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► **To cite this version:**

Saba Gerami, Shi Qi Lim, Vilma Mesa. Undergraduate students' use of interactive mathematics textbooks: A study of user's action paths. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04417413

**HAL Id: hal-04417413**

**<https://hal.science/hal-04417413>**

Submitted on 25 Jan 2024

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# Undergraduate students' use of interactive mathematics textbooks: A study of user's action paths

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*We study the utilization of three interactive mathematics textbooks by college students in calculus (C), abstract algebra (AA), and linear algebra (LA). In biweekly surveys (logs), students reviewed the visual representation of their textbook viewing (heatmaps) and reflected on why and how they were using the textbook on a specific day. Using the instrumental approach, we created 'action paths' based on three students' goals: completing homework assignments, preparing for examinations, and self-studying and looked for commonalities and variations in actions across goals and textbooks.*

*Keywords: Student textbook use, interactive textbook, instrumental approach, college mathematics, mathematics textbooks.*

Interactive textbooks offer undergraduate students different and novel opportunities to engage with the content, yet little is known about how students use these textbooks. We contribute to the current work on student use of interactive textbooks with the goal of informing authors and designers as they improve textbook features. Using viewing data (days, times, and sections viewed) from students who were using three mathematics textbooks and their self-reports that explained why and how they used their textbook on a given date, we mapped out actions related to specific goals in what we call *action paths*. In this paper, we answer the following research questions: What action paths do students report following to achieve specific goals when using interactive C, LA, and AA textbooks? What differences in action paths exist that can be attributed to either the student goals or the textbooks?

As we cannot currently verify that students' use of textbooks varies for different textbook formats (digital and non-interactive vs. static), we review research that pertains to the use of all undergraduate mathematics textbooks, regardless of their format, to provide a broader perspective on the topic.

## Review of literature

The question of how college students use mathematics textbooks has been addressed from multiple fronts. Some research indicates that even though students value reading their mathematics textbooks and find them useful for gaining understanding of the materials (Culvera & Hutchens, 2021), their reading strategies are not effective and that they do not attempt to 'read-the-meaning' behind mathematical symbols or have difficulties with the materials after reading their textbook (Shepherd & van de Sande, 2014; Weinberg et al., 2012). Using a similar approach to that used by Rezat (2013) we have investigated students' use of undergraduate textbooks, but a large scale. In that work we have focused on two areas: 1) use of one interactive feature (questioning devices, which allow students to submit their answers to some questions before class so that their instructor can gauge how well students know the materials, Quiroz et al., 2022), and 2) students' engagement with the textbooks (Castro-Rodríguez et al., 2022). In Quiroz et al. (2022), we used 492 students' self-reports to identify the situations in which students used questioning devices in C, LA, and AA textbooks (e.g., studying, self-evaluating understanding, familiarizing with content before class) and how they used them

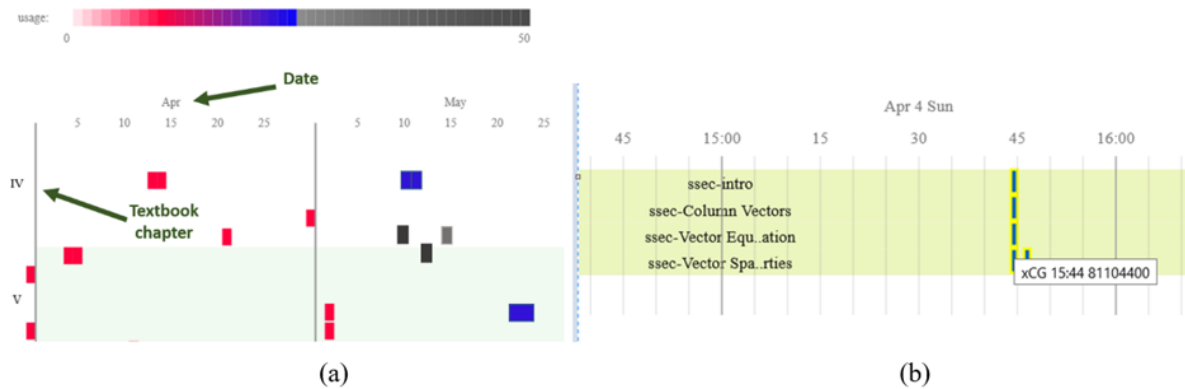
(actions and rationale). In Castro-Rodríguez et al. (2022), we analyzed students' textbook viewing data quantitatively and their descriptions of specific textbook elements qualitatively to find when they used their textbooks more (before homework's due dates and exam days) and how they used various elements and features. Earlier literature suggested that students "exhibit a wide range of reading behaviors" (Shepherd et al., 2012, p. 237), use examples, exercises, and solution manuals to achieve their goals, and that they rarely read expository texts, such as introduction and chapter summary (e.g., Weinberg et al., 2012). We have found more specific details about the students "reading behaviors" and as documented in Castro-Rodríguez et al. (2022), students extensively use various textbook features, which seem to be tied to the course orientation. We believe that a reason for the discrepancy in reading behaviors is that we have captured student actions in real time, rather than in laboratory settings or via a one-time survey. There is also some evidence that students may closely follow their textbooks' instructions and avoid skimming, possibly due to lacking an agentive relationship with their textbooks (Shepherd & van de Sande, 2014; Wiesner et al., 2020). Although collectively the studies suggest that students make use of elements and features of their textbooks and provide some details of such uses, we do not know how they engage with their textbooks when they have a particular goal in mind that is tied to their quotidian use of the textbook in naturalistic settings.

