept. 18 of these tasks were selected for further analysis grounded in a Vygotskian developmental perspective which encourage in-depth investigation of mathematical relationships based, among other, on analytical observation, reflection, and synthesis. The findings show a potential of DEM textbooks to support teachers in enriching students' conceptual understanding of multiplication and provoking their general development.

Keywords: Mathematical tasks, developmental education in mathematics, mathematical thinking, multiplication.

Introduction

In a 21st century classroom teachers are challenged to provide students with opportunities to think, thus increasing and deepening their understanding of mathematical concepts and relationships (Kazemi & Stipek, 2009). While Norwegian teachers are free to choose from a multitude of resources that are available for them to select appropriate tasks for their students, mathematics textbooks remain “a pivotal resource” in the Norwegian “teachers’ resource system” (Pepin et al., 2013, p. 685). One such resource used by Norwegian primary teachers in more than 70 schools since 2009 (Gjære & Blank, 2019), is Arginskaya’s *Mathematics* for grades 1-4 (e.g., Arginskaya et al., 2014). This adapted and translated textbook series is a part of DEM (Developmental Education in Mathematics). Grounded in a Vygotskian perspective on learning and development, DEM is based on a didactical system conceptualised and implemented by Leonid V. Zankov (1901–1977) and his followers to reveal the “potential of [each] pupil and to create favorable conditions for its development” along with their general “abilities, natural gifts, independence, and initiative” (Zankov, 1977, cited by Gjære & Blank, 2019, p. 28).

When one talks about the developmental education in a Vygotskian sense, several approaches and views are to be considered: The Davydov and El’Konin’s curriculum in mathematics (Dougherty & Simon, 2020); a more general approach to education adopted in parts of the Netherlands (van Oers, 2012); and the DEM which is probably less known. While each has its own theoretical base, they all share the Vygotskian idea that teaching developmentally requires the teacher (and students) to work within the *zone of proximal development*, by for instance attending to concepts that are “on the cusp of emergence” (Walshaw, 2017, p. 303). Hence, teaching developmentally implies addressing students’ mental functions “that have not yet matured but are in the process of maturation” (Vygotsky,

1978, p. 86), staying one step ahead of development. To Zankov (1977) this can be achieved by constructing teaching in accordance with five didactical principles underpinning his educational system (see Gjære & Blank, 2019 for more detailed description). We draw on four key concepts Zankov (1977) identified to study students' mental development when implementing these principles in the classroom teaching, namely observation, abstraction, reflection, and synthesis, to frame the analysis of multiplication tasks we describe in the next section.

Gjære and Blank (2019) described how Zankov's didactical principles and DEM tasks are perceived by teachers as appropriate resources to reach students' deeper understanding of the mathematical ideas and connections between topics. Yet, their study did not explain what features of the task bear a potential to engage students into such a developmental path. Hence, a need of in-depth analysis of the tasks motivated us to explore possible patterns in their structure. As an initial step, we posit that DEM textbooks are structured differently to how more traditional textbooks in mathematics usually present their content, with introduction of procedures or theory, then examples or questions for students' practice (Ronda & Adler, 2017). For instance, a common textbook series used in Norway, *Multi 3A* (Alseth et al., 2020), contains multiplication tasks with ready-made blank spaces such as $2+2+2+2 =$ [blank] followed by $4 \cdot 2 =$ [blank], referring to some visual representation (groups of objects). In contrast to this approach, DEM tasks seek to engage students into individual investigation followed by a whole-class discussion (Gjære & Blank, 2019).

Furthermore, according to Gjære and Blank (2019), DEM is perceived by teachers as a way to "engage and challenge their students and provide them with a deeper learning experience in mathematics" (p. 32). However, what within these textbooks aids teachers to facilitate such learning experiences? This question was posed by the first author in his ongoing doctoral study and motivated us to investigate the tasks that introduce multiplication in 2nd Grade¹. Moreover, supporting and developing multiplicative thinking has remained a relevant topic of research for the past two decades (e.g., Park & Nunes, 2001; Polotskaia & Savard, 2021) when highlighting the importance of thought-provoking tasks (Cheeseman et al., 2022). Here, we aim to shed light on which aspects of multiplication, such as multiplicative *structures* (e.g. Greer, 1992) that are introduced. We then seek to apply a developmental perspective on how the tasks themselves are structured in the Arginskaya's *Mathematics 2B* book used in the spring semester during our data collection. In particular, we investigate: (a) Which aspects of multiplication can be identified? (b) How are the tasks structured to provide opportunities to develop students' thinking?