## **Theoretical conceptualization**

We are interested in the *instrumentation* processes of interactive textbooks by students (Rabardel, 2002; Trouche, 2020). A key element to understanding instrumentation processes is the identification of utilization schemes. Utilization schemes are the established and observable manners in which a user engages with an artifact to achieve implicit or explicit goals (Vergnaud, 1998). An artifact can be instrumented in multiple ways because of the users' unique utilization schemes. Vergnaud (1998) indicated that a utilization scheme is composed of four elements: goals; rules of action (observable behaviors of a person using an instrument); operational invariants, as true propositions held by the user that allow them to shape their goals and rules; and propositions for inference (repeated use in similar circumstances in the future). In this study, we used students' narrations of their textbook viewing to identify two elements of the utilization scheme: goals and rules of actions.

## **Methods**

As part of the larger project that investigates the use of three interactive textbooks, *Active Calculus* (Boelkins, 2019), *First Course in Linear Algebra* (Beezer, 2019), and *Abstract Algebra: Theory and Applications* (Judson, 2019), we recruited instructors through nation-wide invitations, and they in turn invited their students to participate in the study; participation was voluntary. Of the 594 students who consented to participate across five semesters (from Spring 2020 to Spring 2022; 199 from C, 275 from LA, and 120 from AA), 366 students responded to the questions we analyzed for this paper (103 from C, 175 from LA, and 88 from AA). In the fourth log, we gave students a link to their individual viewing data, i.e. heatmaps, in which they could see their textbook use (Figure 1). After describing the representation, they had to choose a day within the past two weeks when they used the textbook and narrate their use, given two open-ended prompts: 1) "State why you chose that day" and 2) "Narrate what you were doing with the textbook sections represented by the rectangles of the representation."



**Figure 1. The individual heatmap of a user of the LA textbook during (a) two months of Spring 2020 (Darker colors indicate more viewings); and (b) from 15:00 to 16:00 on April 4th**