DEM curriculum materials

DEM curriculum materials consist of the main textbook, a teacher's guide, and a booklet with additional tasks, the latter usually reserved for homework or individual seat work. In the main textbook, tasks are divided into five categories, two of which are stated by the authors as "red" and "blue" tasks. Further, tasks in the 2nd grade book consist of several sub-tasks labelled from a) to g).

¹ In the textbook used by the teacher in Grade 2 we visited during our study this was the first time when multiplication was introduced. This however has changed in the new edition of the textbook due to the renewal of our national curriculum, where multiplication is not present until 3rd grade.

The authors have described the red tasks as introducing something new, for example a strategy or a concept. This does not imply that red tasks cannot include some kind of prompt for students to revise previous work, but the overall aim is to add something novel. According to the authors, blue tasks serve as standalone tasks or as repetition of previous content, but they too might include some novelty. In fact, while stating that the red tasks are mandatory, the teacher's guide (Melhus, 2016) specifies that teachers may choose freely among the blue ones as long as they carefully identify which of the latter that are adding something novel when deciding to include them in lessons.

Analytical approach

In this paper, tasks from the main textbook are analysed, and tasks that do not fall into the red or blue categories are omitted as they constitute a clear minority. When counting the red and blue tasks, *Mathematics 2B* contains a total of 251, of which 54 are red and 197 are blue. Out of the 251, we identified 82 tasks, 29 red and 53 blue, where multiplication is included. We selected instances where multiplication was present when introducing the concept of division or as an implicit step to solve equations or word problems. Thus, almost one third of the 2B book has tasks dealing with multiplication. They are distributed across three main chapters²: “multiplication and division”, “multiplication table”, and “three-digit numbers”.

To identify which multiplicative aspects the 2B volume presents, we coded the initial 82 multiplication tasks into pre-established categories based on literature on aspects such as structures: *repeated addition* (Steffe, 1994) cartesian *product* and *multiplicative comparison* (Greer, 1992); *multiplication in the order of operations* (Bay-Williams & Martinie, 2015); *multiplication as the inverse of division* (Greer, 2012). The latter includes tasks which prompts students to solve linear equations that contains divisions. Moreover, in concordance to Zankov's principles of high-level difficulty and leading role of theoretical knowledge, the tasks we analysed deal, explicitly or implicitly, with *properties of multiplication* such as commutative or distributive properties, which are also used to construct *multiplication tables*. Other categories, inductively constructed, refer to the tasks asking students, for example, to identify expressions containing multiplication that would correctly calculate or *measure* the perimeter of polygons or represent multiplication as a *product of factors*.

To reveal the structural nature of the tasks from a developmental point of view, a further reduction was required. From a pool of 82 tasks, 18 tasks met the criteria of 1) being a red task, 2) multiplication was the main focus, and 3) contained three or more sub-tasks. That is not to say blue tasks cannot provide the same opportunities for students' mathematical thinking, but for the sake of data reduction we based our final pool of tasks on red tasks only. In accordance with the second criterion, we omitted tasks where for example multiplication was used to illustrate that it is an inverse operation to division and the remainder of the task revolves around division. Tasks where the focus was the order of operations, and multiplication is only one of these operations, were also not included. Finally, one single task was omitted due to the number of sub tasks (2), which would by nature follow a different

² Even though these are the main chapters, topics such as geometry, algebra or even the clock are treated throughout the textbook.


structure than the other tasks. On the 18 tasks we applied a mixed approach of pre-defined categories based on the four key concepts related to Zankov’s (1977) system, and inductively formed categories.

Tasks that either explicitly or implicitly ask students to compare, contrast, notice properties of expressions were coded as *analytical observation*. This is related to observation of particular properties of objects. Zankov viewed the activity of observation as essential to study students’ development. We relate this kind of observation to mathematical *noticing* (Lobato et al., 2013). Tasks that require students to create expressions that model a situation or formulate a general statement about a multiplicative property such as multiplying by 1 or 0 were coded as *abstraction*. We used *synthesis* for tasks that encourage students to create their own expressions, with similar characteristics or requirements to the ones they already have been introduced to. This could mean to follow a descending or ascending pattern, to create more expressions that involve repeated addition and so on. *Abstraction* and *synthesis* are processes that to Zankov were integral to studying students’ mental activity. When students are modelling situations with mathematical expressions it requires some form of abstraction (Mitchelmore & White, 2007). Investigating general aspects of multiplication, such as the commutative property, we also consider as abstraction. According to Zankov, synthesis entails viewing parts “in terms of how they are combined in the finished product” (Zankov, 1977, p. 128). Extending this view further, we view the process of applying recent learned concepts or procedures into new contexts as a form of synthesis. Further, *reflection* was used for tasks that asked students to evaluate why certain approaches of calculating expressions work or not, or what the students themselves would prefer to use and why. It is related to how Zankov described the didactical principle of “insuring pupils’ awareness of the learning process” (Zankov, 1977, p. 58).