After reading student responses to both questions, we pursued a qualitative analysis of the responses and open-coded them for *goals* (the purposes they used the textbooks for, classes of situations) and used an existing codebook (Quiroz et al., 2022) to identify the *rules of actions* (verbs in their descriptions that indicated activities or processes). After revising the codebooks, the first two authors independently coded 15% of the data, reaching a 83% agreement. After resolving discrepancies, the second author coded the rest of the data. Goals usually appeared in responses to the first question and rules of actions in responses to the second question. We identified five goals: to complete homework assignments (*HW Due*), to prepare for examinations (*Test Prep*), to enhance understanding of certain concepts (*Self-study*), to refer to during a lesson (*During Class*), and to prepare before a lesson (*Class Prep*). We identified six rules of actions: checking solutions and exercise explanations (*Check Solution*); doing exercise, activities, examples or answering reading questions (*Doing Exercise/Example*); reading without mentioning specific elements or features in the textbook (*Read*); reading specific elements or features in the textbook (*Read Specific*); referring to or searching the textbook to find something specific (*Reference/Search*); and taking notes (*Take Note*). Next, the second author read the responses again to identify the order in which students took their actions, and whether they referred to specific textbook elements or features (e.g., reading questions, definitions); if so, she captured these orders and textbook elements/features associated with the actions. An *action path* is the set of rules of actions together with the order in which they were used. We represent action paths as tree maps, with separate branches describing different sets of rules of actions that students report taking consecutively to achieve the same goal. The trees include the number of students who took each action and the number of students who associated their action with a textbook element or feature. To examine commonalities and variations across goals and textbooks, we defined four variables: *starting branches*, as the number of different first actions taken by students; *unique branches*, as the number of unique paths taken by students in an action path; *branch width*, as the number of students in a path; and *branch length*, as the number of actions in a path. For example, a student (S3506) using the C textbook chose to narrate a day they used their textbook “before a test” (goal = Test prep). They then explained: “I read the textbook section and compared it to my notes and homework problems. I also did the practice problems and checked my answers in the back of the book”, which we coded for three rules of actions in the following path: Read → Doing Exercises/Examples → Check Solutions (branch length = 3). This student was one of the 63 students

represented in Figure 2 (Figure 4a's top branch), who started using the textbook by reading the textbook to prepare for their test; because this student shared their path with one other student, the branch width for this path is two. After reading the textbook, 33 of the 63 students continued their paths by taking more actions (e.g., eight students did examples and exercises). This action path has 12 unique branches: 1. Read; 2. Read → Read Specific; 3. Read → Read Specific → Take Note; 4. Read → Read Specific → Doing Exercises/Examples; 5. Read → Read Specific → Read; and so on. To facilitate comparison, we use different colors for different actions and use the size of the boxes to indicate the number of students that took that action (these areas are not exact).