We also introduced other categories, such as *exploration*, for tasks asking students to investigate some calculations, sometimes prompted to find multiple or all solutions. Further, we coded as *repetition* tasks starting with something already known to the students before introducing a new concept. The final category addresses the *introduction of a new concept* asking if students agree with the suggested term, and sometimes encouraging them to memorise the definition.

Table 1 presents an example of the task 259 of how multiplicative aspects and the task structure were identified and analysed.

Table 1: Example of analysis task 259/1

Translated to English	Analysis of task structure	Analysis of multiplicative aspect
<p>a) Look at these trees. What is equal? What is not equal?</p> 	<p>Analytical observation. Students are asked to compare the trees, noticing what is equal and not equal.</p>	<p>Repeated addition. Even though the task does not introduce the concept of multiplication yet, it is aimed at</p>

b) Which operation can you use to find out how many apples there are on each tree? Write the expressions and find the values.	Abstraction. Students are asked to model the situation, creating expressions that represent the number of apples and the sum.	introducing students to the idea of the difference between sums that can be written with equal terms and sums that cannot.
c) Compare the equalities you got in b). What is the most important difference between them?	Analytical observation. Students are asked to compare and notice the differences between the expressions they made in b).	
d) Create some more sums with equal terms and find their values.	Synthesis. Students are asked to create their own sums, based on the idea of equal terms.	

Findings and discussion

Across the combined red and blue tasks (N=82), the multiplicative aspects and their frequencies are: *repeated addition* (RA): 14; *multiplication table* (MT): 13; *properties of multiplication* (PM): 13; *multiplication in the order of operations* (MO): 11; *multiplicative comparison* (MC): 8; *multiplication as the inverse operation to division* (MI): 13; *cartesian product* (CA): 4; *multiplication in measurement* (MM): 2; *factorisation* (FA): 1; *other* (OT): 3. Aspects related to the selected 18 red tasks will be presented in Figure 1 with their respective abbreviations.

While the tasks address a variety of aspects related to multiplication, the most frequent categories were repeated addition, multiplication table, properties of multiplication, and multiplication in the order of operations. We stress the fact it is not a matter of memorising the multiplication table that is important, but a systematic process of construction based on understanding of its internal relationships, in connection to other tables. For instance, it is not necessary to construct and memorise $3 \cdot 2$ when students already have constructed the “times 2” table. Further, similar to one of Greer’s (1992) categories, 2B introduces students to multiplication with some concrete situations with *equal groups* which may be modelled using multiplication as repeated addition, but very soon, the authors move away from the concrete illustrations asking students to operate with abstract numbers.

As we mentioned above, each task usually consists of several sub-tasks that may be categorised in developmental terms, i.e., analytical observation, abstraction, synthesis and so on. Figure 1 presents the overall structure of the 18 red DEM tasks using these terms. For the sake of readability, we labelled each task with numbers from 1 to 18. This does not mean however that tasks appear right after each other in the textbook. Namely, task 259(1) and task 264(2) are separated by four blue tasks not related to multiplication. For coding the sub-tasks, we use a colour scheme: analytic observation (green); abstraction (turquoise); synthesis (pink); introduction of a new concept (yellow); reflection (purple); repetition (brown); exploration (orange). Grey cells indicate no sub-tasks for this item. Some tasks consisted of sub-tasks corresponding to different categories and were marked with multiple colours within the same cell. Row MA (multiplicative aspect) was included to show how each aspect was related to the tasks.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
MA	RA	RA	RA	RA	RA	RA	MT	MC	MT	MT	MT	MT	PM	MT	MT	PM	PM	PM
a	Green	Green*	Green*	Green	Brown	Brown	Green*	Orange*	Green	Green*	Cyan	Orange*	Green*	Purple	Green	Green*	Green	Green*
b	Cyan	Green*	Green	Yellow	Brown*	Brown	Green*	Orange*	Purple	Green*	Pink	Orange*	Green*	Pink	Yellow	Green*	Yellow	Purple
c	Green	Pink	Pink	Pink	Brown*	Pink	Purple	Cyan	Purple	Green*	Purple	Green	Cyan*	Purple	Green	Green	Pink	Purple
d	Pink	Pink	Grey	Purple	Pink	Pink	Yellow	Yellow	Purple	Green*	Pink	Pink	Green	Yellow	Pink	Green	Pink	Grey
e	Grey	Grey	Grey	Grey	Pink	Pink	Grey	Pink	Pink	Pink	Grey	Orange	Cyan	Grey	Grey	Grey	Grey	Grey
f	Grey	Grey	Grey	Grey	Grey	Grey	Grey	Grey	Grey	Pink	Grey	Pink	Grey	Grey	Grey	Grey	Grey	Grey