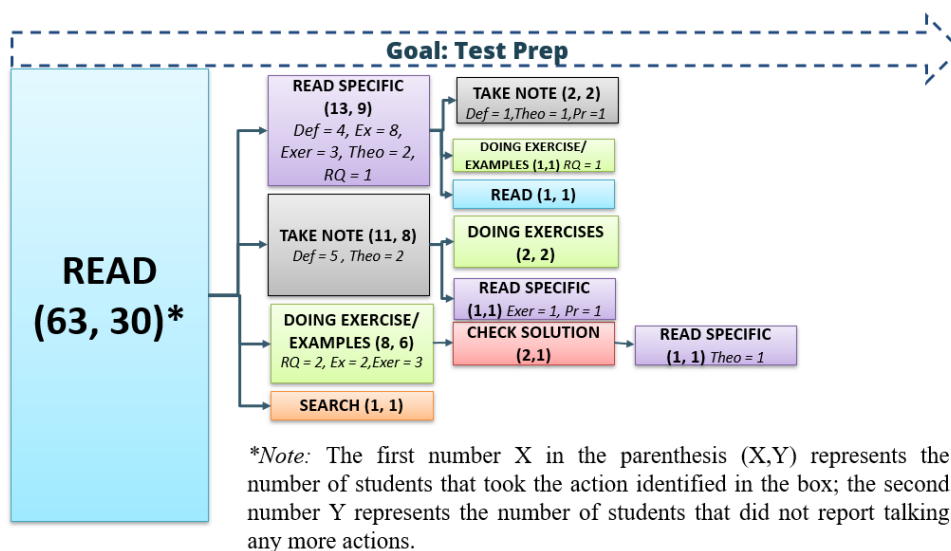


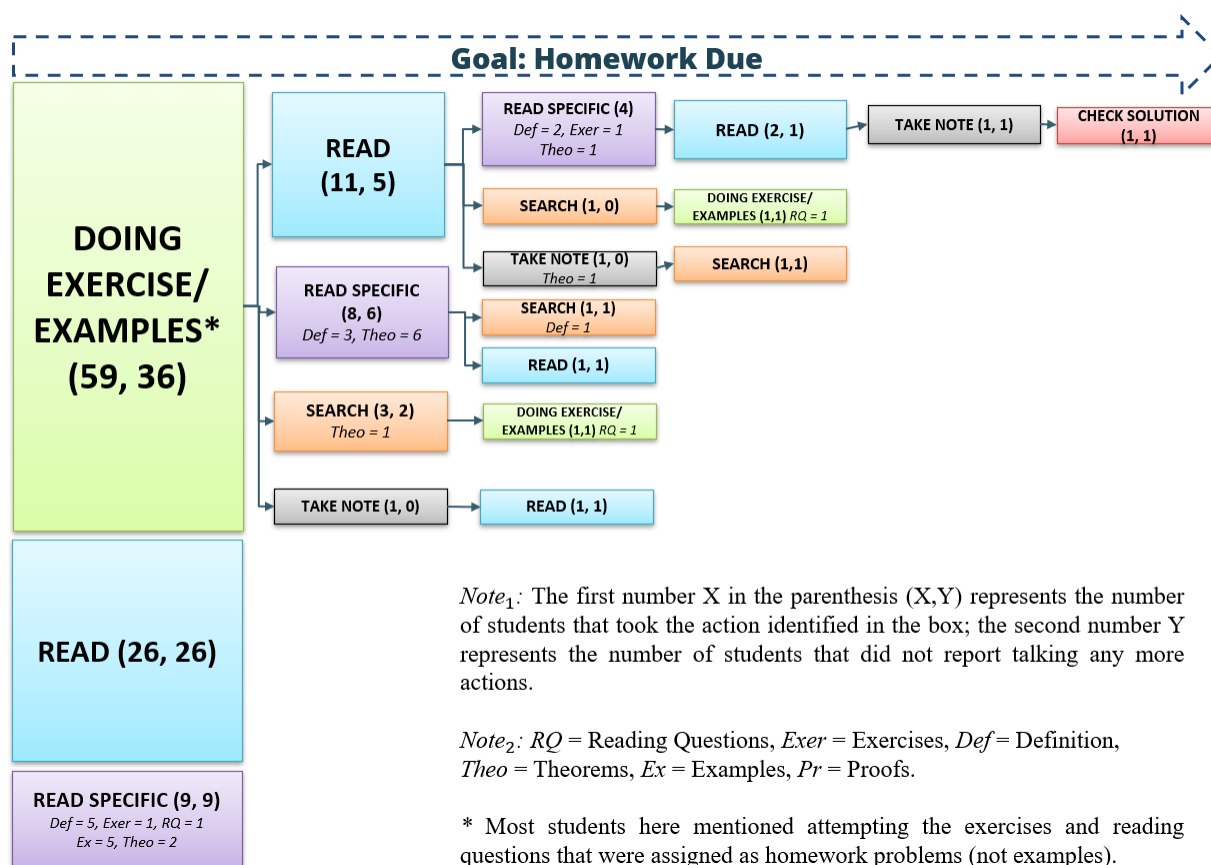
Figure 2. An example of an action path of 63 students when using their textbooks for test prep

## Findings

Most students narrated a time when they used their textbook to complete their homework assignment (HW Due,  $n = 185$ ), prepare for examinations (Test Prep,  $n = 84$ ), or enhance their understanding of certain concepts (Self-study,  $n = 32$ ). The remaining students ( $n = 65$ ) either said they do not use their textbooks or did not answer the log questions, or used them During Class ( $n = 6$ ) or for Class Preparation ( $n = 3$ ). For space reasons, we report on the first three goals.

**HW due.** When narrating their textbook use for completing homework assignments, students mentioned a variety of paths, mostly with length one. The complete action path for all 185 students had six starting branches and 18 unique branches with lengths from one to six rules of actions. The three widest branches that started with nine or more students are shown in Figure 3. The most common unique path was taken by students who only mentioned using the textbook to do the exercises in the homework (as a rule of action), without reporting taking further actions in their action paths ( $n = 36$ ). In the same branch and after attempting the homework, 23 students continued their paths to take one or more actions, such as reading the textbook, reading a specific element/feature, searching the textbook to find specific information, or taking notes from the textbook. Students taking the second most common unique path ( $n = 26$ ) only reported reading the textbook in general terms without mentioning specific elements/features; although we know these students read the textbook because their homework was due, we could not infer from their responses whether they attempted to