Figure 1: Structure of 18 red DEM multiplication tasks

As indicated by Figure 1, it seems that the first sub-task of a red task usually prompts an observational activity, followed by a reflection portion, often ending with synthesis of learnt mathematical content. In fact, only three tasks (4, 13 and 15) ended differently. Sometimes a new concept or word is introduced, but only after students have done some repetition work (task 5 and 6) or been encouraged to notice or explore mathematical objects (7 and 8). As indicated by the asterisk (*), many sub-tasks need to be viewed as one continuous task with the same goal. This was the case with task 10 where all the sub-tasks lead up to students constructing the 4 times table building and expanding on their knowledge of the 3 times table. What is also apparent is the gradual development of the concept of multiplication itself, from presenting aspects related to repeated addition (tasks 1-6), moving towards constructing the multiplication table (tasks 7, 9, 10, 11, 12, 14, 15), and including properties of multiplication at the end (tasks 13, 16, 17, 18).

Overall, we found that the red and blue tasks provide an array of approaches to studying the concept of multiplication, emphasising properties of multiplication equally as much as the fundamental idea of repeated addition. This emphasis is related to Zankov’s (1977) principle of assigning a leading role to *theoretical knowledge* and thus implies that to teach mathematics developmentally one should not solely focus on abilities and skills, but attend to more general properties found in mathematics, achieving “the deepest possible understanding of various concepts, relationships, and dependencies” (Zankov, 1977, p. 57). According to Zankov, this is key to realise the principle of *teaching at a high level of difficulty*, continuously increasing depth of the studied material and making sure students face obstacles to gradually overcome. We observed that the identified ways which the concept of multiplication is treated often was repeated interchangeably throughout the volume, providing students with multiple opportunities to for example encounter multiplication as repeated addition. This overall structure and distribution of multiplicative aspects was not captured in our analysis. Additionally, we could not identify any tasks which addressed other multiplicative structures, such as rate or rectangular area (Greer, 1992). Regarding the latter, we found that DEM multiplication tasks provide little to no visible aids or suggestions of manipulatives for teachers and students. Nevertheless, we argue that these preliminary findings provide a clearer understanding of the inherent nature of the tasks, and in turn the make visible some opportunities for teachers to enrich students’ mathematical thinking.

Conclusion and implications

We identified a variety of multiplicative aspects presented in red and blue tasks from Arginskaya's *Mathematics 2B* textbook, repeated addition, multiplication table, properties of multiplication, and multiplication in the order of operations as the most prominent. We also identified an inherent structure among the selection of red tasks that seem to roughly prompt a triad of student learning activity: observation and/or exploration, reflection, and finally some sort of synthesis of the learning material. We argue, from a developmental point of view, that this structure has potential to provide teachers with opportunities to provoke students' mathematical thinking. We noticed a trend across the red and blue tasks towards a gradual rich development of the concept of multiplication; from repeated addition towards more general properties, operationalising Zankov's principles of developmental teaching. Displaying this complex line of development however was out of scope for this paper and is suggested as an avenue for further research. As this paper represents but a portion of a larger PhD study, a further analysis will include both the 2nd and 3rd grade volumes, investigating possible inter-volume multiplicative aspects. Also, the teachers provided us with a selection of tasks to analyse in depth as they selected them for their teaching and shared their rationale with us in a pre-lesson interview. They were also encouraged to reflect on their enactment in a post-lesson interview, allowing for an even further understanding of the tasks' potential. Hence, tasks will not only be analysed in isolation but put into a greater context of task enactment as was suggested by Haggerty and Pepin (2002).

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