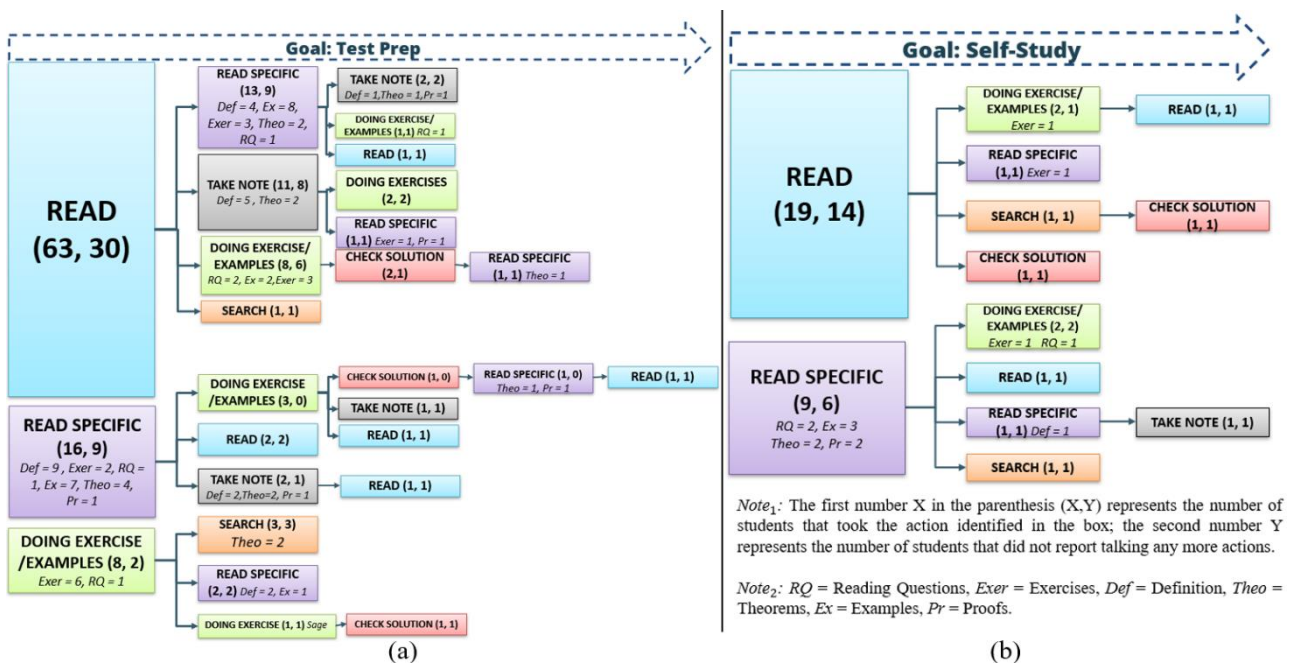
do the exercises in the homework (as a rule of action) before or after reading the textbooks. This is also the case for the nine students who only described reading specific elements in their textbooks, shown as the bottom branch. Although not captured in Figure 3, LA and AA students who mentioned reading specific elements of their textbooks, often explicitly referred to definitions and theorems, which we assume are useful for doing homework given that these courses involve more proofs than doing homework in C. Moreover, only two students across the complete action path of all 185 students mentioned looking at solutions when doing their homework and no student mentioned reading examples explicitly; this is interesting because solutions are often thought of as resources students could use to check or make sense of their work, and examples are solved problems with explanations that students can follow to solve similar problems in their homework.



**Figure 3. Students' action paths with branches with nine or more students when doing homework**

**Test Prep.** The complete action path of all 84 students who narrated their textbook use with the goal of preparing for upcoming examinations had six starting branches and 28 unique branches with lengths from one to five rules of actions. Figure 4a shows the three widest branches that started with at least eight students. The most common unique paths were taken by students who only mentioned using the textbook to read (top branch in the Figure 4a,  $n = 30$ ). In this branch and after reading the textbook, 13 students continued their paths by reading specific elements or features (more explicitly, definitions, exercises, and examples), 11 students took notes, and eight students started doing exercises and examples. The remaining paths of the top and middle branch were only taken by three or less students. In the middle branch in Figure 4a, 16 students started by reading specific parts of the textbook (more explicitly definitions, theorems, and exercises), seven of whom continued to take

other actions, such as doing exercises and examples or taking notes. Shown as the bottom branch, eight students started their paths by doing exercises and examples to prepare for their test, six of whom continued to search the textbook, read specific parts, or check the solutions. It seems that these students' strategy was to start assessing their content knowledge by doing the exercises, and then using the textbook to fill in the holes or confirm their work. This strategy is different than those in the top and middle branches who reported doing exercises and examples as a part of preparing for their tests ( $n = 11$ ), but they read the textbook (generally or specifically) beforehand. Like using the textbook for completing homework, students rarely reported using solutions ( $n = 5$ ); however, they did report acting upon more types of textbook elements/features than definitions and theorems when doing their homework (e.g., examples, reading questions, Sage, exercises). We also noticed in the complete action path that although most students said that they do examples and exercises to prepare for their examinations, the C students, who did examples and exercises, decided to do so after reading their textbooks. Compared to the HW Due action path and despite having 100 less students (185 for HW Due, 84 for Test Prep), the Test Prep action path is more diverse; while both started with the same number of branches, the Test Prep action path has more unique branches (28 vs. 18), suggesting that students' textbook use strategies to prepare for tests may be more complex or idiosyncratic.



**Figure 4. Students' action paths with branches with eight or more students when: (a) preparing for tests, and (b) self-studying**

**Self-study.** Thirty-two students narrated their textbook use for enhancing their understanding of certain concepts, with their complete action path having five starting branches and 15 unique branches with lengths from one to three rules of action. The two widest branches started with 19 and 9 students (Figure 4b), which include the most two common unique paths, both with only one rule of action: reading generally or reading specific parts of the textbook. About a third of students continued their paths after reading generally or specifically ( $n = 10$ ) with eight different actions, meaning that, after reading, these students often had different ways of using the textbook for studying specific concepts.

Overall, the Self-study action path is less complicated than those for HW Due and Test Prep, as it has fewer starting branches (5 vs. 6 for HW Due and Test Prep) and less unique branches (15 vs. 18 and 28 for HW Due and Test Prep respectively), and shorter paths (1-3 rules of actions vs. 1-6 and 1-5 for HW Due and Test Prep respectively). However, given that only 32 students narrated their textbook use for Self-study (vs. 185 and 84 for HW Due and Test Prep respectively), it is possible that the action path is not representative of what students do when they use the interactive textbooks, and that action path of more students could be more complex than the one we arrived at. Like C students who did exercises after reading their textbooks when preparing for examinations, C students who used the textbook for self-studying also started their action paths by reading the textbook first before taking other actions (not shown in Figure 4b). Although the majority of LA and AA students also read their textbooks before doing the exercises, some of them felt comfortable taking other actions (searching, doing exercises, looking at solutions) without reading the textbook first.

## **Discussion and conclusion**

The action paths demonstrate the diverse ways students use their textbooks to achieve three goals: completing homework assignments, preparing for examinations, and studying a specific concept. The action paths related to Test Prep were the most complex of the three, followed by HW Due and then Self-study (based on starting branches, unique branches, branch width, and branch length). Students within the Test Prep action path also mentioned using a more varied set of textbook elements and features. Perhaps the high stakes of college mathematics exams and their extensive content, drives students into more actions and using a variety of textbook elements and features. In contrast, when using the textbook to complete their homework, students may focus on less content, and only on elements and features related to a single section or a specific concept. We only found one obvious difference across the three textbooks: students using the C textbook for Test Prep and Self-studying started their paths by reading the textbook before taking other actions, which can be explained by the design of this textbook; while the LA and AA textbooks follow the typical definition-example-theorem-proof-exercise presentation with added interactive features, *Active Calculus* is organized around activities (that can be done individually or in groups) and require students to read short texts between them. It is also possible that the nature of the students' mathematical work is different because of the content at stake. A future investigation can address whether this is the case by contrasting interactive online textbooks with static textbooks. While we have used heatmaps to describe student textbook use at scale (Castro-Rodríguez et al., 2022; Kanwar & Mesa, 2022) here we used heatmaps as a prompt for students to reflect on their textbook use. Our methodology connects students' goals and actions to the heatmap data (i.e., the parts of the textbook that were viewed), but our findings might not be representative of students' 'typical' textbook use because students chose the day for their narrations. Thus, our findings are representative of textbook use that students find relevant, memorable, or worthy to reflect on.

## **Acknowledgment**

Funding for this work has been provided by the National Science Foundation through Awards IUSE 1624634 and 1821509. Any opinions, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.



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